

Letter to the Editor

Proton-temperature-anisotropy-driven magnetic fields in plasmas with cold and relativistically hot electrons

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Abstract. We present a dispersion relation for a plane-polarized electromagnetic wave in plasmas composed of cold electrons, relativistically hot electrons and bi-Maxwellian protons. It is shown that the free energy in proton-temperature anisotropy drives purely growing electromagnetic modes in our three-component plasma. Expressions for the growth rates and thresholds of instabilities are presented. The present results are relevant for explaining the origin of spontaneously generated magnetic fields in laboratory and astrophysical plasmas.

1. Introduction

About 40 years ago, Weibel [1] discovered a purely growing electromagnetic instability in an unmagnetized plasma with a bi-Maxwellian electron velocity distribution. Thus, the free energy in the electron temperature anisotropy generates quasi-stationary magnetic fields [1, 2]. The thermal Weibel instability may account for spontaneously generated magnetic fields in inertial confinement fusion plasmas [3–8], as well as in interplanetary spaces [9–11] and in astrophysical environments (e.g. cluster of galaxies [12]). The importance of relativistic thermal Weibel instability [13, 14] has also been recognized in the context of large-scale magnetic fields in a number of astrophysical sources, such as the gamma-ray bursts and relativistic

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jets [15, 16]. Recently, large-scale toroidal magnetic fields have been observed in the Galactic Centre [17], accompanied with splendid filamentary radio arcs [18].

Recent simulations [19] and laboratory experiments [20] have revealed that the absorption of intense laser pulses into electrons produces relativistically hot electron components within a dense plasma. Our objective here is to show that the pre-existing proton-temperature anisotropy can generate quasi-stationary magnetic fields in plasmas with cold and relativistically hot electrons. For this purpose, we present a dispersion relation for a plane-polarized electromagnetic wave in our multi-species plasma. The new dispersion relation admits purely growing instabilities. The growth rates and thresholds of the instabilities are presented.

The propagation of the electromagnetic wave is governed by the Faraday law

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} \quad (1)$$

and the Maxwell equation

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} (\mathbf{J}_c + \mathbf{J}_h + \mathbf{J}_i) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (2)$$

where c is the speed of light in vacuum; \mathbf{B} and \mathbf{E} are the wave magnetic and electric fields, respectively; and \mathbf{J}_c , \mathbf{J}_h and \mathbf{J}_i are the current densities of cold electrons, relativistically hot electrons and hot ions, respectively. For the electromagnetic fields, we express $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -c^{-1} \partial \mathbf{A} / \partial t$, where \mathbf{A} is the vector potential. The Coulomb gauge is $\nabla \cdot \mathbf{A} = 0$.

We are interested in obtaining a dispersion relation for a plane-polarized electromagnetic wave (with $\mathbf{A} = \hat{\mathbf{x}} A_x \exp(-i\omega t + ikz)$, where $\hat{\mathbf{x}}$ is a unit vector along the x -axis in a Cartesian co-ordinate system and A_x is the x -component of the vector potential, ω is the frequency, and k is the wave number along the z axis) in our three-species plasmas. The equilibrium distribution functions of cold electrons, hot electrons and hot protons are a delta function, a relativistic Maxwellian distribution function [21] and a bi-Maxwellian distribution function [22, 23], respectively. The current density associated with cold electrons is

$$\mathbf{J}_e = -en_{c0} \mathbf{u}_c, \quad (3)$$

where e is the magnitude of the electron charge; n_{c0} is the unperturbed number density of cold electrons; and the cold electron fluid velocity is

$$\mathbf{u}_c = \hat{\mathbf{x}} \frac{eA_x}{m_e c}. \quad (4)$$

Here m_e is the rest mass of electrons.

Furthermore, for $m_e c^2 \ll T_h$, where T_h is the hot-electron temperature, the current density associated with relativistically hot electrons is [21]

$$\mathbf{J}_h = -\hat{\mathbf{x}} \frac{n_{h0} e^2 c A_x}{3T_h}. \quad (5)$$

The proton current density \mathbf{J}_i in the presence of an equilibrium bi-Maxwellian proton distribution function reads [23]

$$\mathbf{J}_i = \hat{\mathbf{x}} \frac{Z_i^2 n_{i0} e}{m_i c^2} \left[1 + \frac{T_{i\perp}}{T_{i\parallel}} W(\xi) \right] A_x, \quad (6)$$

where Z_i is the proton charge state; n_{i0} is the unperturbed proton number density; $Z_i n_{i0} = n_{c0} + n_{h0}$; m_i is the ion mass; $T_{i\perp}$ and $T_{i\parallel}$ are the proton temperatures across and along the propagation vector $\mathbf{k} = \hat{z}k$, where \hat{z} is the unit vector along the z -axis. Furthermore, we have denoted $W(\xi) = -1 - \xi Z(\xi)$, where $Z(\xi)$ is the plasma dispersion function [24], with $\xi = \omega/kV_{T\parallel}$ and $V_{T\parallel} = (2T_{i\parallel}/m_i)^{1/2}$.

By using the definition of the electromagnetic fields \mathbf{B} and \mathbf{E} , we can combine (1) and (2) and Fourier transform the resultant equations to obtain the dispersion relation

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pc}^2}{\omega^2} - \frac{m_e c^2}{3T_h} \frac{\omega_{ph}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \left[1 + \frac{T_{i\perp}}{T_{i\parallel}} W(\xi) \right], \tag{7}$$

to obtain which we have used (3), (5) and (6). We have denoted $\omega_{p\alpha} = (4\pi n_{\alpha 0} e^2 / m_e)^{1/2}$ and $\omega_{pi} = (4\pi n_{i0} Z_i^2 e^2 / m_i)^{1/2}$, where the subscript α equals c for cold electrons and h for relativistically hot electrons.

We analyze (7) in two limiting cases. First, consider the limit $\xi \gg 1$, so that $G(\xi) = 1/2\xi^2$. Here we have from (7)

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pc}^2}{\omega^2} - \frac{m_e c^2}{3T_h} \frac{\omega_{ph}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \left(1 + \frac{k^2 T_{i\perp}}{m_i \omega^2} \right), \tag{8}$$

which for $\omega \ll kc$ yields

$$k^2 c^2 + \Omega_p^2 = -\frac{\omega_{pi}^2}{\omega^2} \frac{k^2 T_{i\perp}}{m_i}, \tag{9}$$

where $\Omega_p = (\omega_{pc}^2 + \omega_{pi}^2 + m_e c^2 \omega_{ph}^2 / 3T_h)^{1/2}$. Equation (9) admits a purely growing mode ($\omega = i\gamma$), with the growth rate

$$\gamma = \frac{kV_{T\perp}\omega_{pi}}{\sqrt{2}(k^2 c^2 + \Omega_p^2)^{1/2}}. \tag{10}$$

The threshold is

$$\frac{T_{i\perp}}{T_{i\parallel}} \gg \frac{(k^2 c^2 + \Omega_p^2)}{\omega_{pi}^2}. \tag{11}$$

Second, consider the limit $|\xi| \ll 1$, so that $W(\xi) = -1 - i\sqrt{\pi}\xi$. Here (8) with $\omega \ll kc$ yields

$$1 + \frac{\omega_{pi}^2}{(k^2 c^2 + \Omega_p^2)} \left[1 - \frac{T_{i\perp}}{T_{i\parallel}} \left(1 + i\sqrt{\pi} \frac{\omega}{kV_{T\parallel}} \right) \right] = 0, \tag{12}$$

which admits a purely growing solution, with the growth rate

$$\gamma = \frac{kV_{T\parallel}}{\sqrt{\pi}} \frac{T_{i\parallel}}{T_{i\perp}} \left[\frac{T_{i\perp}}{T_{i\parallel}} - 1 - \frac{(k^2 c^2 + \Omega_p^2)}{\omega_{pi}^2} \right], \tag{13}$$

provided that

$$\frac{T_{i\perp}}{T_{i\parallel}} > 1 + \frac{(k^2 c^2 + \Omega_p^2)}{\omega_{pi}^2}. \tag{14}$$

In summary, we have shown the existence of purely growing electromagnetic instabilities in a plasma with cold electrons, relativistically hot electrons and non-relativistic bi-Maxwellian protons. It is found that the free energy in proton-temperature anisotropy drives purely growing magnetic fields. Proton-anisotropy-driven instabilities may saturate when the gyrofrequency in the saturated magnetic

field B_s is comparable to the growth rate of instabilities. The saturated magnetic field can be associated with a large-scale magnetic field which may coexist with cold electrons, relativistically hot electrons and protons having a bi-Maxwellian distribution function. Such a scenario might occur in intense laser-plasma interaction experiments [8] and in astrophysical environments [22]. Finally, we note that the present investigation can be readily generalized for multi-component plasmas with a relativistic bi-Maxwellian proton distribution function. Here we should follow the analysis of Yoon [14] and Mart'yanov et al. [25] for investigating relativistic proton-anisotropy-driven magnetic fields in plasmas.

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