

The stochastically subordinated Poisson normal process for modelling financial assets

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For pricing financial derivatives many market practitioners use models, such as the Black–Scholes model, that rely on the hypothesis that financial asset returns follow a multiplicative Wiener process. The evidence against the normal hypothesis is overwhelming – empirical studies have shown that asset returns possess leptokurtic distributions and exhibit high degrees of heteroscedasticity. Additionally, market prices for financial derivatives show significant deviations from model prices.

The method of stochastic subordination, or random time indexing, has been successfully used since 1973 to model the returns on financial assets. Previous emphasis in stochastic subordination models has involved explicitly identifying the subordinating process with an observable quantity such as number of trades, volume of trades or the like. For prediction under the stochastic subordination hypothesis, the exogenous variables are as difficult to estimate as the underlying returns, although studies in the past have supported the hypothesis for estimation.

In contrast, the approach taken here does not depend on the specific identification of the subordinated time variable, but rather assumes a class of time models and estimates parameters from data. In addition, a simple hidden Markov process is proposed for the characteristic parameter of the subordinating distribution to explain the significant autocorrelation of the squared returns. Parameter estimation by maximum likelihood is shown to be efficient and accurate.

Compared to the Wiener process models, the proposed model with few additional parameters, fits selected financial time series particularly well. In particular, the model accurately replicates the observed autocorrelation structure and heavy tails common in financial time series. Moreover, it is shown that the proposed model is a significant improvement over the more usual generalised autoregressive conditional heteroscedasticity (GARCH) models favoured by researchers in the field.

Option pricing by analytical approximation or Monte Carlo technique is shown to generate option prices more consistent with those observed in liquid markets than the Black–Scholes model.

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Various extensions to the model are considered such as the incorporation of stable deviates, expansion of the hidden Markov process and multivariate modelling.

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