where A, B, C are the angles of a triangle, was proposed in an examination paper, and it has been suggested to me that it might be of interest to teachers. It is rather easier than that given on p. 344 of "New Trigonometry for Schools" by Lock and Child.

O is the circumcentre of the triangle ABC and B' and C' are the mid points of AC and AB. From the triangle $OB'C$ since $\angle COA = 2B$, $OB' = R\cos B$. Similarly $OC' = R\cos C$.

$$B'C' = \frac{a}{2} = R\sin A.$$

But from the triangle $B'C'O$, since $\angle B'OC' = 180° - A$,

$$B'C'^2 = OB'^2 + OC'^2 + 2OB' \,.\, OC'\cos A.$$

$$\therefore \quad \sin^2 A = \cos^2 B + \cos^2 C + 2\cos B\cos C\cos A$$

which gives the result.

R. J. T. Bell.

Note on students and original work.—I have sometimes been laughed at when I have ventured the opinion that University students of mathematics ought to do original work in mathematics as part (and an important part) of their degree work. But this will not prevent me from maintaining that original work should be done by every scholar in a junior class of geometry. Of course in one sense this is a mere platitude, for every "problem" and "rider" requires for its solution original work on the part of the scholar. He or she that would solve a geometrical problem must "intend the mind continually" on it, like Sir Isaac Newton, and acquire the faculty of trying different points of view till the right one is found. For him or her, at any rate, the Pragmatists' test of truth is appropriate. He asks of every hypothesis "will it work?" and finds the answer by actual trial.

But besides problems and riders, why not "Dissertations"? The following little exercise has been used with good effect in a junior class of geometry.

Consider the following properties of the parallelogram $ABCD$:

(1) AB parallel to CD
(2) AD parallel to BC
(3) $AB = CD$
(4) $AD = BC$

(5) angle A = equal angle C
(6) angle B = angle D
(7) diagonal AC bisected at its intersection with BD
(8) diagonal BD bisected at its intersection with AC.

By Euclid's definition properties (1) and (2) determine a parallelogram. By Euclid I. 34 (1) and (3) determine a parallelogram. It is required to find which combinations of two out of the eight properties above listed are sufficient to ensure that $ABCD$ is a parallelogram.

We find that there are 18 combinations that are sufficient, while 10 are insufficient. To prove sufficiency we have in general to prove the equality in all respects of two triangles: in the insufficient cases, these triangles are analogous to the "ambiguous case" in the solution of triangles. But to be unable to prove sufficiency is not to prove insufficiency. The most direct way to do that is to show that it is possible to construct an $ABCD$ satisfying the conditions considered, but still not a parallelogram. This part of the investigation is only within the reach of riper scholars.

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