New Cosmological Data and the Values of the Fundamental Parameters
IAU Symposium, Vol. 201, 2005
Anthony Lasenby and Althea Wilkinson, eds.

H_0 from Gravitational Lenses: Recent Results

Paul L. Schechter
Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139-4307, USA

Abstract. The 0^{th}, 1^{st} and 2^{nd} derivatives of a "Fermat potential" give the three D's of gravitational lensing: delay, deflection and distortion. Observations of these delays, deflections and distortions for doubly and quadruply imaged quasars give estimates of Hubble's constant, H_0. The single largest contribution to the uncertainty in H_0 arises from the difficulty in constraining the degree of central concentration of the lensing potential. Fortunately, astronomers have spent a good deal of effort over the past quarter century addressing just this question. If galaxies at $z = 0.5$ are like nearby galaxies, the associated systematic uncertainty in H_0 is less than 10%. The expected lens-to-lens scatter is 20%. Results from three particularly well constrained systems are reported.

1. Introduction

Refsdal's (1964) method for measuring the Hubble constant using time delays of gravitationally lensed quasars avoids all of the systematic errors described by Freedman, Feast, Gibson, and Dressler in their discussion of Cepheid based measurements: small parallaxes, metallicity variations, reddening, crowding, blending, detector non-linearities, unmodelled effects in secondary distance indicators, and local deviations from pure Hubble flow. Instead Refsdal's method has systematic errors of its own, of which the best that can be said is that they are completely different.

In what follows we first give a brief of outline[1] of gravitational lensing. We report recent measurements of time delays and discuss the uncertainties associated with predicting time delays from models of gravitational potentials. Using the three systems for which these predictions are least uncertain, we report a result for H_0.

2. Gravitational Lensing

In the weak field limit, gravitational potentials introduce an effective index of refraction n (e.g. Binney & Merrifield 1998), which increases the light travel

[1]Narayan and Bartelmann (1999) give an excellent and thorough introduction to gravitational lensing. The paper by Blandford and Kundić (1997) on the measurement of H_0 is likewise highly recommended.

https://doi.org/10.1017/S0074180900216276 Published online by Cambridge University Press

time for photons traveling through a gravitational potential, Φ_{3D},

$$n = 1 - \frac{2\Phi_{3D}}{c^2} \quad . \tag{1}$$

A photon considering travel from a quasar at the edge of the universe to the Milky Way will therefore plan on detouring around mass concentrations if it wishes, consistent with Fermat's principle, to take the path which takes the least time. In the thin lens approximation, the extra travel time τ will be proportional to a dimensionless projected two-dimensional potential,

$$\psi_{2D} = \frac{D_{LS}}{D_S} \int_{observer}^{source} \frac{2\Phi_{3D}}{c^2} \frac{d\ell}{D_L} \quad , \tag{2}$$

where D_{LS} and D_S are angular diameter distances to the source from the lens and the observer, respectively. The extra travel time is given by

$$\tau = \frac{1+z_L}{c} \frac{D_L D_S}{D_{LS}} \left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi_{2D}(\vec{\theta}) \right] \quad , \tag{3}$$

where $\vec{\beta}$ is the angular position of the source and $\vec{\theta}$ is the angular position at which the path intersects the plane of the lens. Fermat's principle tells us that images form at stationary points (minima, maxima and saddle points) of the extra travel time. These can be found by setting $\partial\tau/\partial\vec{\theta} = 0$, giving the famous "lens equation",

$$\vec{\theta} - \vec{\beta} - \frac{\partial \psi_{2D}}{\partial \vec{\theta}} = 0 \quad , \tag{4}$$

the solutions of which we denote, $\vec{\theta}_A$, $\vec{\theta}_B$, *et cetera*. A small displacement of the source produces a small displacement of the image. The ratio of these is the inverse magnification. As both quantities are vectors, differentiating the lens equation with respect to $\vec{\theta}$ gives the inverse magnification matrix:

$$\frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} 1 - \frac{\partial^2 \psi_{2D}}{\partial \theta_x^2} & -\frac{\partial^2 \psi_{2D}}{\partial \theta_x \partial \theta_y} \\ -\frac{\partial^2 \psi_{2D}}{\partial \theta_x \partial \theta_y} & 1 - \frac{\partial^2 \psi_{2D}}{\partial \theta_y^2} \end{pmatrix} \quad , \tag{5}$$

which is symmetric and therefore can be diagonalized. Its inverse, the magnification matrix, maps a circular source of unit area into an elliptical image whose area is given by the absolute value of its determinant. Saddle points have one negative eigenvalue and maxima have two. Saddle points produce images with reversed handedness.

Taking the projected potential ψ_{2D} and the source positions $\vec{\beta}$ to be known, we can solve equation (4) for the image positions $\vec{\theta}_A$ and $\vec{\theta}_B$ and obtain $\tau_A - \tau_B$ from equation (3). This difference depends on the difference of the square bracketed dimensionless delays and some angular diameter distances, which depend upon the Hubble constant and the redshifts of the lens and source.

All that is needed to determine the Hubble constant is a) a measurement of the difference in travel time and b) a model for the two dimensional potential which can be determined, in principal, from the deflections and distortions of the observed images, and from any surplus delays. We shall see that in practice there are daunting challenges both in the measurement of delays and in the modelling of potentials.

3. Time Delay Measurements

The quantities outside the square brackets in equation (3) can be rewritten as a dimensionless quantity (of order unity for quasars) divided by H_0. The scale of the square bracketed quantity is set by the square of the image deflection, $\vec{\theta} - \vec{\beta}$, which for lensed quasars is of order 3×10^{-6} radians. The typical difference in travel time is therefore of order $10^{-11}/H_0$ and is measured in weeks or months.

This is at best a mediocre match to the intrinsic variability timescale for quasars, which is of order several years (e.g. Cristiani et al. 1996). To make matters worse, the typical variation on that typical timescale is only of order 10%, with smaller variations on shorter timescales. One therefore needs high photometric or radiometric accuracy to measure a time delay. It is somewhat misleading to talk of "typical" variability, since some quasars (e.g. 3C279 and BL Lac) are atypically variable while many others, to the chagrin of those who would measure time delays (e.g. Saust 1991, Moore & Hewitt 1997), are atypically steady.

Lens delay timescales are also poorly matched to more local phenomena. Delays of 2 weeks are difficult to measure in the optical if the object can only be observed during the dark run. Delays of 6 months are difficult to measure in the optical if the object lies near the equator. Delays of more than 6 months can be difficult to measure with the VLA because it changes configuration.

Yet another obstacle is microlensing by the stars in the intervening galaxy (Chang and Refsdal 1979). The rms amplitude for microlensing can be as high as 0.5-1.0 magnitudes for an unresolved source (Witt et al. 1995). Predictions of microlensing on a timescale of years for source velocities of a few hundred km s^{-1} have been dramatically confirmed (Wozniak et al. 2000). But microlensing on a timescale of days has now been observed both at optical (Burud et al. 2000) and radio wavelengths (Koopmans and de Bruyn 2000), and may be more general than has heretofore been appreciated. While microlensing may be signal for investigators who want to measure the mass fraction in compact objects, it is noise for those who wish to measure time delays.

There are also sociological hurdles. An optical monitoring campaign might require one 15 minute observation every other night for 3 months, for a total 11 hours of observing time. Instrument changes and scheduling policies make such programs exceedingly difficult, even at observatories which attempt to accommodate them. Try entering a fraction of an hour in the ESO proposal form on the "time requested" line – the number will be rejected! Successful monitoring campaigns have relied on a combination of pleading, arm-twisting and horsetrading. They also produce substantial telephone bills.

These difficulties notwithstanding, time delays have now been measured for 8 systems, as summarized in Table 1. The efforts reported there have been truly heroic, with the observers having gone to extraordinary lengths to produce accurate photometry and radiometry. Of particular note is the effort reported by FASSNACHT ET AL., at this meeting, who have measured all three independent time delays for the system CLASS 1608+656. Considerable good work has also gone into the difficult measurement of redshifts for the lensing galaxies. But it is in the nature of the enterprise that we spend most of our time on the bad news rather than the good, so we turn to modelling potentials.

Table 1. **Measured time delays**

Lens	Delay	Investigator
B0957+561	$417^{d} \pm 3$	Kundić et al. 1997
	$403^{d} \pm 30$	Haarsma et al. 1999
PG1115+080*	$25\overset{d}{.}0 \pm 1.6$	Schechter et al. 1997
		Barkana 1997
B1608+656*	$73^{d} \pm 3$	Fassnacht et al. 1999, 2000
B0218+357	$10\overset{d}{.}5 \pm 0.3$	Biggs et al. 1999
PKS1830-211	$26^{d} \pm 4.5$	Lovell et al. 1998
	$24^{d} \pm 6$	Wiklind & Combes 1999
HE1104-1805	$267^{d} \pm 90$	Wisotzki et al. 1998
B1600+434	$47^{d} \pm 5$	Koopmans et al. 2000
	$51^{d} \pm 2$	Burud et al. 2000a
RXJ0911+0551	$200^{d} \pm 40$	Burud et al. 2000b†

* Multiple delays measured; longest reported.
† Author's handwritten notes, 1999 Boston University lens conference.

4. Modelling Potentials

One can, with little effort, find very different values for H_0 in the literature based on the same measured time delay. Not surprisingly, the differences can be traced to differences in the model for the lens potential. Unfortunately, models are unavoidable, since lens potentials can never be determined with arbitrary accuracy. For the sake of consistency we adopt here a simple "yardstick" model which one may then compare with more refined models as the data permit. The yardstick also suffices to illustrate the principal systematic errors associated with model predictions.

4.1. A "yardstick" model

We take the lensing galaxy to be circularly symmetric in projection on the sky, with a two dimensional potential given by

$$\psi_{gal} = \frac{b^2}{(1+\alpha)} \left(\frac{\theta}{b}\right)^{1+\alpha} \tag{6}$$

where b gives the angular size of the "Einstein ring" that the lens would produce for a source on axis. The singular isothermal sphere corresponds to $\alpha = 0$, in which case $b = 4\pi\sigma^2/c^2$, where σ is the one dimensional velocity dispersion. For sources close to the axis (Refsdal and Surdej 1994) we have

$$\tau_B - \tau_A \approx \frac{1+z_L}{H_0} \frac{d_L d_S}{d_{LS}} \frac{1}{2} (\theta_A^2 - \theta_B^2)(1-\alpha) \quad , \tag{7}$$

where the dimensionless parts of the angular diameter distances are given by $d = DH_0/c$. Witt, Mao and Keeton (2000) have shown that strict equality holds for $\alpha = 0$ and for related self-similar potentials.

An important lesson to be learned from equation (7) is that one cannot tolerate much uncertainty in the center of the lensing potential, particular for images which are nearly equidistant. This has unhappy consequences for several of the lenses for which time delays are reported in Table 1.

In two cases the lensing galaxy is barely visible or not at all, making it difficult to determine where its center lies. In another the lens appears compound, comprised of merging components. It seems risky, for such a lens, to assume that the mass (in particular the dark matter) is comprised of components which are centered on the light. While such close pairs might at first seem unlikely, there are other cases known (Wisotzki et al. 1999, Rusin et al. 2000). A close pair of galaxies will have a larger quadrupole moment than a single galaxy, and is therefore more likely to give a highly magnified and therefore overrepresented (Turner et al. 1984) quadruple image configuration.

But we've gotten ahead of ourselves. The potential of equation (6) will only produce pairs of images, or occasionally triples if $\alpha > 0$ (Rusin and Ma 2000). Quadruple systems, which are frequently seen, require that the potential have a quadrupole, possibly attributable to a tide,

$$\psi_{tide} = \frac{\gamma}{2}\theta^2 \cos 2(\phi - \phi_\gamma) \quad , \tag{8}$$

where γ (sometimes called the shear) measures the strength of the tide and ϕ_γ measures its orientation on the plane of the sky. If the observed quadrupole is strong, one can, assuming a tidal origin, look for higher order terms and solve for the position of the tidal perturber. In the cases of PG1115+080, RXJ0911+0551, B0957+561, B1422+231, CLASS 2045+265 and B1413+113 a group or cluster of galaxies is observed with its center at the computed position and with a measured or plausible redshift identical to that of the lensing galaxy.

The sense of satisfaction one gets from identifying the group of galaxies responsible for the tide quickly becomes dismay when one realizes that the group extends beyond the lens and therefore projects a mass surface density onto it. A mass sheet of uniform density Σ produces a projected potential

$$\psi_{sheet} = \kappa\theta^2 \quad , \tag{9}$$

where the dimensionless surface density (sometimes called the convergence) is given by $\kappa = 4\pi G\Sigma D_L D_{LS}/D_S c^2$. Adding a mass sheet to a lens produces an image configuration which is consistent with a scaled version of the original lens potential. The time delays predicted by this scaled potential are longer than those predicted by the actual (lens plus sheet) potential by a factor of $1/(1-\kappa)$. But unless one knows the intrinsic size (or luminosity) of the lensed object, one cannot tell from the observed deflections, distortions, and delays whether a mass sheet is present or not (Saha 2000). This "mass sheet degeneracy" makes it impossibile to predict a time delay unambiguously.

Different investigators have chosen different approaches to this problem. One can estimate the mass surface density of the cluster through measurements of its velocity dispersion (Angonin-Willaime et al. 1994) or through weak lensing (Fischer et al. 1997). Alternatively one can estimate the expected deflection due to the lens alone through measurement of the velocity dispersion of the lensing galaxy (Tonry & Franx 1999). Absent such measurements, one can still estimate the effect of such a cluster if one is willing to assume that the cluster is isothermal. One then finds that the convergence is equal to the shear,

$$\kappa = \gamma \quad \text{(for an isothermal cluster).} \tag{10}$$

Operationally, one finds the best model using equations (6) and (8) and corrects for the effect of the projected mass sheet by multiplying the predicted time delay by a factor $1 - \gamma$.

4.2. Constraining models[2]

Our yardstick model for the potential has four free parameters associated with it: the lens strength, b, the tidal shear γ, its orientation, ϕ_γ and the radial exponent of the potential α. But there are free parameters associated with the source as well. For example the source position, $\vec{\beta}$ has two components which cannot be directly observed and therefore must count as model parameters.

Table 2. **Count of Source Parameters and Constraints**

observable	source parameters	constraints (2 images)	constraints (4 images)	fractional accuracy
delay	1	1	3	.01-.1
position	2	4	8	.001
flux*	1	2	4	.01-.5
shape*	3	6	12	.1

*Shape uses the full magnification matrix; flux uses only its determinant.

In Table 2 we show how different observable quantities contribute additional source parameters and constraints. We show delay observations adding one free parameter, H_0. We count "shapes" as having a size, an axis ratio and an orientation. Not all constraints are equally useful. Position constraints are typically accurate to one part in a thousand. Fluxes might be good to a part in one hundred, but microlensing may make these uncertain by as much as 50%. Surplus delays (beyond the one needed to measure H_0) constrain the projected potential. Positions constrain its first derivatives. Fluxes or shapes (one cannot use both simultaneously) constrain second derivatives.

The simplest lenses, those with two unresolved images, present a problem. Measuring their positions and (possibly microlensed) fluxes we have 4 parameters associated with the yardstick model and 3 source parameters, and only 6 constraints. At this point we introduce a "prior". We know a good deal about the shapes of galaxy potentials from studies of nearby galaxies. These appear to be approximately isothermal. Short of discarding doubles, we may reasonably assume that the lensing galaxies are isothermal and take $\alpha = 0$.

Extended sources can, in principal, be decomposed into a set of finite sources, for each of which one can measure delays, deflections and distortions. In practice the success in using extended sources to constrain models has been

[2]The material in this subsection was omitted in the 16 minutes allotted to the spoken version of this review.

spotty (Chen et al. 1995; Kochanek et al. 2000) with the successes involving high surface brightness sources.

Even without the benefit of Table 2, it is clear that systems with more images and more sources are better than those with fewer. But a system with fewer constraints for which one suspects that the potential is relatively simple may be more useful than one for which the potential is likely to be complicated. CLASS 1359+154 has one source with six images, but the lens is a triplet of galaxies. How would one expect the dark matter to be distributed such a system?

4.3. Radial exponent: the return of the mass sheet degeneracy

Perhaps the largest source of variance among estimates of H_0 which use the same underlying observations can be traced to differing treatments of the radial exponent α. Almost universally, investigators have found it exceedingly difficult to constrain α and its equivalents. The reader is referred to figure 4a in Williams & Saha (2000), figure 3 in Bernstein & Fischer (1999) and figure 7 in Cohn et al. (2000), which show the wide the range of allowable values of α and the strong dependence of H_0 thereon. For the case of the quadruple PG1115+080, the difference between the best fitting isothermal model ($\alpha = 0$) and the best fitting point mass model ($\alpha = -1$) amounts to only several parts in a thousand in the positions of the images.

Over the relatively narrow range of radii over which one is likely to see four images, a change in power law is not very different from adding (or subtracting) a mass sheet. Even in the case of 1933+503, with two quadruply imaged sources and one doubly imaged source, Cohn et al. (2000) find it difficult to constrain the concentration.

At this point the pragmatist will return to a prior based on the accumulated knowledge of nearby galaxies. Studies of spiral galaxies show their rotation curves to be very nearly flat, corresponding to $\alpha = 0$. Studies of the line of sight velocity distribution as a function of radius in ellipticals have yielded slightly steeper potentials. Romanowsky & Kochanek (1999) have collected and averaged these results. Application of a ruler to their figure 1 yields $< \alpha >= -0.2$, with a galaxy-to-galaxy scatter of roughly 0.2.

For a handful of lenses (only one of which has a measured time delay) the image configuration does suffice to constrain α (Chen & Kochanek 1995; Barkana et al. 1999; Cohn et al. 2000). In all of cases the results are consistent with the isothermal hypothesis, with uncertainties in α of 0.1-0.2. Rusin & Ma (2000) note that the absence of third images restricts α to values $\lesssim 0.1$.

As most observed lensing galaxies appear to be elliptical (Keeton, Kochanek & Falco 1998) we shall proceed under the assumption that $< \alpha >= -0.2$, with a guess at the uncertainty in the mean of 0.1, and a corresponding systematic uncertainty in H_0 of 10%. Given the observed scatter in α of 0.2 in nearby ellipticals, we expect scatter in individual values of H_0 of 20%.

5. The Hubble Constant

At the risk of deeply offending some large fraction of the local organizing committee we shall not apply our "yardstick" model to all the lens systems for which delays have been measured. In particular we shall set aside a) those systems for which the center of the lensing galaxy cannot be reliably identified from optical/IR images and b) those systems for which there are only two unresolved images, and which therefore depend on magnifications to constrain the model. For both unresolved doubles, B1600+434 and HE1104-1805, there is substantial evidence that microlensing affects the fluxes. The case of B1600+434 is further complicated by the possible effects of the bright galaxy which lies just a few arcseconds from the system.

There is considerable danger in deciding *post hoc* where to draw the line on including or not including systems. While other investigators have chosen not to exclude those systems excluded here, I suspect there is general agreement on the relative reliability of the predicted time delays for the lenses in Table 1.

Table 3. **Predicted and observed delays**

system	γ	$\Delta\tau_{pred}$	$\Delta\tau_{obs}$	h
B0957+561	0.279	$274^{\rm d}/h$	$417^{\rm d}$	0.66
PG1115+080	0.110	$10^{\rm d}\!.2/h$	$25^{\rm d}\!.0$	0.41
RXJ0911+0551	0.307	$83^{\rm d}\!.5/h$	$200^{\rm d}$	0.42
average			$< h >=$	0.494
$\times < 1-\alpha >$			$< h >=$	0.592
$(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$			$< h >=$	0.622

Applying our isothermal yardstick model to the three remaining systems gives the predicted time delays in Table 3. The individual predictions have had the factor $1-\gamma$ applied on the assumption that the cluster responsible for the shear is also isothermal with $\kappa = \gamma$. Defining $h \equiv H_0/(100\text{km s}^{-1}\text{Mpc}^{-1})$, its average value is then corrected for the assumed deviation of the lensing galaxy from isothermality. Our angular diameter distances, D, were computed using an Einstein-deSitter model. Adopting $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ gives another small correction.

¿From the variations in α in nearby galaxies we expect a dispersion in the derived values of h of 20%. The uncertainties in the measured time delays add (in quadrature) as much as 20%. If you want to bet on the statistics of three objects, it is straightforward to derive a statistical uncertainty in h.

There are two paths to a better Hubble constant, both of which require finding new lensed systems. The purists seek a few "golden" lenses which are well constrained by the image configuration. The pragmatists are willing to work with less well constrained systems, and are willing to take advantage of "prior" information gleaned from nearby galaxies.

Astronomers in both camps can take heart in the fact that there are many more bright lenses waiting to be discovered. By way of illustration, there are some 5500 quasars brighter than $B = 18.5$ in the 7500 square degrees of the Hamburg-ESO survey (Wisotzki et al. 2000). Imaging these at high resolution should yield (in addition to the six already found) roughly 18 as yet undiscovered lenses for a conservative, Einstein-deSitter model. Whether by brute force or elegance, gravitational lenses will give a competitive value for H_0.

Acknowledgments. This work was supported in part by the U.S. National Science Foundation, grant AST96-16866.

References

Angonin-Willaime, M.-C., Soucail, G. & Vanderriest, C. 1994, A&A, 291, 411

Barkana, R. 1997, ApJ, 489, 21

Barkana, R., Lehár, J., Falco, E. E., Grogin, N. A., Keeton, C. R. & Shapiro, I. I. 1999, ApJ, 520, 479

Bernstein, G. & Fischer, P. 1999, AJ, 118, 14

Biggs, A. D., Browne, I. W. A., Helbig, P., Koopmans, L. V. E., Wilkinson, P. N. & Perley, R. A. 1999, MNRAS, 304, 349

Binney, J. and Merrifield, M. 1998, Galactic Astronomy (Princeton: Princeton University Press), 732

Blandford, R. D. & Kundić, T. 1997, in The Extragalactic Distance Scale, ed. M. Livio (Cambridge: Cambridge University Press), 60

Burud, I., Hjorth, J., Jaunsen, A. O., Andersen, M. I., Korhonen, H., Clasen, J. W., Pelt, J. Pijpers, F. P., Magain, P. & OEstensen, R. 2000a, preprint, astro-ph/0007136

Chang, K. & Refsdal, S. 1979, Nature, 282, 561

Chen, G. H., Kochanek, C. S. & Hewitt, J. N. 1995, ApJ, 447, 62

Cohn, J. D., Kochanek, C. S., McLeod, B. A. & Keeton, C. R. 2000, preprint, astro-ph/0008390

Cristiani, S., Trentini, S., La Franca, F., Aretxaga, I., Andreani, P., Vio R. & Gemmo, A. 1996, A&A, 306, 395

Fassnacht, C. D., Pearson, T. J., Readhead, A. C. S., Browne, I. W. A., Koopmans, L. V. E., Myers, S. T. & Wilkinson, P. N., 1999, ApJ, 527, 498

Fassnacht, C.D., Xanthopoulos, E., Koopmans, L. V. E., Pearson, T. J., Readhead, A. C. S. & Myers, S. T. 2000, in Gravitational Lensing: Recent Progress and Future Goals, ed. T. G. Brainerd & C. S. Kochanek (San Francisco: ASP), astro-ph/9909410

Fischer, P., Bernstein, G., Rhee, G., Tyson, J. A. 1997, AJ, 113, 521

Haarsma, D. B., Hewitt, J. N., Lehár, J. & Burke, B. F. 1999, ApJ, 510, 64

Koopmans, L. V. E., de Bruyn, A. G., Xanthopoulos, E. & Fassnacht, C. D. 2000, A&A, 356, 391

Koopmans, L. V. E. & de Bruyn, A. G. 2000, A&A, 358, 793

Kundić, T., Turner, E. L., Colley, W. N., Gott, J. R. III, Rhoads, J. E., Wang, Y., Bergeron, L. E., Gloria, K. A., Long, D. C., Malhotra, S. & Wambsganss, J. 1997, ApJ, 482, 631

Lovell, J. E. J., Jauncey, D. L., Reynolds, J. E., Wieringa, M. H., King, E. A., Tzioumis, A. K., McCulloch, P. M. & Edwards, P. G. 1998, ApJ, 508, 51L

Keeton, C. R., Kochanek, C. S. & Falco, E. E. 1998, ApJ, 509, 561

Kochanek, C. S., Keeton, C. R., & McLeod, B. A. 2000, preprint, astro-ph/0007458

Moore, C. B. & Hewitt, J. N. 1997, ApJ, 491, 451

Narayan, R. & Bartelmann, M. 1999, in Formation of Structure in the Universe, ed. A. Dekel & J. P. Ostriker (Cambridge: Cambridge University Press), 360

Refsdal, S. 1964, MNRAS, 128, 307

Refsdal, S. & Surdej, J. 1994, Rep. Prog. Phys., 57, 117

Romanowsky, A. J. & Kochanek, C. S. 1999, ApJ, 516, 18

Rusin, D. & Ma, C.-P. 2000, ApJL, submitted

Rusin, D., Hall, P. B., Nichol, R. C., Marlow, D. R., Richards, A. M. S. & Myers, S. T. 2000, ApJ, 533, 89L

Saha, P. 2000, preprint, astro-ph/0006432

Saust, A. B. 1991, A&A, 243, 33

Schechter, P. L., Bailyn, C. D., Barr, R., Barvainis, R., Becker, C. M., Bernstein, G. M., Blakeslee, J. P., Bus, S. J., Dressler, A., Falco, E. E., Fesen, R. A., Fischer, P., Gebhardt, K., Harmer, D., Hewitt, J. N., Hjorth, J., Hurt, T., Jaunsen, A. O., Mateo, M., Mehlert, D., Richstone, D. O., Sparke, L. S., Thorstensen, J. R., Tonry, J. L., Wegner, G., Willmarth, D. W. & Worthey, G. 1997 ApJ475, 85L

Tonry, J. L. & Franx, M. 1999, ApJ, 515, 512

Turner, E. L., Ostriker, J. P., & Gott, J. R. III 1984, ApJ, 284, 1

Wiklind, T. & Combes, F. 2000, in Gravitational Lensing: Recent Progress and Future Goals, ed. T. G. Brainerd & C. S. Kochanek (San Francisco: ASP), astro-ph/9909314

Williams, L. L. R. & Saha, P. 2000, AJ, 119, 439

Wisotzki, L., Wucknitz, O., Lopez, S. & Sorensen, A. N. 1998, A&A339, 73L

Wisotzki, L., Christlieb, N., Liu, M. C., Maza, J., Morgan, N. D. & Schechter, P. L. 1999, A&A, 348, 41L

Wisotzki, L., Christlieb, N., Bade, N., Beckmann, V., Köhler, T., Vanelle, C. & Reimers, D. 2000, A&A, 358, 77

Witt, H. J., Mao, S. & Keeton, C. R. 2000, preprint, astro-ph/007011

Witt, H. J., Mao, S. & Schechter, P. L. 1995, ApJ, 443, 18

Woźniak, P. R., Alard, C., Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G. & Zebruń, K. 2000, ApJ, 531, 25L