

RAMSIFICATION AND SEMANTIC INDETERMINACY

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Abstract. Is it possible to maintain classical logic, stay close to classical semantics, and yet accept that language might be semantically indeterminate? The article gives an affirmative answer by *Ramsifying classical semantics*, which yields a new semantic theory that remains much closer to classical semantics than supervaluationism but which at the same time avoids the problematic classical presupposition of semantic determinacy. The resulting *Ramsey semantics* is developed in detail, it is shown to supply a classical concept of truth and to fully support the rules and metarules of classical logic, and it is applied to vague terms as well as to theoretical or open-ended terms from mathematics and science. The theory also demonstrates how diachronic or synchronic interpretational continuity across languages is compatible with semantic indeterminacy.

Since the publication of Ramsey's [56] 'Theories', the Ramsification of scientific theories has become a major tool in theory reconstruction and interpretation. It has been applied to argue for the instrumental character of theories (which is one way of reading 'Theories', cf. [58]), to determine the synthetic content of theories [17] or the empirical claim made by theories [66], to explicate structural realism about science [82] and various kinds of functionalism, including functionalism about mental terms [43] and about truth (see, e.g., [45, 85]).

In what follows, I will argue that the Ramsification of *classical semantics* can also help us overcome problems of semantic indeterminacy that result from the vagueness of natural language or from the theoreticity or open-endedness of mathematical and scientific language. Call the result of that Ramsification *Ramsey semantics*, which, I want to show, saves all of classical logic and almost all of classical semantics, while embracing semantic indeterminacy without going down the epistemicist or supervaluationist road. The upshot will be: if one wants to be prepared for semantic indeterminacy—unlike epistemicism—and one aims to stay closer to classical semantics than supervaluationism, one should Ramsify classical semantics.¹

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¹ This motivation for Ramsifying classical semantics is orthogonal to instrumentalist or functionalist motivations: the point of Ramsey semantics is neither to show that talk of interpretation is merely instrumental nor to convey insights into the 'nature' of truth, but to deal with semantic indeterminacy. In contrast, e.g., Wright's [85] paper on Ramsification and monism-vs.-pluralism-about-truth does not apply Ramsification for the sake of doing semantics and in fact presupposes semantic *determinacy* (see [85], p. 272).

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§0. Introduction. Before developing the theory in full detail, let me first present the idea in a nutshell by means of an example, which will also allow me to introduce the different parts of the paper.

Say, we are interested in stating the semantics of the language of the real number calculus. At some point we want to introduce the semantic interpretation of the function symbol ‘ $\sqrt{}$ ’ for the (principal) square root of a real number, or, alternatively, the interpretation of the predicate ‘*Sqrt*’, where ‘*Sqrt*(y, x)’ is meant to express that y is the (principal) square root of x . Within the calculus of real numbers, mathematicians standardly define (principal) square roots only conditionally (see [68], Section 8.6 for a survey), that is, under the condition that x is a non-negative real:

For all x, y : if *Real*(x), *Real*(y), and $x \geq 0$, then

$$\sqrt{x} = y \text{ (that is, } Sqrt(y, x) \text{) if and only if } y^2 = x \text{ and } y \geq 0.$$

Clearly, this leaves *the square roots of negative reals* undefined: nothing is said whatsoever about ‘ \sqrt{x} ’ or ‘*Sqrt*(y, x)’ in the case when ‘ x ’ denotes a negative real number. Accordingly, on the metalevel, ‘the’ intended interpretation of ‘ $\sqrt{}$ ’ and ‘*Sqrt*’ should only be partially determined by what might be called the ‘metasemantic facts’ of mathematical concept formation. And although classical (meta-)semantics does not recognize the possibility of a partially determined intended interpretation (about which more in Section 1), mathematicians still manage to reason successfully about square roots in classical logic. How is that possible?

If one were to extend epistemicism about vague terms (see e.g., [67, 75, 77]) to the present case of semantic indeterminacy without vagueness, the answer would be: there might *seem* to be a factual gap left by the conditional definition of ‘ $\sqrt{}$ ’ and ‘*Sqrt*’, but really the usage of these terms by mathematicians—and possibly some further facts—somehow conspire to determine a unique fully defined function on the reals. There is no guarantee anyone might know how that gap is filled in, but nevertheless classical semantics can be applied just as usual, and hence it is no wonder that mathematicians are able to reason classically about square roots.

But how plausible is that? As far as the predicate ‘*Sqrt*’ is concerned, one might try to adapt a suggestion by Williamson [76] and argue that, since the conditional definition does not do enough to make *Sqrt*(y, x) true for any negative real x , ‘it *thereby* does enough to make it false’ ([76], p. 213): if so, *for every negative real* x , there is a fact of the matter that is correctly described by $\forall y(\textit{Real}(y) \rightarrow \neg \textit{Sqrt}(y, x))$ and the semantic interpretation of ‘*Sqrt*’ is determined completely after all. But what would be the argument for this other than that it saves the classical presupposition of a unique fully determined intended interpretation? And wouldn’t the proposal suggest too much? For, by the same token, the same conditional definition ‘For all x, y : if *Real*(x), *Real*(y), and $x \geq 0$, then...’ interpreted over the domain of *complex* numbers would not do enough to make *Sqrt*(y, x) true for any *negative real-valued complex number* x either, but thinking that thereby $\forall y(\textit{Complex}(y) \rightarrow \neg \textit{Sqrt}(y, x))$ would be determined true would run counter to the standard conservative extension of the conditional definition of square roots to the full definition of square roots on the complex numbers, according to which negative reals x *do* have (complex) square roots.

When I will summarize the tenets of classical semantics in Section 1, I will reconsider its central metasemantic presupposition of the existence of a unique factually determined intended interpretation in more detail, and I will present three

examples to the effect that the presupposition is likely to be false: one is about semantic indeterminacy by vagueness in natural language, while the other two examples concern the semantic indeterminacy of mathematical and scientific languages without vagueness. Since it would be dangerous to base semantics on what is likely to be a false presupposition, we should find an alternative way of making semantic indeterminacy compatible with a semantics that saves classical logic and its applications—which is the goal of this paper.

As I will argue from Section 2, Ramsifying classical semantics with respect to its theoretical term ‘intended interpretation’ is going to deliver that combination. While it is less common for semanticists to reflect on the logical form in which their semantic theory is stated, Ramsifying a theory (in this case, classical semantics/metasemantics) by replacing some of its terms by variables (in this case, ‘intended interpretation’ by ‘ F ’), and closing the resulting formula existentially (in this case, by ‘there is an admissible interpretation F , such that...’) is a well-known procedure in the philosophy of science. The point of the paper is to put these insights from the philosophy of science to good semantic use. Sections 2 and 3 will show that the semantic consequences of the resulting Ramsey semantics resemble those of classical semantics. Sections 4, 5, and the Appendix will demonstrate how Ramsey semantics copes successfully with the semantic indeterminacy of vague terms, such as in the Sorites Paradox and in higher-order vagueness. Section 6 will demonstrate the same for cases of semantic indeterminacy that result from theory-ladenness and conceptual change in mathematics and science. The same section will also show that Ramsey semantics offers a new understanding of inter-theoretical interpretational continuity that is compatible with semantic indeterminacy. In particular, Ramsey semantics is compatible with conceptual extensions, such as extending the interpretation of ‘ $\sqrt{}$ ’ and ‘ $Sqrt$ ’ from the reals to the complex numbers.

But maybe it would not even be necessary to beat a new path towards an ‘almost classical’ semantics while allowing for semantic indeterminacy: for isn’t supervaluationism (see e.g., [25, 35]) doing precisely that? In the square root example, supervaluationists might reconstruct the conditional definition of real-valued square roots model-theoretically with the help of a space Adm of semantically ‘admissible’ classical interpretations: all interpretations that satisfy the conditional definition from above. Admissible interpretations serve more commonly as ‘precisifications’ of imprecise vague terms, but they can be used to fill other kinds of metasemantic gaps, too, including those left by precise conditional definitions. On that basis, supervaluationists would proceed to re-define classical truth as super-truth: truth with respect to *all interpretations* in Adm . Consequently, they would count $Sqrt(2, 4)$ as (super-)true, $Sqrt(2, 9)$ as (super-)false, and $(Sqrt(2, -4) \vee \neg Sqrt(2, -4))$ as (super-)true, even though $Sqrt(2, -4)$ and $\neg Sqrt(2, -4)$ would neither be (super-)true nor (super-)false and hence lack a truth value.

Ramsey semantics will start from the same class Adm of admissible interpretations as supervaluationism, but it will postulate *the existence of an interpretation in Adm , such that truth and falsity for all sentences of the object language are determined from that very interpretation in the standard Tarskian manner*. This metalinguistic existence postulate will turn out to be the Ramsey sentence for classical semantics, as will be explained in Section 2. Section 3 will prove the resulting Ramsey semantics to remain much closer to classical semantics than supervaluationism: in particular, in contrast with supervaluationist entailment, the rules and metarules of classical logic will be validated by Ramsey semantics in the same way as they are validated by

classical semantics. And unlike super-truth, the concept of truth will remain classical by satisfying all T(ruth)-biconditionals, being compositional with respect to all classical logical operators, and avoiding truth value gaps. For instance, in the example above, $Sqrt(2, 4)$ will be evaluated as true, $Sqrt(2, 9)$ as false, $(Sqrt(2, -4) \vee \neg Sqrt(2, -4))$ as true, $Sqrt(2, -4)$ will be either true or false, and $\neg Sqrt(2, -4)$ will be either true or false, too, even though neither $Sqrt(2, -4)$ nor $\neg Sqrt(2, -4)$ will be determined to be true by the metasemantic facts and thus will not be determinately true. In Ramsey semantics, *truth may outrun the facts*.

Where epistemicists have a classical concept of truth and maintain that the metasemantic facts fit that concept by determining a unique intended interpretation, and where supervaluationists accept that the metasemantic facts might not determine a unique intended interpretation and fit a non-classical concept of truth to these incomplete states of affairs, Ramsey semantics will occupy a position ‘in between’: the metasemantic facts might not determine a unique intended interpretation but the concept of truth will still remain classical. The details of the theory, including explanations of its terms (such as ‘metasemantic fact’), a formalization of its postulates with the help of metalinguistic Ramsey sentences and epsilon terms, and a discussion of its consequences and extensions (in the final Section 7) will be developed from Section 1.

Along the way, there will various pointers to overlaps with the existing literature, in particular to: Carnap [14, 15, 17] on the reconstruction of scientific theoretical terms by Ramsification and epsilon terms; McGee and McLaughlin [47] on semantic determination and the combination of supervaluationism for vague terms with a disquotationalist theory of truth; and Breckenridge and Magidor [6] on arbitrary reference and irreducible semantic facts. Although none of these references actually describe or advocate the metalinguistic Ramsification of classical semantics, they come close to it, and in many ways the present theory is going to continue threads of reasoning begun by them.

§1. Classical semantics and the challenge from semantic indeterminacy. Let me start by summarizing classical semantics. By that I mean classical formal semantics in the Tarskian-Carnapian-Montagovian-... model-theoretic tradition that includes both classical semantics in the narrow sense of the term (the statement of the classical semantic rules) and classical metasemantics (the explanation in virtue of what these classical semantic rules hold). Not every semanticist will agree with every detail of my summary, but they should at least agree with something in that ballpark. For simplicity, I will concentrate on extensional semantics, though a similar story could be told for intensional semantics.² For linguists, intensional semantics is much more interesting than extensional semantics, as only intensional semantics allows for stating the truth conditions of sentences with *intensional operators* and for the formal reconstruction of the update of a conversational common ground by the *intension* of an asserted sentence. At the same time, extensional semantics still serves as the basis of intensional semantics and continues to attract attention for its own sake in philosophical semantics

² The Ramsey sentence for classical intensional semantics would begin with: ‘There is an admissible *intensional* interpretation, that is, an assignment of references and *intensions* to the descriptive terms of the object language, such that...’.

(as in discussions of truth), which is why restricting our attention to it will leave us with an interesting enough case to consider. I will have to postpone topics such as intensional indeterminacy, the assertion of intensionally indeterminate sentences, and the pragmatic aim of such assertions to follow-up work on intensional Ramsey semantics.

Classical extensional semantics builds upon the analysis and formalization of the syntax of some fragment of natural, mathematical or scientific language as given at some point in time; let the result of that formalization be L , which—as all other languages in this paper—is supposed to be an *interpreted* formal language. For illustration, let us take the syntax of L to be that of a standard first-order extensional language with identity, and let us assume that L is not itself concerned with semantic matters, so that the vocabulary of the object language L does not itself include semantic terms, such as ‘true’.

A (classical) interpretation F of L is a function that assigns references/extensions to the members of the descriptive vocabulary of L , based on a universe $Uni(F)$ (which is a non-empty class); e.g., the extension $F(P)$ of a unary predicate P is a subclass of $Uni(F)$. A variable assignment s relative to F is a mapping that assigns values to the variables of L , where these values are taken from $Uni(F)$. (I will ignore complex singular terms.) I am going to leave open here whether the metalinguistic variable ‘ F ’ is a first-order variable for set-theoretic functions with set-sized domains, or whether it is a first-order or second-order variable for functions in a more general sense, such that the domain of a function might have the cardinality of a proper class. The same holds for ‘ $Uni(F)$ ’, which might denote a set or a proper class of individuals.³

Subsequently, the truth conditions of formulas A in L are specified by (classical) semantic rules by which the (classical) truth or satisfaction of a formula A is defined relative to an interpretation F and a corresponding variable assignment s . In particular:

For all F, s :

$$\begin{aligned} F, s \models P(a) &\text{ iff } F(a) \in F(P); \\ F, s \models \neg A &\text{ iff } F, s \not\models A; \\ F, s \models C \vee D &\text{ iff } F, s \models C \text{ or } F, s \models D; \\ F, s \models \exists x A &\text{ iff } \exists d \in Uni(F), \text{ such that } F, s \stackrel{d}{x} \models A. \end{aligned} \tag{1}$$

(Analogously for n -ary predicates with $n > 1$ and for the other classical logical connectives. $s \stackrel{d}{x}$ is like s but d is assigned to x ; for *sentences* A , that is, for closed formulas, reference to s may be omitted. ‘iff’ is short for ‘if and only if’.)

The semantic rules are classical in virtue of their compositional format and the classical manner in which they treat the classical logical operators in the logical vocabulary of L . The logic of semantics itself, that is, of the semantic metatheory to which e.g., definition (1) belongs, is assumed to be classical, too, and a sufficiently strong deductive system of classical set/class theory or higher-order logic is presupposed as well.

The logical consequence relation of (classical) logic (as applied to L) results from quantifying universally over all interpretations of L and all corresponding variable

³ $Uni(F)$ might even be the class of *all first-order individuals whatsoever*, if classical semantics is formulated in a suitable higher-order language: see Williamson [78].

assignments:

$$A_1, \dots, A_n \models C \text{ iff } \forall F, s : \text{if } F, s \models A_1, \dots, A_n, \text{ then } F, s \models C. \quad (2)$$

Next, amongst all possible interpretations F of L , one presupposes there to be *the intended or actual (classical) interpretation* I that involves *the intended or actual universe* $Uni(I)$ of objects and which assigns *the intended or actual references/extensions* to the members of the descriptive vocabulary of L . The actual truth values of formulas A in L are defined with the help of I and the semantic rules from before. In particular, one defines:⁴

$$\text{For all sentences } A \text{ in } L : A \text{ is true iff } I \models A. \quad (3)$$

Obviously, this does not mean that semanticists are interested in *all* aspects of truth—e.g., *finding out* which sentences of L are true normally involves empirical or mathematical investigation beyond semantics—but they are interested in *defining* or *axiomatizing* truth and in its *semantic properties*. It is for such purposes that I is important, at least so far as extensional semantics is concerned, since I does not just fix the intended references and extensions of terms but also the intended extensions of sentences, that is, their (actual) truth values.

I itself is supposed to be determined jointly by

- (i) all linguistic facts concerning the competent usage of predicates and singular terms (individual constants, individual variables, function terms) in L and
- (ii) all non-linguistic facts that are relevant as to whether the atomic formulas in L are satisfied.

The facts in (i) determine the truth conditions of atomic formulas and the universe of discourse over which the individual variables (attached to quantifiers) range. Or in other words: these facts determine the *intensions* of predicates and singular terms in L (including, possibly, rigid intensions). For instance, in the example of the principal-square-root predicate ‘ $Sqrt$ ’ from the introduction, the linguistic fact in question would be that mathematicians understand ‘ $Sqrt$ ’ to be partially defined on the set of real numbers by the conditional definition stated in Section 0.

The facts in (ii) determine whether the truth conditions determined by (i) are actually met over the universe that is determined by (i). These non-linguistic facts yield the ‘worldly’ contribution that, if taken together with intensions determined by the linguistic facts in (i), determines the extensions and references of predicates and singular terms in L at the ‘actual world’. In the square-root example, the relevant facts would be mathematical facts concerning which real numbers y are such that $y^2 = x$ and $y \geq 0$ for a given non-negative real number x : e.g., one such fact would be described by ‘ $2^2 = 4$ and $2 \geq 0$ ’. More generally, since the best present-day science and mathematics

⁴ More precisely, for each context c , one assumes there to be the actual or intended interpretation I_c of L in c , and then one uses I_c to define truth of A in c . For simplicity, I will suppress reference to contexts in what follows, which is of course not to say that classical semantics (or supervaluationist semantics or Ramsey semantics) could not or should not be combined with contextual parameters, such as those relevant to the extension of vague predicates, or with corresponding operations of context change. (See e.g., [62] for a contextualist version of supervaluationist semantics of vague terms, and see e.g., [50] for a contextualist version of a classical semantics of vague terms.)

are our best present approximations of what the actual world is like, one may think of the facts in (ii) as obtaining states of affairs which, at least in principle, can be described by the best available language(s) of mathematics and science. For the same reason, it should be possible to describe the facts in (ii) in a manner that is precise enough according to the best present mathematical or scientific standards. This does not mean that (ii) would require an absolute notion of ‘precise fact’ (as [48], p. 231 worry)—if anything, it would be the other way around: one might call the facts in (ii) ‘precise (enough at this point in time)’ in virtue of mathematicians and scientists describing them by the best linguistic means they have available (by their present standards).

The metasemantic determination of I by (i) and (ii) above is assumed to be governed by

- (iii) all metasemantic laws taken together that concern the atomic formulas, and hence the predicates and singular terms, of L .⁵

In the square-root example, the respective metasemantic law might be expressed by: *For all objects d and d' , if mathematicians understand ‘Sqrt’ to be defined on the set of real numbers by the conditional definition ‘For all x, y : if $Real(x)$, $Real(y)$, and $x \geq 0$, then: $Sqrt(y, x)$ if and only if $y^2 = x$ and $y \geq 0$ ’, and d' (for ‘ y ’) and d (for ‘ x ’) are such that the condition and definiens of that conditional definition apply to them, then the pair $\langle d', d \rangle$ is a member of the intended interpretation $I('Sqrt')$.* E.g., for $d = 4$ and $d' = 2$ it follows: since the first conjunct of the embedded antecedent describes a fact in (i) (the fact that mathematicians use the relevant conditional definition), and since the second conjunct of the embedded antecedent describes a fact in (ii) (in particular, the fact that $2^2 = 4$ and $2 \geq 0$), the law implies that $\langle 2, 4 \rangle$ is a member of $I('Sqrt')$, which yields the expected constraint on the intended interpretation of ‘Sqrt’. (Another law would concern the exclusion of pairs of numbers from $I('Sqrt')$.)

Considering some more prominent examples, another metasemantic law in (iii) might be the following quasi-Kripkean one (cf. [37]): *for all proper names a in L , for all objects d , if present usage of a in L is suitably causally connected to an act of baptism in which d was named a , then $d \in Uni(I)$ and $I(a) = d$.* Consider any concrete instantiation of that universal claim: if the corresponding ‘if’-part describes one of the linguistic facts in (i), then the metasemantic law jointly with that fact determines the constraint on the intended interpretation I of L that is described by the ‘then’-part. Or yet another law in (iii) might be expressed in the following quasi-Putnamian way (cf. [54]): *for all kind terms K in L , for all objects d and d' , if K was collectively specified in L by pointing at d while being interested in the physical structure of d , and if d' has the same physical structure as d , then: if $d' \in Uni(I)$ then $d' \in I(K)$.* Once again, consider any concrete instantiation of that universal statement: if the first conjunct of the corresponding ‘if’-part describes one of the linguistic facts in (i), whilst the second conjunct of

⁵ This account closely resembles McGee & McLaughlin’s [47] ‘psycholinguistic’ account of ‘definite application/satisfaction’. More about determination can be found in McGee & McLaughlin (1998). I prefer ‘metasemantic determination’ over their term ‘semantic determination’, but I will use their term ‘constraint’ ([47], p. 225) in much the same way as they do. Most authors working on vagueness and semantic indeterminacy would at least accept that semantic meaning supervenes on, or is determined by, use (but see [34] for a contrary view). Use corresponds to (i) above, (ii) adds what is needed to determine referential/extensional semantic meaning, and the laws in (iii) govern the determination.

the ‘if’-part describes one of the non-linguistic facts in (ii), then the metasemantic law and the two facts jointly determine the constraint on I that is described by the ‘then’-part.

In terms of an analogy, metasemantic determination in classical semantics is supposed to be much like a physical or economic quantity z being determined from other such quantities x and y in the sense that z is a function f of x and y : z is like the intended interpretation I , x and y are respectively like (i) and (ii) above, and the law $z = f(x, y)$ corresponds to (iii). E.g., x being of some value, say, 3, and y being of some value, say, 1, jointly determine that z is of the value $f(3, 1)$, which is analogous to (i)–(iii) determining I .

Let us call the facts in (i) and (ii) *metasemantic facts* (for L). They are metasemantic in the sense that the linguistic expressions in L have their intended references and extensions in virtue of them and the metasemantic laws in (iii). Sometimes I will also drop the qualification ‘metasemantic’ and just refer to *facts*.

Let us call the constraints that (i)–(iii) jointly impose on the interpretation of L *the existing metasemantic constraints* on the interpretation of L . One may think of these constraints as being summed up by a theory that states what I must be like in view of (i)–(iii). It may be possible to (at least approximately) express some of these constraints linguistically, as in the examples for (iii) above, and if one does so, one might use terms, such as ‘baptize’, ‘specify’, ‘interest’, ‘point to’, ‘describe’, ‘rule’, ‘practice’, ‘intention’, ‘intension’, ‘think’, ‘cause’, ‘historical’, ‘kind’, ‘same as’, ‘physical structure’, ‘natural’, ‘division of labor’, ‘expert’, ‘intuition’, ‘define’, ‘axiom system’, and more. (I am not saying all these terms are sufficiently clear or that each of them is required for that purpose, just that classical semantics is in principle compatible with all sorts of metasemantic constraints.) But there is no guarantee that every existing metasemantic constraint can be expressed easily in such terms or in others. Indeed, Williamson [76] worries that ‘Meaning may supervene on use in an unsurveyably chaotic way’: while interpretation may be determined by the metasemantic facts, the ways in which this comes about—the metasemantic laws—might be extremely sensitive to how certain parameters are set (like parameters in a complex chaotic dynamic system of differential equations) and therefore also difficult to express in language. For that reason, it is more helpful to summarize the existing metasemantic constraints taken together as a theory in the sense of the *non-statement* view of scientific theories (see e.g., [69]), that is, as a *class of interpretations (models)* of L : let us call that class ‘*Adm*’. (‘*Adm*’ is short for ‘admissible’, which I borrow from [25] and supervaluationist semantics, about which more from Section 2.) The existing metasemantic constraints on the interpretation of L show up in what *Adm* is like. Exploring these constraints and hence *Adm* goes beyond extensional semantics: the study of (i) belongs to pragmatics and intensional semantics, the study of (ii) is normally not the subject matter of linguistics at all (but, e.g., of mathematics or physics), and the study of (iii) lies at the interface of all the previous subjects. Following a reasonable divide-and-conquer strategy, classical semantics is therefore not itself concerned with formulating or testing metasemantic hypotheses on ‘*Adm*’ but merely *presupposes* that the metasemantic constraints (whatever they are like) yield a particularly strong and restrictive theory: the *singleton* set $Adm = \{I\}$, from which the intended interpretation I can be defined as its sole member. Nor is the claim that it is *known* exactly what I is like—one merely *presupposes* that there is a classical interpretation I , such that I conforms to the existing metasemantic constraints ($I \in Adm$), and where I is in fact determined uniquely by these

constraints ($Adm = \{I\}$). Let us put these metasemantic presuppositions of classical semantics on record:

$$\exists! F(F \in Adm) \text{ and } I \in Adm \text{ (where 'Adm' is understood as explained before)}. \quad (4)$$

What classical extensional semanticists do is to formulate and test hypotheses about the (classical) semantic rules of L and about semantically salient aspects of the (classical) intended interpretation I of L , while presupposing (4) in the background. If all goes well, these hypotheses lead to empirically successful predictions and explanations of some linguistic phenomena concerning L . That is classical extensional semantics explained in a nutshell.⁶

So far, so good. But there is a problem: while it is common to speak of *the* intended interpretation I (to which the term ' I ' supposedly refers), and hence to presuppose that there is one and only one intended classical interpretation, *we lack good reasons for believing that the existing metasemantic constraints on the interpretation of a language will always determine a unique such interpretation*. That is: Adm might well happen not to be a singleton class. Let me illustrate this by means of three examples: one from natural language, another one from mathematical language, and a third one from scientific language.

EXAMPLE 1: *Vague* terms may cause trouble: terms for which there seem to be 'indeterminate' or 'imprecise' borderline cases beyond the clear-cut positive and clear-cut negative ones. I will concentrate on what Alston [2] calls *degree vagueness*, but the general conclusions should apply to all kinds of semantic vagueness.⁷ Let B be a predicate of L that formalizes the English term 'bald', or rather, 'is the number of hairs on the head of a bald person', where I will presuppose that two people with the same number of hairs either both count as bald or both do not, and where I will ignore the distribution of hairs. (' $Bald(Peter)$ ' may then be understood as: there exists an x , such that $number-of-hair-on-head-of(Peter) = x$ and $B(x)$.)

Plausibly, the following claims express some existing metasemantic constraints on the intended interpretation of B , that is, the actual extension of B : any person with 0

⁶ More precisely, that is how classical semantics works as a descriptive empirical discipline. There is also the more normative philosophical project of semantics as the rational reconstruction of meaning with the aim of improving language and interpretation through formal languages (see [51], p. 21). The main part of the present paper deals with classical semantics in the former sense and hence belongs to the philosophy of semantics as a part of the philosophy of science. I will turn briefly to the rational reconstruction of meaning at the end of Section 6.

⁷ I will not deal with vague *objects*, which would constitute a form of *metaphysical* vagueness that is orthogonal to the aim of this paper. Assuming that objects themselves are not a source of vagueness, and assuming the same holds for predicates and singular terms as syntactic objects (e.g., the string *b-a-l-d*), mappings from predicates and singular terms to (sets of tuples of) objects will be perfectly unproblematic, too. Hence, classical interpretation mappings will not themselves be affected by vagueness, only *which interpretation is meant to be 'the' intended one*. Similarly, intensions (functions from possible worlds to references/extensions) in intensional semantics are unproblematic, so long as worlds do not themselves give rise to vagueness either. If so, concepts/properties in the sense of intensions will not themselves be vague, though the predicates expressing them may be so in the sense that *which of their intensional interpretations is meant to be 'the' intended one* may be affected by vagueness.

hairs on their scalp belongs to the extension of B ($0 \in I(B)$); any person with 100000 hairs does not ($100000 \notin I(B)$); if one person has more hair on their head than a second one, then, if the former belongs to the extension of B , the same holds for the latter (for all m, n , if $m > n$ and $m \in I(B)$ then $n \in I(B)$)⁸; $Uni(I)$ includes the required natural numbers (e.g., $0 \in Uni(I)$); and so on.

But even if one combines all linguistic competent-use facts concerning L (e.g., pertaining to all rules for B -usage and all competent B -assertions ever made) with all relevant non-linguistic facts (e.g., concerning the number of hairs those people have had about whom competent B -assertions have or could have been made), and if one takes these facts together with all metasemantic laws for L , the following seems *unlikely*, at least at first glance: for each number n , either the existing metasemantic constraints determine that $n \in I(B)$ (and hence, by (1), the truth of $B(n)$), or they determine that $n \notin I(B)$ (and hence, by (1) again, the truth of $\neg B(n)$). (I am using ‘ n ’ both as a metalinguistic and an object-linguistic symbol here.) For, at least *prima facie*, it is *plausible* that there are borderline cases n to which one may competently ascribe B , but to which one may also competently refrain from ascribing B and indeed ascribe $\neg B$. It does not just seem to be our knowledge of I that is incomplete or empirically underdetermined: rather, it seems that no competent speaker of L would be right in correcting or criticizing either of the two ascriptions even if they knew all metasemantic facts whatsoever. Thus, there should be at least two different classical interpretation functions F and F' that conform to *all* existing metasemantic constraints, but which fill the gaps left by them differently: for some n , $n \in F(B)$ whereas $n \notin F'(B)$. Consequently, the corresponding class Adm of admissible interpretation functions includes more than just one member, and hence there isn’t a uniquely factually determined intended interpretation I of L . Let us call this phenomenon the *semantic indeterminacy* of ‘the’ intended interpretation of L : vagueness seems to be ‘semantic indecision’ ([44], p. 213; see [74] for more on vagueness as indeterminacy.)

Of course, ‘unlikely’ is not ‘impossible’ and ‘plausible’ does not mean ‘guaranteed’: indeed, epistemicists about vagueness *do* believe the reference of ‘ T ’ to be determined uniquely by the metasemantic facts, and hence that vagueness does not entail semantic indeterminacy. And they might be right about this—who knows? *But what if not?* Epistemicists might also rightly complain that not enough has been said above about facts and metasemantic determination. For instance, they might ask: are the non-linguistic facts mentioned under (ii) above so that for each n , it is either a fact that $B(n)$ or it is a fact that $\neg B(n)$? For if so, these ‘baldness facts’ would determine the interpretation of B uniquely after all. If not, which would seem more likely, as ‘baldness-facts’ would not be countenanced to be precise enough according to the best present mathematical or scientific standards (recall our description of (ii)): what exactly counts as a fact in (i) and (ii) above, and how do these facts constrain semantic interpretation by means of the laws in (iii)? Will the explication of ‘determination’ and ‘ Adm ’ possibly reveal some epistemic components in the special case of *vague* terms? (In contrast, the way in which the interpretation of the *non-vague* term ‘ $Sqrt$ ’ was determined from the mathematicians’ usage of a conditional definition and from mathematical facts did not seem epistemic at all.)⁹

⁸ In the terminology of Fine [25], this would correspond to a so-called ‘penumbral connection’.

⁹ For more on such worries, see Williamson [75, 77] and Williamson’s [79] criticism of McGee & McLaughlin [48]. For a reply, see McGee & McLaughlin [49]. E.g., McGee & McLaughlin

These are reasonable questions, and more should be said indeed. *But shouldn't the previous considerations at least cast enough doubt on semantic determinacy that the burden of proof is switched to the classical semanticist?* Wouldn't we simply tempt the fate of semantics by building it on presupposition (4) for which it seems difficult to cite empirical evidence and which, at least *prima facie*, sounds doubtful with respect to the interpretation of vague terms?

Instead of continuing the philosophical debate on metasemantic determination, which is not itself the topic of this paper, I will rather add to the pressure on (4) by presenting further examples of semantic indeterminacy, though this time without vagueness. Accordingly, the focus of the paper is not vagueness *per se* but really the *general* phenomenon of semantic indeterminacy. And I will continue to understand '*Adm*' in terms such as 'determinately' and 'metasemantic facts' rather than, say, 'definitely' and 'knowledge'.

(The present example of the language L that includes vague terms, such as '*B*' for 'bald', will remain the central example from Section 3 with the sole exception of Section 6. In what follows, any reference by means of ' L ' is a reference to the language from Example 1.)

EXAMPLE 2: Let L' be a second-order formalization of the language of *arithmetic* (so here I am deviating from the previous assumption of a first-order language L): let $N, 0, s, +, \cdot$ be terms in L' that formalize the corresponding arithmetical terms for 'natural number', 'zero', 'successor', 'sum', and 'product', as used by number theorists.

It is plausible that the following claim expresses an existing metasemantic constraint on 'the' intended interpretation I' of L' : $I'(N), I'(0), I'(s), I'(+), I'(\cdot)$ jointly satisfy the *second-order Dedekind-Peano axioms* for arithmetic. (That is: $I'(0) \in I'(N)$; for all $d \in I'(N)$ it holds that $I'(s)(d) \in I'(N)$; and the other axioms, including second-order induction.) As proven by Dedekind [21], the axiom system of second-order Dedekind-Peano arithmetic is categorical, that is, it pins down the *structure* of the natural number sequence uniquely. If structuralists about arithmetic are right (see [28] for a survey), there do not exist any other metasemantic constraints on the interpretation of arithmetical symbols than getting the structure of the natural number sequence right; hence, the second-order Dedekind-Peano axioms taken together might actually express *all* the metasemantic constraints on I' there are. But, at the same time, these constraints do not pin down I' uniquely: e.g., if $I'(N)$ is identified with the set of *finite von-Neumann-ordinals*, there are suitable and easily definable choices for $I'(0), I'(s), I'(+), I'(\cdot)$, such that the axioms are satisfied; but the same is true also of the set of *finite Zermelo-ordinals* and the interpretations of the arithmetical symbols that are suitable to those ordinals (as famously highlighted by [4]). The corresponding class *Adm* should therefore include at least two distinct classical interpretations F and F' —perhaps even *all* (infinitely many) interpretations isomorphic to the two previous ones—which is why there isn't a uniquely factually determined interpretation of L' . The intended interpretation of the language of second-order arithmetic is semantically

[49] criticize Williamson's [79] argument that determinate truth would have to collapse into truth and determinate falsity into falsity. One can use the example of the conditional definition of '*Sqrt*' from Section 0 to show which of the premises of Williamson's argument are likely to be false; but I will not do so here.

indeterminate, and since arithmetic terms do not have borderline cases, this is a case of semantic indeterminacy without vagueness.¹⁰

On behalf of classical semantics, one might try to argue that set-theoretic structuralists about arithmetic are simply wrong—there *do* exist metasemantic constraints beyond those of second-order Dedekind-Peano arithmetic, and these *do* restrict the interpretation of arithmetical symbols, say, to the interpretation with the finite von-Neumann-ordinals. However, other than in the context of a textbook in foundational set theory in which natural numbers are indeed often defined as the finite von-Neumann-ordinals, number theorists may not feel bound by any specification of the intended interpretation of arithmetic over and above the Dedekind-Peano axioms. Once made aware of the existence of different set-theoretic interpretations of their axioms, some of them might simply not accept that one such interpretation is taken to be intended *by fiat* while all other interpretations are rejected as inadmissible; they might regard the assumption of a uniquely factually determined intended interpretation of the arithmetical symbols to be unfaithful to the mathematical content of arithmetic. Again, these number theorists might be wrong, but the mere existence of corresponding verdicts by number theorists and structuralist philosophers of number theory (Dedekind would count as a concrete instance on both sides) should suffice at least for a *prima facie* case against semantic determinacy. For the same reason, the burden of proof is again on the classical semanticists to justify their presupposition of a unique factually determined intended interpretation of arithmetical terms (even when doing so would lead them beyond semantics).

EXAMPLE 3: Let L' formalize the language of *Newtonian mechanics*—no matter whether first-order or second-order—and let m formalize the term ‘mass’ as used in Newtonian physics. (m is a function symbol, but let us put questions about syntax to one side now.) Field [23] has argued, using the language of modern relativistic mechanics on the metalevel, that there are two interpretations of m , such that there is no fact of the matter which of them delivers ‘the’ actual or intended reference of m : according to the one, $I'(m)$ coincides with relativistic mass (total energy/ c^2), while according to

¹⁰ The discussion here presupposes an *eliminative set-theoretic* structuralism (see [28], Chapter 3) that restricts the purely mathematical resources of the metalanguage of L' to those of set theory. *Non-eliminative* structuralists do not find that satisfying: they claim that N *does* have a uniquely determined intended interpretation, it is just that $I'(N)$ does not coincide with the set of finite von-Neumann-ordinals or with the set of finite Zermelo-ordinals or with *any* set of set-theoretic entities for that matter. Instead, the intended interpretation of arithmetical terms is given by a uniquely determined structure *sui generis*—the abstract structure of natural numbers—that cannot and should not be eliminated in favor of sets; and if the metalanguage of L' offers ways of talking about that non-set-theoretic structure, the alleged semantic indeterminacy of ‘natural number’ dissolves.

However, even if one agreed with non-eliminative structuralists, worries about semantic indeterminacy would reemerge as far as *singular* terms for *single* objects in some abstract structures are concerned: e.g., the imaginary unit i in the structure of the complex field of numbers is structurally indistinguishable from its numerically distinct ‘sibling’ $-i$, as there exists a field automorphism that maps the one to the other. Therefore, for non-eliminative structuralists, there does not seem to be any fact of the matter whether the numeral ‘ i ’ actually denotes i or whether it rather denotes $-i$ (see [5, 63]). The debate about these matters is ongoing and some authors (see [40, 41, 52, 59, 61, 63, 64]) have suggested ways of developing a structuralist semantics that resemble aspects of Ramsey semantics. (I will return to part of that literature later in Section 2.)

the other, $I''(m)$ coincides with proper mass (non-kinetic energy/ c^2), and relativistic mass and proper mass come apart in value and physical properties. Each of the two interpretations saves *some* of the central claims of the Newtonian theory from falsity (and *many* of Newton's empirical predictions about slowly-enough-moving objects), neither of the two interpretations saves *all* central claims of Newton's theory, and the theoretical roles played by relativistic mass and proper mass in modern relativity theory are equally salient and important. Since these properties seem to exhaust, in present terminology, the existing metasemantic constraints on the interpretation of m in Newtonian mechanics, Field's diagnosis concerning 'mass' is: 'the situation is not that we don't *know* what Newton's word denoted, but that Newton's word was referentially indeterminate' ([23], p. 467, his emphasis).

What is going on here may be viewed as an application of the principle of charity: the Newtonian language is to be interpreted so that the truth of the sentences asserted by the Newtonian theory gets maximized; and as often the case when one maximizes a function, there might not be a unique maximum but two of them (such as for 'mass') or three...or even infinitely many. If so, the class Adm of admissible interpretation functions of L'' includes more than one member, and hence there is no unique factually determined intended interpretation of L'' : 'the' intended interpretation of Newtonian mechanics is semantically indeterminate.¹¹

This argument for indeterminacy does not concern vagueness or structuralism but 'incongruencies' between, on the one hand, the language of Newtonian mechanics, and on the other hand, what *we* think the world is like as described by present-day relativistic mechanics. As Field argues against Kuhn [38], these 'incongruencies' do not require incommensurability or complete referential discontinuity between the Newtonian language and the relativistic language but rather manifest themselves as semantic indeterminacies. (I will return to the discussion of referential change and continuity in science in Section 6.)

Once again, the classical semanticist might claim this analysis to be wrong: there *are* existing metasemantic constraints of which Field is unaware, which determine the intended interpretation of m to coincide with, say, proper mass. If so, the defender of classical semantics should supply arguments and data in favour of this thesis (even when doing so would lead them beyond semantics): does proper mass possess some 'objective naturalness' that relativistic mass lacks, and if so in what sense? Is there something in the experimental practice of Newtonian physicists that rules out relativistic mass as an interpretation of 'mass'? In view of Field's arguments to the contrary, the presupposition that 'mass' in Newtonian physics has a unique factually determined intended interpretation would seem to rest on shaky grounds unless backed up by evidence. Presupposition (4) of classical (meta-)semantics is challenged by examples like these.

§2. Ramsey semantics. In the last section I have described the gist of classical semantics. I ended up pointing out that classical semantics is risky business by

¹¹ Field [23] continues to develop a variant of supervaluationist semantics for semantically indeterminate expressions such as m . See Chapter 2 of Button and Walsh [7] for more on referential indeterminacy and supervaluation. I will deal with supervaluationism in Sections 2–4.

building on a presupposition—the existence of a unique factually determined intended interpretation I —that might not be met: the metasemantic facts seem to leave ‘bald’, ‘natural number’, and ‘mass (in Newtonian mechanics)’ without uniquely determined intended interpretations, and each time for different reasons. The defender of classical semantics, it seems, will have to fend off each worry on separate grounds: *no, vague terms* are not semantically indeterminate, their vagueness can be explicated otherwise (e.g., counterfactually), and complete extensions are somehow determined by the facts even though we might not know how. *No, structuralism* about arithmetic is wrong, there is more to arithmetical terms than their structural content, whatever it is. *No, ‘mass’ in Newtonian physics* does have a uniquely determined reference, even when it is hard to say what it refers to.¹²

Classical semanticists might reply that there might still be a global *abductive* argument for semantic determinacy: how else should one explain the success of classical semantics as applied to certain fragments of language (when successful there), if not by trusting its presupposition of a unique factually determined intended interpretation of that fragment? And even more so, when the alternatives of classical semantics seem to have their own problems!

In what follows, I want to demonstrate that this kind of inference to the best explanation does not go through either. For there exists a minor retreat from classical semantics that has similar theoretical virtues as classical semantics and which should therefore be similarly successful when and where classical semantics is successful: *Ramsey semantics*. At the same time, the new semantics will be less risky than classical semantics by not presupposing semantic determinacy. Ramsey semantics does not thereby claim semantic determinacy to be false, it only avoids presupposing it, and yet it keeps classical logic and truth on board. For these reasons, overall, Ramsey semantics should be preferable over classical semantics. What is more, by approximating classical semantics much more closely than supervaluationist semantics, Ramsey semantics should also win over supervaluationists who are attracted by classical semantics apart from not wanting to rule out semantic indeterminacy. While the present section will explain the new semantics, Section 3 will draw the comparison with supervaluationist semantics and classical semantics.

Returning to the semantics of our first-order language L (with ‘ B ’ for baldness) from Section 1, let us accept for now that ‘the’ intended interpretation of L is subject to metasemantic constraints without necessarily being pinned down by them uniquely: Adm may be a singleton class, as presupposed by classical semantics, but it may also include *more than just one admissible classical interpretation mapping*. Either way, by being members of Adm , all F in Adm have something in common; e.g., $0 \in F(B)$; $100000 \notin F(B)$; for all m, n , if $m > n$ and $m \in F(B)$ then $n \in F(B)$; $0 \in Uni(I)$; and so on. I will also assume, for simplicity, that all members F of Adm are based on one and the

¹² Further arguments against semantic determinacy may be distilled from the literature, though all of them are controversial and none of them constitutes a knock-down argument: cf. Willard van Orman Quine on the indeterminacy of reference, Hilary Putnam on the Löwenheim–Skolem theorem and non-standard interpretations, Saul Kripke on rule-following, Solomon Feferman on the semantic indeterminacy (in his terms, ‘vagueness’) of the Continuum Hypothesis and the set-theoretic membership predicate (see also [27] on that topic), and Wilson [80] on the indeterminacy and open-endedness of terms from applied mathematics and science.

same universe U , which is a non-empty set (rather than a proper class): for all $F \in Adm$, $Uni(F) = U$.

In terms of the mathematical analogy from the last section: the quantity z (which was analogous to ' T ') might be *constrained* by x and y , but not in the sense of being a function of them ($z = f(x, y)$) but by there being an equation that is to be satisfied jointly by z, x, y : say, $eq : z^2 = x + y$. And that equation might offer more than one solution for ' z ' when given concrete values for ' x ' and ' y ': e.g., $x = 3$ and $y = 1$ constrain the value of ' z ' to the effect that *either* $z = 2$ *or* $z = -2$, without imposing further constraints that rule out either of the two solutions. The totality of all metasemantics laws that support the existing metasemantic constraints on the interpretation of L might be more like eq than like $z = f(x, y)$, and the resulting class Adm of admissible interpretations of L might be more like $\{2, -2\}$ than like $\{f(3, 1)\}$.

Instead of presupposing that all the facts taken together determine metasemantically a unique intended classical interpretation I and hence what is true in virtue of I , a less risky way of proceeding in semantics should therefore be: to merely presuppose that *there exists a classical interpretation F that conforms to all existing metasemantic constraints and from which truth is defined by means of the classical semantic rules*. For that existence statement will be true both in case the intended interpretation of L is factually determined—when there is a uniquely determined F for which $F \in Adm$ —and in the case of semantic indeterminacy—when there is more than one F , such that $F \in Adm$. (When there is no F at all such that $F \in Adm$, classical semantics suffers from an even deeper-going problem than indeterminacy, about which I will have nothing to say here. The obvious way out of that case would be to consider a larger class of 'approximately admissible' interpretations and work with that.)

But this means we enter well-trodden territory, since what is going on now may be viewed as an instance of *Ramsification*: first, one regards the terms ' T ' ('intended interpretation') and 'true' from classical semantics as theoretical terms, the meanings of which are determined by ' $I \in Adm$ ' and the definition (3) of truth, that is, 'for all sentences A in L : A is true iff $I \models A$ '. That is plausible because I was meant to be intended precisely in the sense of belonging to Adm , that is, of conforming to all existing metasemantic constraints, and at the same time delivering the domain, references, and extensions that feed into the definition of truth by means of the standard semantic rules (1). Secondly, one replaces ' T ' and 'true' in the metalinguistic sentence

$$'I \in Adm \text{ and for all sentences } A: A \text{ is true iff } I \models A'$$

by the function variable ' F ' and the class variable ' T ', respectively, which yields the open metalinguistic formula

$$'F \in Adm \text{ and for all sentences } A: A \in T \text{ iff } F \models A'$$

Finally, one does *not* presuppose any longer

(4) and (3) $\exists! F(F \in dm)$ and $I \in Adm$, and for all sentences A : A is true iff $I \models A$ (equivalently, $\exists! F(F \in Adm)$ and $I = \iota F(F \in Adm)$)¹³, and for all sentences A : A is true iff $I \models A$).

¹³ This is an instance of Lewis' [42] definition of a theoretical term by a definite description ($T = \iota XTh[X]$). Lewis' account builds historically on Ramsey's and Carnap's work; Lewis also cites Carnap's definition of theoretical terms by epsilon terms that will become important later in this section.

Instead, one merely claims the *existence* of an F and a T , such that $F \in Adm$ and where F ‘defines’ T by means of satisfaction, that is:

$$\exists F \exists T (F \in Adm \text{ and for all sentences } A: A \in T \text{ iff } F \models A). \quad (5)$$

But (5) is nothing but the *Ramsey sentence* of ‘ $I \in Adm$ and for all A : A is true iff $I \models A$ ’ with respect to the theoretical terms ‘ T ’ and ‘true’.¹⁴

The classical presupposition ‘ $\exists! F (F \in Adm)$ ’ from (4) is dropped rather than Ramsified, because it is exactly what Ramsey semantics intends to avoid. The remaining parts of classical semantics, that is, the definition of ‘interpretation’, ‘variable assignment’, and (1) and (2) from Section 1, are explicit model-theoretic definitions that are presupposed by classical semantics and Ramsey semantics (and supervaluationist semantics) and do not need to be Ramsified. ‘ Adm ’ is regarded as an $O(Id)$ -term in the terminology of Lewis [42], which is left invariant by Ramsification: as explained in Section 1, the characterization of Adm does not belong to semantics *proper*, and the term ‘ Adm ’ (or something in the same ballpark it, such as ‘intended’ or ‘actual’) needs to be presupposed equally by classical (meta-)semantics, supervaluationist semantics, and Ramsey semantics. (As in [42], the distinction between *observational* and *non-observational terms* known from more traditional applications of Ramsification does not play any role in Ramsey semantics.)

Since, for given F , ‘for all sentences A in L : $A \in T$ iff $F \models A$ ’ uniquely characterizes T so far as its sentence members in L are concerned (as can be shown set-theoretically or in higher-order logic), one may also formulate (5) alternatively as the combined existence/unique-existence claim

$$\exists F \exists! T (F \in Adm \text{ and for all sentences } A: A \in T \text{ iff } F \models A). \quad (5')$$

Either way, the idea is:

- classical semantics consists in the definition of ‘interpretation’, ‘variable assignment’, and (1)–(4) and
- while Ramsey semantics consists in the same definitions of ‘interpretation’ and ‘variable assignment’, (1), (2), and the *Ramsey sentence* (5)/(5'). (I am going to add yet another alternative formulation of Ramsey semantics below, which may be understood as a reformulation of the present one.)

As the term ‘*Admissible*’ had already suggested before, Ramsey semantics shares the assumption of a non-empty class Adm of admissible interpretations with supervaluationism (see [25, 35, 47]), such that neither of them presupposes that Adm is a singleton, and where ‘ Adm ’ is interpreted similarly in both semantics.

¹⁴ I want to leave open here how (4) and (5) would be stated in a fully formalized language. One option would be to understand them as second-order formulas, such that, e.g., ‘ F ’ would be a variable for functions as second-order entities and ‘ $F \in Adm$ ’ would really be an instance of higher-order predication ($Adm(F)$). Another option would be to treat (4) and (5) as first-order formulas, in which case, e.g., ‘ F ’ would be a variable for functions as first-order individuals, and ‘ $F \in Adm$ ’ would either be an instance of standard predication ($Adm(F)$) or it would actually invoke the set-theoretic membership predicate ($\in(F, Adm)$), in which case ‘ Adm ’ would be a singular term.

However, that is also where paths will diverge: in particular, Ramsey semantics does *not* introduce a supervaluationist notion of *super-truth* (cf. [71]), that is,

for all sentences A in L : A is super-true iff for all $F \in Adm$, $F \models A$,

to the effect that super-truth takes over some, if not all, conceptual roles that truth *simpliciter* plays in classical extensional semantics. (I am going to compare Ramsey semantics with supervaluationism in more detail in Section 3.)

Nor should the present theory be mistaken for so-called subvaluationism about vague terms (cf. [20, 32, 33]), which defines

for all sentences A in L : A is sub-true iff $\exists F \in Adm$, such that : $F \models A$,

and which assigns *sub-truth* the roles that truth *simpliciter* plays in standard semantics.

Instead, Ramsey semantics maintains (reformulating (5') just a bit)

$\exists F \in Adm \exists! T$, such that for all sentences A in L : $A \in T$ iff $F \models A$,

in which ' F ' is bound *existentially*, but in which the existential quantifier expression ' $\exists F \in Adm$ ' takes *wide scope* (instead of occurring on the right-hand side of the embedded equivalence), and in which T (truth relative to F) is determined uniquely from F by the semantic rules for *classical* satisfaction. The claim is: *there exists* a classical admissible interpretation in terms of *which* the truth values of all sentences of L are given in a classical manner. For instance, while there may be sentences A , such that neither A nor $\neg A$ is *super-true*, and where both A and $\neg A$ are *sub-true*, neither of this could possibly happen in Ramsey semantics, as there is no classical interpretation mapping F of *which* it would hold that $F \not\models A$ and $F \not\models \neg A$ (and hence $A \notin T$ and $\neg A \notin T$), or of *which* it would hold that $F \models A$ and $F \models \neg A$ (and thus $A \in T$ and $\neg A \in T$).

Ramsey semantics should not be misunderstood either as requiring that the theoretical terms ' I ' and 'true' would need to be eliminated—Ramsified *away*—when doing semantics: semantic statements including them merely need to be used and interpreted more cautiously than in classical semantics, so that they can always be understood as *stand-ins* for *existential* statements. There are two ways of achieving that.

One is to start from the existential statement (5) or (5'), to apply to it the elimination rule of natural deduction for existential quantifiers, thereby 'picking' one of the relevant F s and calling 'it' ' T ', calling the members of the set T that is given uniquely by I the 'true sentences (of L)', and introducing ' $I \in Adm$ and for all sentences A : A is true iff $I \models A$ ' as a *temporary assumption*. Of course, what is 'picked' by ' T ' in this way is really left indeterminate— I could be *any* of the relevant F s.

That being in place, the Ramsey semanticist is able to reformulate in their own terms every metalinguistic semantic statement

$$S[I, \text{true}] \tag{6}$$

involving the terms ' I ' or 'true' that the classical semanticist might want to put forward. This does not mean that each single statement (6) would require a new piecemeal application of Ramsification: instead, one application of Ramsification (the result of which is (5) or (5')) may be used to reconstruct various possible metalinguistic instances of (6) simultaneously. Of course, at some point this application of the existential elimination rule will have to be terminated by discharging the assumption involving ' I ' and 'true' from above and by deriving a statement that does not involve the 'newly

introduced' terms anymore. For instance, in a case in which the classical semanticist would logically derive (6) from the classical *thesis* ' $I \in Adm$ and for all sentences A : A is true iff $I \models A$ ', the Ramsey semanticist would derive (6) in the same manner from the *temporary assumption* ' $I \in Adm$ and for all sentences A : A is true iff $I \models A$ ', hence derive the conjunction of (6) with that assumption, then derive from that conjunction that

$\exists F \in Adm \exists T$, such that: for all sentences A , $A \in X$ iff $F \models A$, and also $S[F, T]$ (7)

by existential introduction, and finally discharge the temporary assumption and thus derive (7) from the Ramsey sentence (5) or (5') that is part of the theory of Ramsey semantics.¹⁵

The downside of deriving semantic statements from the Ramsey sentence (5)/(5') in this manner is that the terms ' I ' and '*true*' are only introduced temporarily in the context of an application of existential elimination in which the assumption ' $I \in Adm$ and for all sentences A : A is true iff $I \models A$ ' will need to be discharged eventually. Fortunately, there is also a second, more stable, way of presenting Ramsey semantics that allows the Ramsey semanticist to use the terms ' I ' and '*true*' continuously without having to make or discharge assumptions: one recycles Carnap's [14, 15] treatment of theoretical terms as Hilbertian epsilon terms (see [59] for a recent survey, discussion, and application) and employs it in the present semantic context. The rest of this section will be devoted to this topic.

Epsilon terms ' $\varepsilon F \dots$ ' are indefinite descriptions, which are much like definite descriptions ' $\iota F \dots$ ' (*the F, such that...*), except that the uniqueness presupposition of definite descriptions is dropped. Accordingly, one may read ' $\varepsilon F \dots$ ' as '*an F, such that...*': analogously to the application of existential elimination explained before, ' $\varepsilon F C[F]$ ' is supposed to 'pick' one of the F s that satisfy ' $C[F]$ ', such that what gets 'picked' by ' $\varepsilon F C[F]$ ' is left indeterminate—once again, $\varepsilon F C[F]$ could be *any* of the relevant F s. The Hilbert school (see [1, 30]) introduced such epsilon terms as tools in metamathematics by which 'ideal' mathematical entities could be denoted and with the help of which proof-theoretic reductions were meant to be carried out.

In the present context, epsilon terms are made formally precise by adding, first, the primitive epsilon operator ' ε ' to the logical vocabulary of the metalanguage of L . (For the logicity of the ε -operator, see [81].) If ' F ' is a metalinguistic variable, then the result of replacing ' $C[F]$ ' in ' $\varepsilon F C[F]$ ' by a metalinguistic formula yields a metalinguistic term ' $\varepsilon F \dots$ ' of the same type as ' F '. (Abusing notation just a bit, I will denote the resulting term by ' $\varepsilon F C[F]$ ' again.) E.g., just as ' F ' can occur on any side of the metalinguistic equality symbol (as in ' $F = \dots$ '), the same holds for ' $\varepsilon F C[F]$ ' (as in ' $\varepsilon F(C[F]) = \dots$ '). If ' F ' is the *only* free variable in (the metalinguistic formula replacing) ' $C[F]$ ', then the epsilon term ' $\varepsilon F C[F]$ ' is a *closed term*, since the free variable ' F ' of ' $C[F]$ ' is bound by ' εF ' within ' $\varepsilon F C[F]$ ' in the same way in which ' $\exists F$ ' would bind the free variable ' F ' of ' $C[F]$ ' within the *closed formula* (sentence) ' $\exists F C[F]$ '.

¹⁵ As mentioned in Example 2 of Section 1, the literature on structuralism in the philosophy of mathematics discusses similar strategies of introducing terms ('parameters') for numerically distinct but structurally indistinguishable objects in structures or for numerically distinct but structurally indistinguishable set-theoretic systems: see Shapiro [63, 64], Pettigrew [52] for more on this.

Secondly, one extends the classical logic of the metalinguistic semantic theory by the axioms of the so-called (extensional) epsilon calculus, that is, all statements of the form

$$\exists FC[F] \leftrightarrow C[\varepsilon FC[F]]$$

$$\text{Extensionality : } \forall F (C[F] \leftrightarrow C'[F]) \rightarrow \varepsilon FC[F] = \varepsilon FC'[F],$$

which, if added to classical first-order predicate logic or to the standard deductive (axiomatized) system of classical second-order predicate logic, is known to yield a conservative extension thereof (as follows from the Second Epsilon Theorem proven in [31]). The left-to-right direction of the axiom scheme ‘ $\exists FC[F] \leftrightarrow C[\varepsilon FC[F]]$ ’ states that, by the logic of ‘ ε ’, if some F is such that C , then $\varepsilon FC[F]$ (‘an F , such that C ’) is such that C , too. The rationale for this is that the epsilon term ‘ $\varepsilon FC[F]$ ’ is understood to ‘pick’ one of the F s that exists according to ‘ $\exists FC[F]$ ’. (If it is not the case that $\exists FC[F]$, there are no constraints whatsoever on what member of the universe of discourse is ‘picked’ by ‘ $\varepsilon FC[F]$ ’.) Extensionality adds that what is ‘picked’ by $\varepsilon FC[F]$ only depends on the extension of ‘ $C[F]$ ’, that is, on the class of all F s for which $C[F]$ is the case.

Carnap [14, 15] suggested to logically reconstruct theoretical scientific terms by defining them explicitly as epsilon terms, which he presented as a variation on Ramsification for theoretical terms. ([42] suggested much the same, except that he defined theoretical terms as definite descriptions.) I will return to this in Section 6. In the present context, the idea is to apply Carnap’s procedure to the theoretical terms ‘ T ’ and ‘true’ of semantics. Instead of thinking of Ramsey semantics to be given by the definitions of ‘interpretation’ and ‘variable assignment’, (1), (2), and the Ramsey sentence (5)/(5’), one puts forward instead the ‘simplified’ Ramsey sentence

$$\exists F(F \in Adm), \tag{8}$$

which follows logically from the Ramsey sentences (5)/(5’) from before. Afterwards, one defines ‘ T ’ explicitly with the help of the *indefinite description*

$$I = \varepsilon F(F \in Adm), \tag{9}$$

and one concludes by defining ‘true’ explicitly based on the previously defined term ‘ T ’:

$$\text{for all sentences } A \text{ in } L : A \text{ is true iff } I \models A \text{ (with ‘} T \text{’ being understood as in (9))}. \tag{10}$$

The ‘simplified’ Ramsey sentence (8), the epsilon-term definition (9) of ‘ T ’, and the definition (10) of ‘true’ are stated as semantic eigenaxioms in this reformulation of Ramsey semantics. The previous definition of ‘interpretation’ as well as definitions (1) and (2) from Section 1 are presupposed, too. Overall, this yields:

- Ramsey semantics (in its ε -operator version) consists in the definitions of ‘interpretation’ and ‘variable assignment’, (1), (2), (8)–(10), and the logic of the ε -operator.

If formulated this way, Ramsey semantics can employ the same terms ‘ T ’ and ‘true’ as in classical semantics without making temporary assumptions. The resulting version of Ramsey semantics has the following consequences, which are also known from

classical semantics: by combining (8) with the logical law

$$\exists F(F \in Adm) \text{ iff } \varepsilon F(F \in Adm) \in Adm,$$

which is an instance of the epsilon calculus axiom scheme from above, one can derive

$$\varepsilon F(F \in Adm) \in Adm. \quad (8')$$

And that statement, by definition (9), can be reformulated as

$$I \in Adm. \quad (8'')$$

Taking this together with definition (10) yields

$$I \in Adm \text{ and for all sentences } A: A \text{ is true iff } I \models A, \quad (11)$$

which constitutes a part of the classical (3) and (4) from above, and which thus becomes derivable in Ramsey semantics based on (8)–(10). Indeed, (11) may be viewed as just an analytically equivalent variant of the simplified Ramsey sentence (8) on which its derivation relies (since (9) and (10) are just definitions). For the same reason, the Ramsey sentences (5) and (5') can also be derived with the help of (8)–(10).

Equivalence (10) (the second conjunct of (11)), which coincides syntactically with the classical definition (3) of truth based on the term '*I*' for 'intended interpretation', is now short for

$$\text{for all sentences } A: A \text{ is true iff } \varepsilon F(F \in Adm) \models A, \quad (10')$$

in which ' ε ' is tied logically to existential quantification by the logical axioms of the epsilon calculus. What (10') makes transparent is that truth according to Ramsey semantics is just standard Tarskian truth relative to *an* admissible interpretation mapping that might not be determined uniquely by the existing metasemantic constraints on the interpretation of *L*, as *Adm* might include more than one member: all linguistic facts concerning competent usage of terms in *L* taken together with all non-linguistic facts concerning the satisfaction of atomic formulas in *L* might not, even with the help of all metasemantic laws, determine one and only one intended/admissible interpretation. $I(= \varepsilon F(F \in Adm))$ may still be said to be *unique* in the sense that $\exists! F(F = I)$, which is of course derivable in the classical logic of Ramsey semantics. If understood in that sense, it is even fine to speak of *the* intended interpretation *I*, so long as by that one does not mean *the by the metasemantic facts uniquely determined* interpretation: for *I* may not be the only member of *Adm*. As I have done before, I will sometimes use scare quotes for '*the*' intended interpretation to signal this ambiguity.

By deriving *I* to be a member of *Adm*, Ramsey semantics still maintains that truth *simpliciter* is preserved by classical logic; that is: if $A_1, \dots, A_n \models C$ holds in classical logic, then it follows that

$$\text{if } A_1, \dots, A_n \text{ are true, then } C \text{ is true.} \quad (12)$$

That is because: (12) may be unpacked, with (10), as

$$\text{if } I \models A_1, \dots, A_n, \text{ then } I \models C. \quad (12')$$

And (12') holds because it abbreviates, with (9),

$$\text{if } \varepsilon F(F \in Adm) \models A_1, \dots, A_n, \text{ then } \varepsilon F(F \in Adm) \models C, \quad (12'')$$

which indeed follows logically from $A_1, \dots, A_n \models C$, definition (2), and $\varepsilon F(F \in Adm)$ being a member of *Adm* (recall (8')) and therefore also being a classical interpretation function.

More generally, the role of the Ramsey sentence (8) in the ε -operator formulation of Ramsey semantics is now to claim the existential presupposition of the indefinite description in definition (9) to be satisfied, since (8) claims *Adm* is to be non-empty, which implies (8'), that is, $\varepsilon F(F \in Adm) \in Adm$.

The role of (9) and (10) is to make sure that any instance of (6) involving ‘*T*’ and/or ‘true’ can be understood as being short for

$$S [\varepsilon F(F \in Adm), \varepsilon F(F \in Adm) \models \dots], \tag{6'}$$

which from now I will take to be the official way of rendering statements $S[I, \text{true}]$ from classical semantics in the terms of Ramsey semantics.

For instance, consider the metalinguistic sentence $S[I, \text{true}]$ to be ‘*B*(*n*) is true’, where *B*(*n*) is a sentence in *L* from Example 1 of Section 1 (‘*n*’ is the number of hairs on the head of a bald person’): according to (10), *B*(*n*) is true iff $I \models B(n)$; by (9), $I \models B(n)$ iff $\varepsilon F(F \in Adm) \models B(n)$; and (1) entails that $\varepsilon F(F \in Adm) \models B(n)$ iff $(\varepsilon F(F \in Adm))(n) \in (\varepsilon F(F \in Adm))(B)$, or more briefly, $\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$. So we have that

$$\text{‘}B(n) \text{ is true’ } (S[I, \text{true}])$$

may be understood to be short for

$$\varepsilon F(F \in Adm) \models B(n) \ (S [\varepsilon F(F \in Adm), \varepsilon F(F \in Adm) \models \dots]),$$

which in turn reduces to

$$\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B).$$

Analogously, using (1) again, $\neg B(n)$ is true iff $\varepsilon F(F \in Adm)(n) \notin \varepsilon F(F \in Adm)(B)$; $B(n) \wedge \neg B(n+1)$ is true iff $\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$ and $\varepsilon F(F \in Adm)(n+1) \notin \varepsilon F(F \in Adm)(B)$, etc.

It is easy to see that such metalinguistic claims including ‘ $\varepsilon F(F \in Adm)$ ’ are sometimes logically entailed by sentences without epsilon terms, and sometimes they logically entail sentences without epsilon terms. For instance, if for all $F \in Adm$ it holds that $F(n) \in F(B)$, then this logically implies that $\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$, that is, *B*(*n*) is true in Ramsey semantics. More generally, when an arbitrary sentence *A* is true at all *F* in *Adm*—when *A* is (super-)true in supervaluationism—it follows that *A* is true (simpliciter) in Ramsey semantics, and when *A* is false at all *F* in *Adm*, then *A* is also false (simpliciter) in Ramsey semantics. Furthermore, when e.g., *B*(*n*) is true in Ramsey semantics, this logically implies that there is an $F \in Adm$, such that $F(n) \in F(B)$. In these ways and others, statements involving ‘*I*’ and ‘true’ as understood by Ramsey semantics can be predicted by, and can predict, statements without ‘*T*’ and ‘true’ (but perhaps including ‘*Adm*’). Indeed, with respect to sentences that do not involve the ‘theoretical’ terms ‘*I*’ and ‘true’, the Ramsey sentence (5) itself logically implies the same conclusions as the classical thesis ‘ $I \in Adm$ and for all sentences *A*: *A* is true iff $I \models A$ ’ does (as follows from general considerations on Ramsification: see [17]).

In all of that, usage of the epsilon operator ‘ ε ’ should not signal anything beyond the existential interpretation of ‘ ε ’ that is required by the logic of epsilon terms itself: in particular, usage of ‘ $\varepsilon F(F \in Adm)$ ’ is not meant to suggest the existence of

metasemantic facts in virtue of which ' $\varepsilon F(F \in Adm)$ ' would be able to 'pick' a *factually* uniquely determined member of *Adm* after all. Unless *Adm* is factually determined to be a singleton class, of course, in which case ' $\varepsilon F(F \in Adm)$ ' must denote that member by the epsilon calculus; but if *Adm* is *not* a singleton class, there is simply no metasemantic fact of the matter what member of *Adm* is being 'chosen' by the epsilon term ' $\varepsilon F(F \in Adm)$ '.¹⁶

In the terminology of Breckenridge and Magidor [6], one might say that the reference of ' $\varepsilon F(F \in Adm)$ ' is *arbitrary*, where 'we do not know and cannot know' ([6], p. 377) what a linguistic expression refers to when its reference has been fixed arbitrarily. But it is not merely that we do not know or could not know the particular interpretation function that is 'picked' by ' $\varepsilon F(F \in Adm)$ ': there is simply no metasemantic fact to be known in that case—it is in that sense that 'choice' is indeterminate. Breckenridge and Magidor [6] would describe this as follows: if an expression with arbitrary reference refers to *x*, then this is not determined by any non-semantic facts but only by the semantic fact that the expression refers to *x*, where that semantic fact does not supervene on non-semantic use-facts (see [34] for more on this). Whereas I would say: there is no metasemantic fact concerning which admissible interpretation ' $\varepsilon F(F \in Adm)$ ' refers to, and *there is no other kind of fact what ' $\varepsilon F(F \in Adm)$ ' refers to either, not even a semantic one*. The disadvantage of my way of expressing oneself will be that I will have to acknowledge the possibility of true sentences that are not determinately true and hence (by my understanding of 'determinate') not true in virtue of any facts. (See Section 4.) The disadvantage of their way of talking is that they have to acknowledge the possibility of semantic facts that do not obtain in virtue of any non-semantic facts. In the final Section 3 of Breckenridge and Magidor [6], they also add that there is a 'determinate fact of the matter' what a term with arbitrary reference refers to—which contradicts the way in which 'determinate' is used in the present paper in which it is reserved for a sentential operator or a semantic predicate that characterizes truth at all admissible interpretations (see Sections 3–5 and the Appendix). But these might be merely terminological differences.¹⁷

¹⁶ The choice semantics of epsilon terms [39] interprets epsilon terms by choice functions, such that each model includes a unique choice function. Thinking of such a choice function to be *uniquely determined by the facts* is precisely how one should *not* think of epsilon terms in Ramsey semantics: it is *not* presupposed that each epsilon term of the language of Ramsey semantics possesses a unique intended interpretation that is given by a factually uniquely determined choice function. Schiemer and Gratzl [59] use choice semantics when they state two truth definitions by means of epsilon terms that are interpreted by choice functions. Neither of these definitions corresponds to the definition of truth in Ramsey semantics ((9) and (10)): their notion of plain *truth* involves an existential quantification over choice functions and thereby yields a variant of sub-truth as in subvaluationism, while their notion of *universal truth* involves a universal quantification over choice functions, which yields a variant of super-truth as in supervaluationism. (Compare the previous discussion of how super-truth and sub-truth differ from truth in Ramsey semantics.)

¹⁷ Breckenridge and Magidor [6] do not develop their semantic account of arbitrary reference in formal terms, which makes it difficult to compare it to Ramsification and epsilon terms in Ramsey semantics in more detail. On the one hand, they acknowledge in their Footnote 36 the parallels between their account and Hilbertian epsilon terms in the epsilon calculus, and they also mention in their final Section 3 potential applications to structuralism, structurally indistinguishable objects, and to vagueness, which is perfectly in line with Examples 1 and 2 from Section 1 of the present paper. On the other hand, Breckenridge and Magidor [6] claim at the very end of their Section 3 that the arbitrary choice of the boundary of the vague term

What we do know according to Ramsey semantics is: if Adm is non-empty (which is claim (8) of Ramsey semantics), then $\varepsilon F(F \in Adm)$ is one of the members of Adm , by the logic of epsilon terms.¹⁸ Whereas if Adm is empty after all, the epsilon calculus does not tell us anything about the reference of ' $\varepsilon F(F \in Adm)$ ', except that, obviously, it could not be a member of the (then empty) class Adm . And that is all there is to be known about the reference of ' $\varepsilon F(F \in Adm)$ '. For the same reason, if Adm is not a singleton, we will understand the language in which Ramsey semantics is formulated, that is, the metalanguage of L , as itself including semantically indeterminate linguistic expressions, such as ' $\varepsilon F(F \in Adm)$ '. (The Appendix will explain how the indeterminacy of the metalinguistic term ' $\varepsilon F(F \in Adm)$ ' could itself be expressed formally at the metametalevel.)

Hence, (8)–(10) taken together should only be read as requiring that

$\exists F \in Adm$, such that for all sentences A , A is true iff $F \models A$,
and ' T ' refers to *that very F* in all metalinguistic contexts:
' $S[I, \text{true}]$ ' expresses that $S[F, \text{true}]$,
' $S'[I, \text{true}]$ ' expresses that $S'[F, \text{true}]$,
' $S''[I, \text{true}]$ ' expresses that $S''[F, \text{true}]$, ... ,

in which

$S[I, \text{true}], S'[I, \text{true}], S''[I, \text{true}], \dots$

enumerate all metalinguistic statements whatsoever involving ' T ' (and possibly also 'true'). Carnap's ε -method is simply a variant of Ramsification (about which [14, 15] is perfectly explicit) that allows the Ramsey semanticist to understand the theoretical term ' T ' to be Ramsified *uniformly in all metalinguistic contexts*. And when it was said before that ' $\varepsilon F(F \in Adm)$ ' 'picks an interpretation', this should only be understood as a more picturesque way of expressing the existential quantification from above.

With the help of the metalinguistic epsilon term for ' T ', Ramsey semanticists may develop their semantics while using the term ' T ' for 'intended interpretation' and the defined term 'true' just as much as their classical colleagues do. I will take that existential epsilon-term understanding of ' T ' and 'true' for granted now when comparing the semantics in the next section with its immediate rivals: supervaluationist semantics and classical semantics.

'tall' 'is consistent with the epistemicist claim that "tall" determinately refers to a particular sharp property'. If they meant by that there is a boundary point x , such that it is (by the metasemantic facts) determinate that being x cm is not tall while being $x+1$ cm is tall, then this would *not* correspond to what Ramsey semantics has to say about the Sorites Paradox in the case in which there is more than one admissible interpretation for 'tall': see Section 4.

¹⁸ Woods [81] conveys precisely that understanding of epsilon terms, which is also how Carnap [14] used and understood epsilon terms in his reconstruction of theoretical terms as epsilon terms. Carnap gives the example of the term $\varepsilon n(n = 1 \text{ or } n = 2 \text{ or } n = 3)$ about which one might wonder whether it is the case that $\varepsilon n(n = 1 \text{ or } n = 2 \text{ or } n = 3) = 1$ or not: 'there is no way of finding out the truth of this. Not because of lack of factual knowledge... Its meaning... has been specified by the ε -operator only up to a certain point', that point being that 'it is not any of those numbers which are outside of the class consisting of 1, 2, 3' ([14], p. 171).

§3. Comparison with supervaluationist semantics and classical semantics. Ramsey semantics does not only maintain the key merits of supervaluationism—validating the laws of classical logic, allowing for semantic indeterminacy, and offering a conception of semantic indeterminacy as an incomplete constraint on classical interpretations—it also avoids those features of supervaluationism that should count as shortcomings at least for the classically minded. I will argue for this by considering first the concept of truth and secondly the concept of logical consequence.¹⁹

First of all, there is just one concept of *truth* in Ramsey semantics and it is still classical, in the sense that: Ramsey semantics derives T(ruth)-biconditionals for all sentences of *L*; truth is compositional; and for every sentence, either it is true or its negation is true. None of this applies generally to (super-)truth in supervaluationist semantics. Of course, supervaluationists might add a disquotational truth predicate to their metalanguage by which they would be able to express a classical concept of truth over and above their non-classical concept of super-truth (see [47]). But then the single concept of truth from classical semantics would have to bifurcate into two concepts, and it would need to be determined for which purposes which of them is to be applied and how the two concepts relate to each other. (For instance, expanding supervaluationism and super-truth by a theory of disquotational-truth does not just by itself suffice for the derivation of ‘bridge laws’ such as ‘what is (super-)true is (disquotationally-)true’.)²⁰

Here is why truth is classical in Ramsey semantics. So far as *atomic* formulas in *L* are concerned, combining (10′) with (1) yields T-biconditionals for them; e.g.,

$$\begin{aligned} P(a) \text{ is true iff } \varepsilon F(F \in Adm) \models P(a) \\ \text{iff } \varepsilon F(F \in Adm)(a) \in \varepsilon F(F \in Adm)(P), \end{aligned}$$

¹⁹ Smith [65] suggests to distinguish supervaluationism and plurivaluationism: both rely on a class *Adm* of admissible interpretations, but supervaluationism invokes three-valued evaluations based on *Adm* whereas plurivaluationism does not. In any case, since neither of the two approaches puts forward metalinguistic Ramsey sentences, both of them differ from Ramsey semantics.

²⁰ McGee & McLaughlin’s [47] combined theory of disquotational-truth and super-truth comes close to Ramsey semantics, even though they only deal with vagueness while Ramsey semantics is devoted to all kinds of semantic indeterminacy. What their account lacks if compared to Ramsey semantics is: Ramsification. They distinguish between determinate (or definite) truth and truth *simpliciter*, as Ramsey semantics does, the only difference being that they think of ‘determinately/definitely true’ as a second type of truth predicate, whilst Ramsey semantics does not. That might be more a terminological issue, although in their review of Williamson (1994) it seems at first that McGee & McLaughlin [48] reject the T-biconditionals for ‘true’ for standard super-valuationist reasons, which would indeed conflict with Ramsey semantics. (It is only afterwards that they acknowledge that the T-biconditionals hold for disquotational truth.) More importantly, returning to McGee & McLaughlin [47], their theory assumes that determinate/definite truth entails truth *simpliciter*, just as in Ramsey semantics, which is why one would expect truth to be given by *some* admissible interpretation. However, McGee and McLaughlin do not say much about this (they only discuss this in Appendix), and in particular, they do not tell us *which* admissible interpretation/model is meant to define truth. The closest they get to Ramsey semantics is when they state “there is no model that is distinguished as the actual model” (p. 240) and when they prove at the end of their appendix that there must be *some* admissible interpretation/model, such that truth in that model corresponds to truth *simpliciter* (p. 242). Ramsey semantics *starts* from that statement and works it out on systematic grounds.

in which $(\varepsilon F F \in Adm)(a) \in (\varepsilon F F \in Adm)(P)$ may be regarded as the metalinguistic translation of the object-linguistic formula $P(a)$.

If a and P are in fact semantically *determinate*, that is, when there are d and X , such that for all $F \in Adm$ it holds that $F(a) = d$ and $F(P) = X$, and (talking *metametalinguistically* now) when d is denoted by the semantically determinate metalinguistic term ‘ a ’ and X is the extension of the semantically determinate metalinguistic predicate ‘ P ’, the biconditional from above can be replaced *salva veritate* by

$$P(a) \text{ is true iff } P(a),$$

which constitutes a more common form of a T-biconditional. (The same would also hold in supervaluationist semantics in that case.) But for atomic object-linguistic statements with semantically *indeterminate* terms, their metalinguistic translations in Ramsey semantics involve epsilon-terms that *could not* be replaced by semantically determinate terms that would determinately have the same interpretation as the epsilon terms: if $P(a)$ is semantically indeterminate, and hence there are at least two members F in Adm , then $(\varepsilon F(F \in Adm)(a) \in \varepsilon F(F \in Adm)(P))$ is indeterminate on the metalevel, too, since there are at least two values that $(\varepsilon F(F \in Adm))$ can take. Indeed, the metalinguistic epsilon terms make that potential indeterminacy of P and a explicit: Ramsey semantics is meant to clarify semantic indeterminacy and to reconcile it with classical logic and classical truth, not to eliminate it.

With respect to *complex* formulas in L , truth in Ramsey semantics is still compositional, by (1) and (10') again: e.g.,

$$\begin{aligned} C \vee D \text{ is true iff (by (10'))} \\ \varepsilon F(F \in Adm) \models C \vee D \text{ iff (by (1))} \\ \varepsilon F(F \in Adm) \models C \text{ or } \varepsilon F(F \in Adm) \models D \text{ iff (by (10'))} \\ C \text{ is true or } D \text{ is true.} \end{aligned}$$

Taking the two previous points together, it becomes clear that Ramsey semantics also proves T-biconditionals for all complex formulas in L , such as

$$\begin{aligned} P(a) \vee \neg P(a) \text{ is true iff} \\ \varepsilon F(F \in Adm) (a) \in \varepsilon F(F \in Adm)(P) \\ \text{or } \varepsilon F(F \in Adm)(a) \notin \varepsilon F(F \in Adm)(P), \end{aligned}$$

and

$$\exists x P(x) \text{ is true iff } \exists d \in U, \text{ such that } d \in \varepsilon F(F \in Adm) (P).$$

Why is that important? Tarski [70] intended all T-biconditionals for the object language (L) to be derivable from his theory of truth(-in- L) (the so-called criterion of *material adequacy*) in order to make sure the theory got the extension of ‘true(-in- L)’ right. The same may be said in the present context, except that, as mentioned above, some of the T-sentences include epsilon terms that cannot be eliminated in favor of metalinguistic expressions with a unique factually determined intended interpretation. In partial compensation, Ramsey semantics may still derive *semantic constraints* on ‘the’ intended truth conditions of such sentences based on descriptions of the *existing metasemantic constraints* on the interpretation of L , so long as these descriptions are derivable metatheoretically as constraints on Adm .

For instance, assume that one can derive in one's metatheory that for all $F \in Adm$: $F(0) = 0$; $F(100000) = 100000$; $0 \in F(B)$; $100000 \notin F(B)$; for all m, n , if $m > n$ and $m \in F(B)$ then $n \in F(B)$; and $0, 100000 \in Uni(I) = U$. Then, from this, jointly with the T-biconditional

$$B(0) \text{ is true iff } \varepsilon F(F \in Adm)(0) \in \varepsilon F(F \in Adm)(B)$$

and $\varepsilon F(F \in Adm) \in Adm$ ((8')), Ramsey semantics proves that

$$B(0) \text{ is true;}$$

analogously, it proves that $\neg B(100000)$ is true, and also the semantic constraint:

$$\text{for all } x, y, \text{ if } x > y \text{ and } B(x) \text{ is true, then } B(y) \text{ is true.}$$

Of course, all these statements would be derivable in a supervaluationist metatheory, too, if based on the same statements concerning all $F \in Adm$.

However, Ramsey semantics proves additional semantic theorems for truth that supervaluationist semantics cannot derive for (super-)truth: e.g., in virtue of its compositional semantic rules, Ramsey semantics proves the semantic law

$$\text{for all } x, \text{ either } B(x) \text{ is true or } \neg B(x) \text{ is true (but not both),}$$

by which ' $B(x)$ is true' is provably equivalent to ' $\neg B(x)$ is not true'. Hence, one may replace the former by the latter in the previously derived constraint and prove that

$$\text{for all } x, y, \text{ if } x > y \text{ and } \neg B(x) \text{ is not true, then } B(y) \text{ is true.}$$

None of these additional theorems is derivable in a supervaluationist semantics in which 'true' expresses super-truth. For the same reason, no sentence A of L lacks a classical truth value in Ramsey semantics, since one can derive from (1) and (10') the classical semantic law

$$\text{for all sentences } A \text{ in } L: \text{ either } A \text{ is true or } \neg A \text{ is true,}$$

which cannot be derived in supervaluationist semantics either, as supervaluationism countenances (super-)truth value gaps.

In a nutshell, truth in Ramsey semantics is classical, whilst (super-)truth in supervaluationist semantics is not: super-truth is *not* compositional (e.g., the super-truth of $P(a) \vee \neg P(a)$ does *not* entail that $P(a)$ is super-true or that $\neg P(a)$ is super-true), it does *not* support (super-)T-biconditionals for all sentences (in the sense that A may be true at some F in Adm without being super-true), and it may well leave a sentence A without (super-)truth/(super-)falsity values (when neither A nor $\neg A$ is super-true).

Secondly, logical consequence is defined classically as *truth preservation* in Ramsey semantics and validates *classical logic* completely: not just the theorems and rules of classical logic (such as the Excluded Middle or Modus Ponens) but also its metarules (such as Conditional Proof, Reductio ad Absurdum, Proof by Cases, and metarules for sequents with multiple conclusions). In contrast, either supervaluationism does not define logical consequence as (super-)truth preservation, or if it does, logical consequence does not fully validate classical logic.

Here is why logical consequence and truth relate classically in Ramsey semantics: as explained in Section 2, logical consequence is still defined classically ((2)) as preservation of truth relative to all (classical) interpretations: $A_1, \dots, A_n \models C$ iff $\forall F, s$, if $F, s \models A_1, \dots, A_n$, then $F, s \models C$. What Ramsey semantics adds to this is to define

I as $\varepsilon F(F \in Adm)$ (9), where Adm is a non-empty class of classical interpretations (8), from which it follows that $I \in Adm$ (recall Section 2). Thus, I (that is, $\varepsilon F(F \in Adm)$) is one of the interpretations F over which the universal quantifier in (2) ranges and by which both the formulas to the left and to the right of the logical consequence symbol ‘ \models ’ are interpreted. Since truth is defined by ‘ $I \models \dots$ ’ in Ramsey semantics (see (10)), it is itself an instance of ‘ $F \models \dots$ ’ in (2), which is why truth and logical consequence ‘fit together’ in Ramsey semantics in precisely the same way in which they do in classical semantics. In contrast, super-truth does not ‘fit together’ with so-called *local* supervaluationist definitions of logical consequence that are not based on the preservation of supervaluationist (super-)truth but which are instead defined, for each admissible interpretation, to preserve truth-at-it: see e.g., Williamson [76] for a criticism.²¹

What is more, the classical metarules also remain logically valid in Ramsey semantics if one expands the logical vocabulary of L by a new sentential operator *Det* (for ‘determinately’): in order to do so, change ‘ $F, s \models \dots$ ’ everywhere into an ‘ $F, s, X \models \dots$ ’ format (where ‘ X ’ denotes a non-empty class of interpretations) and augment the semantic rules by

$$F, s, X \models Det(A) \text{ iff } \forall F' \in X : F', s, X \models A.$$

(If A is a sentence, ‘ s ’ may be dropped again.) The original definition (10) of truth can be adapted to the presence of the new logical operator *Det* by means of: A is true iff $I, Adm \models A$.

The operator *Det* is familiar from supervaluationist theories in which it expresses the metalinguistic concept of (super-)truth by object-linguistic means. But adding it to the object language is known to *undermine* the logical validity of some classical metarules if logical consequence is defined *globally* as preservation of super-truth: e.g., $A \models Det(A)$ holds in supervaluationist semantics with global logical consequence, while $\models A \rightarrow Det(A)$ fails, thus invalidating Conditional Proof (see [25], p. 290, [76], pp. 151–3).

In contrast, *Det(A)* does not express in Ramsey semantics that A is true but rather that competent-usage facts and relevant non-linguistic facts, governed by metasemantic laws, determine A to be true: the truth of A is entailed by what the existing metasemantic constraints are like. Indeed, for all sentences A in L it holds: if *Det(A)* is true, then A is true,²² but not necessarily the other way around. When n is a borderline case of the vague term B , and thus it is neither the case that for all $F \in Adm$ $n \in F(B)$ nor that for all $F \in Adm$ $n \notin F(B)$, it follows that the conjunction $\neg Det(B(n)) \& \neg Det(\neg B(n))$ is true, while either $B(n)$ is true or $\neg B(n)$ is true according to Ramsey semantics. But what is most important for present logical purposes: the presence of *Det* in the object language does *not* affect the logical validity of any of the classical metarules. E.g., the metalinguistic proof of ‘if $A \models B$, then $\models A \rightarrow B$ ’ goes through in Ramsey semantics in precisely the same way in which it does in classical semantics, independently of

²¹ Field [24] has argued recently that logical consequence does *not* necessarily preserve truth, but that was in the special context of a semantically closed language with paradoxical sentences, such as the Liar sentence.

²² That is because *Det(A)* is true just in case $I, Adm \models Det(A)$, which is equivalent to $\forall F' \in Adm: F', Adm \models A$, which entails by universal instantiation that $I, Adm \models A$ and hence by (10) from Section 2 again that A is true.

whether the object language (and hence A or B) includes Det or not. Such are the logical advantages of Ramsey semantics over supervaluationist semantics. (But see [35, 72] for defenses of supervaluationism against these challenges, and see [19, 72] for further supervaluationist notions of consequence beyond the local and the global one.)

By preserving classical truth, classical logic, and their classical relationship, Ramsey semantics is much closer to classical semantics than supervaluationism, and perhaps as close as is possible without committing oneself to the unique satisfiability of ' $F \in Adm$ '. This should put Ramsey semantics ahead of supervaluationism in the eyes of anyone who is attracted by classical semantics, who wants to grant the possibility of semantic indeterminacy, and who therefore aims to approximate classical semantics while being prepared for semantic indeterminacy.

So far as the comparison with classical semantics itself is concerned, it is easy to see what is gained by switching from classical semantics to Ramsey semantics: the problematic metasemantic presupposition of semantic determinacy ($\exists!F(F \in Adm)$) is avoided. It is much more difficult to see what is lost by moving to Ramsey semantics, since it preserves so many of the central theoretical features of classical semantics. Indeed, other than ' $\exists!F(F \in Adm)$ ', the only apparent difference between the two semantics concerns the presentation and structure of their axioms and the appearance of epsilon terms in Ramsey semantics on the right-hand side of T-biconditionals for atomic formulas. Since, given the epsilon calculus, postulates (8)–(10) prove (11), that is, ' $I \in Adm$ and for all sentences A : A is true iff $I \models A$ ' (recall Section 2), every statement that a classical semanticist logically derives from that thesis must also be derivable in Ramsey semantics. (Though these consequences may not always have the same meaning, as ' I ' is understood differently in Ramsey semantics than in classical semantics.). In other words: on purely deductive grounds, one is never going to notice the difference between classical semantics and Ramsey semantics so long as the classical semanticist does not explicitly invoke its only additional presupposition ' $\exists!F(F \in Adm)$ '. It is precisely that assumption by which classical semantics exceeds Ramsey semantics in deductive strength: but semanticists do not normally exploit that assumption deductively in their semantic work but merely presuppose it, and Ramsey semantics happily abandons that presumption in order to accommodate semantic indeterminacy if and when necessary.

In the remaining sections, I will describe the ramifications that Ramsey semantics has for the Sorites, higher-order vagueness, and interpretational change and continuity. This, I hope, will add to the attractiveness of the semantics. Taking everything into account, Ramsey semantics should thus cross the finishing line as the top contender amongst theories that aim to meet the challenge from semantic indeterminacy on (quasi-)classical grounds.

§4. The Sorites paradox. What does Ramsey semantics predict concerning the infamous Sorites paradox?

If formulated for the predicate B (for baldness) in L from Example 1, and with quantifiers ranging over the natural numbers, the paradoxical argument is:

$$B(0) \tag{13}$$

$$\forall x(B(x) \rightarrow B(x + 1)) \tag{14}$$

$$\text{Therefore, } B(100000). \tag{15}$$

Whilst (13) is obvious and (14) sounds plausible at least at first glance, (15) is absurd, and yet the argument is logically valid in classical logic—hence the paradox.

Indeed, in Ramsey semantics, (13) is true ($I \models B(0)$) and (15) is false ($I \models \neg B(100000)$): recall the description of the metasemantic constraints at the beginning of Section 2 which hold for all members F of Adm , and use $I = \varepsilon F(F \in Adm)$ (9) and $\varepsilon F(F \in Adm) \in Adm$ ((8')). Since Ramsey semantics also preserves the classical logical validity of the argument (see Section 3), (14) must be the culprit: the Sorites is not sound since (14) is false, that is,

$$\varepsilon F(F \in Adm) \not\models \forall x(B(x) \rightarrow B(x + 1)), \tag{16}$$

which, by (1), is equivalent to

$$\varepsilon F(F \in Adm) \models \neg \forall x(B(x) \rightarrow B(x + 1)), \tag{17}$$

or equivalently again:

$$\varepsilon F(F \in Adm) \models \exists x(B(x) \ \& \ \neg B(x + 1)). \tag{18}$$

That is like the supervaluationist diagnosis, except that supervaluationists regard the existential object-linguistic claim from (18) as (super-)true without necessarily regarding any of its *instances* as (super-)true, which conflicts with the pre-theoretically plausible principle that ‘a true existential generalization has a true instance’ ([76], p. 154).

In contradistinction, *there is a true instance of the true existential claim* $\exists x(B(x) \ \& \ \neg B(x+1))$ according to Ramsey semantics, since definition (1) implies that (18) is equivalent to

$$\exists n \in U, \text{ such that: } \varepsilon F(F \in Adm), s \frac{n}{x} \models B(x) \ \& \ \neg B(x + 1) \tag{19}$$

and also to

$$\exists n \in U, \text{ such that: } n \in \varepsilon F(F \in Adm) (B) \text{ and } n + 1 \notin \varepsilon F(F \in Adm) (B), \tag{20}$$

both of which state the existence of a true instance of the true existential claim from (18).

On the negative side: doesn't (20) say that there is a sharp cut-off point for B , which would seem counterintuitive again? This depends on what is meant by ‘sharp cut-off point’: Ramsey semantics is certainly not committed to the existence of a sharp *factually determined* cut-off point, for (20) does not mean that the *existing metasemantic constraints on the interpretation of L would determine* a particular instance of the existential claim in (18) to be true. That is, using the *Det* operator from Section 4: Ramsey semantics does *not* assume that there is an $n \in U$, such that $\varepsilon F(F \in Adm), s \frac{n}{x}, Adm \models Det(B(x) \ \& \ \neg B(x+1))$, as the boundary of B may be semantically indeterminate. Where supervaluationists interpret *Det* as expressing (super-)truth and hence regard metalinguistic statements about *Det* as statements about (super-)truth, Ramsey semanticists keep ‘determined to be true by the facts’ and ‘true’ separate: the former is about metasemantic determination, only the latter is about *truth*. While an existential statement is true just in case one of its instances is, the existing metasemantic constraints might determine Adm to be non-empty—here: determine the existence of various admissible cut-off points for B —without determining which admissible cut-off point is ‘the’ intended one. Or in more logical terms: ‘true’ commutes with the

existential quantifier, whereas *Det* does not necessarily do so (as pointed out by [47], p. 212): if *B* is semantically indeterminate, $Det(\exists x(B(x) \ \& \ \neg B(x+I)))$ will be true in Ramsey semantics while $\exists x(Det(B(x) \ \& \ \neg B(x+I)))$ will be false.

But doesn't (20) at least say that there must be a sharp cut-off point for *B* in the following weaker sense that does not involve the '*Det*' operator: there is a natural number *n*, such that every number of hairs less than or equal to *n* counts as bald according to the intended interpretation of *B*, and every number of hairs greater than *n* counts as non-bald?

Now the answer is of course *yes*, since this is just a restatement of (20) (for $I = \varepsilon F$ ($F \in Adm$)), to which Ramsey semantics is committed. But one should add that Ramsey semantics uses '*T*' ('intended interpretation') as a short-hand for ' $\varepsilon F(F \in Adm)$ ', and statements involving ' $\varepsilon F(F \in Adm)$ ' are themselves just stand-ins for existential claims. (Recall Section 2.) In particular, deriving a statement such as (20) should merely be regarded as a short-hand for deriving the existential claim

$$\begin{aligned} \exists F \in Adm, \text{ such that for all sentences } A, A \text{ is true iff } F \models A, \\ \text{and } \exists n \in U, \text{ such that: } n \in F(B) \text{ and } n + 1 \notin F(B), \end{aligned} \quad (21)$$

from (8)–(10), the epsilon calculus, and the fact that for all $F \in Adm$ it holds that $\exists n \in U$, such that $n \in F(B)$ and $n + 1 \notin F(B)$ (which follows from the assumptions on *Adm* that were described at the beginning of Section 2).

Jointly with the semantic determinacy of *B*, that is, if the existing metasemantic constraints were to determine the extension $I(B)$ uniquely—if all members *F* of *Adm* were to assign the same extension to *B*—the plain existential statement (21) would indeed commit the Ramsey semanticist to the existence of a sharp cut-off point for *B* in every sense of the term: which is fine, since in that case '*I(B)*' would refer to a uniquely factually determined set-theoretic extension with a precise cut-off. In that case, Ramsey semantics for *B* would simply collapse into epistemicism about *B*, assuming epistemicists would still be right that we would not know exactly what that extension $I(B)$ of *B* would be like.

However, if the existing metasemantic constraints do *not* determine $I(B)$ uniquely, and hence *Adm* includes more than one *F*, (21) is still claimed true by Ramsey semantics but is *only* committed to the *existence of an admissible interpretation with a sharp cut-off point for B*. And that reduces to a mere commitment to the existence of an admissible interpretation, as *all* interpretations in *Adm* assign—by their classical nature—a sharp cut-off point to *B*. In this way, Ramsey semantics takes the bite out of (21) and hence (20). Statement (20) is accepted by Ramsey semanticists but only expresses the *Ramsification* of what classical semanticists would mean by 'the intended interpretation involves a sharp boundary for baldness'.

Perhaps the tolerance intuition (cf. [83]) that lends support to the major premise (14) of the Sorites is itself but a reflection of there being no fact of the matter about which member of *Adm* is denoted by ' $\varepsilon F(F \in Adm)$ ' in the case of semantic indeterminacy. For assume again the interpretation of *B* is semantically indeterminate, that is, there exist at least two members of *Adm* that assign different extensions to *B*: in that case, there must also be interpretation functions *F* and F^+ in *Adm*, such that

$$\text{for all } n, \text{ if } n \in F(B) \text{ then } n + 1 \in F^+(B).$$

(This follows easily from semantic indeterminacy and from ‘for all m, n , if $m > n$ and $m \in F(B)$ then $n \in F(B)$ ’ expressing an existing metasemantic constraint on all $F \in Adm$.) Since both F and F^+ are members of Adm , each of them could serve as the referent of ‘ $\varepsilon F(F \in Adm)$ ’, that is: the existential statements for which statements of the form ‘ $S[\varepsilon F(F \in Adm, \text{true})]$ ’ serve as stand-ins quantify both over F and F^+ (and all other members of Adm). It is therefore not far off the truth to summarize the situation by (14), even though strictly speaking the B in the antecedent of the embedded conditional of (14) needs to be interpreted by a different member of Adm (say, F) than the B in the consequent (say, F^+).

One may summarize the different verdicts by classical semantics, supervaluationist semantics, and Ramsey semantics concerning the existence of borderline cases of the vague term B (‘bald’) as follows:

- Classical semantics takes the relevant borderline facts to be complete (‘ $\exists x Det(B(x) \& \neg B(x+I))$ ’ is true) and employs a *classical* concept of truth (‘ $\exists x(B(x) \& \neg B(x+I))$ ’ is true, hence $\exists n$, s.t. ‘ $B(x) \& \neg B(x+I)$ ’ is true of n).
- Supervaluationist semantics takes the borderline facts to be incomplete (‘ $\exists x Det(B(x) \& \neg B(x+I))$ ’ is super-*false*) and invokes a *non-classical* concept of truth (‘ $\exists x(B(x) \& \neg B(x+I))$ ’ is super-true, but *not* $\exists n$, s.t. ‘ $B(x) \& \neg B(x+I)$ ’ is super-true of n).
- Ramsey semantics takes the borderline facts to be incomplete (‘ $\exists x Det(B(x) \& \neg B(x+I))$ ’ is *false*) but uses a *classical* concept of truth (‘ $\exists x(B(x) \& \neg B(x+I))$ ’ is true, hence $\exists n$, s.t. ‘ $B(x) \& \neg B(x+I)$ ’ is true of n).

The position taken by Ramsey semantics vis-à-vis the borderline cases of vague terms is therefore ‘half way in between’ classical semantics and supervaluationism: it agrees with supervaluationism that the existing metasemantic constraints are likely to leave vague terms with *factual* borderline gaps, while agreeing with classical semantics that the *concept* of truth is classical, such that, e.g., it obeys the standard compositional clauses. Classical semantics presupposes that there are no metasemantic gaps, supervaluationism acknowledges metasemantic gaps and translates them into semantic ones, while Ramsey semantics acknowledges metasemantic gaps but avoids translating them into semantic gaps.

The Ramsey semanticist would therefore summarize the situation concerning the vague term B as follows: there are sentences involving B , such as $B(0)$, which the metasemantic facts determine to be true and which are thus true. There are sentences involving B , such as $B(100000)$, which the metasemantic facts determine to be false and which are hence false. The Sorites argument is logically valid but not sound: its major premise is false. Since it is likely that the metasemantic facts do not determine the intended interpretation of B uniquely, it is likely that there is a sentence of the form $B(n)$ for some number n in between 0 and 1000000, which is not determined to be true by the metasemantic facts but which is still true, while $B(n+1)$ is not determined to be false by the metasemantic facts but still false. Both n and $n+1$ would be available as admissible cut-off points for baldness (and maybe others) and there is no metasemantic fact of the matter which of them (if either of them) is ‘picked’ by the epsilon term ‘ $\varepsilon F(F \in Adm)$ ’ by which the intended interpretation I is defined in Ramsey semantics. That is also why the major premise of the Sorites sounded plausible initially.

These should be reasonable verdicts. Whether or not the existing metasemantic constraints determine ‘the’ intended interpretation of L uniquely, Ramsey semantics

stays on the safe side and yet points to a way out of the Sorites that should be attractive to anyone who is inclined towards classical semantics at all.

§5. Higher-order vagueness. If the first-order extensional object language L from Example 1 in Section 2 is indeed semantically indeterminate in virtue of terms such as B being vague: does that mean that there must also be higher-order vagueness?

I have already explained in Section 2 that if Adm is not a singleton set, ' $\varepsilon F(F \in Adm)$ ' is meant to be semantically indeterminate, so that there is no fact of the matter which member of Adm is 'picked' by that epsilon term. Thus, in that case, there will certainly be semantically indeterminate linguistic expressions in the very metalanguage of L in which Ramsey semantics is developed. I will deal with the formalization of that kind of metalinguistic indeterminacy in the Appendix.

For now I only want to deal with the potential *indeterminacy of the indeterminacy of the object language L with vague terms*, which corresponds to what is more usually understood by higher-order vagueness. In other words: consider metalinguistic semantic predicates such as 'is determinate(-in- L)' and 'is an (L -)borderline case of', which express semantic properties/relations of sentences of the vague object language L , as in: ' $B(0)$ is determinate', ' $\neg B(100000)$ is determinate', and ' n is a borderline case of B ', that is, ' $B(n)$ is not determinate and $\neg B(n)$ is not determinate'. Unlike the previous sections, let us make such claims precise by a metalinguistic determinacy predicate ' Det ' of sentences instead of the previous sentential determinacy operator, in order not to leave the realm of standard first-order extensional languages. (Otherwise we would first have to extend Ramsey semantics for the strictly extensional language L from Section 2 to non-extensional sentential operators.) The question is thus: are semantic predicates such as ' Det ' semantically indeterminate themselves? (See [84] for arguments to the contrary.) And if so, how would Ramsey semantics deal with that?

In the present context, in which such a metalinguistic ' Det ' predicate for sentences in L will be defined metalinguistically in analogy with the determinacy operator from previous sections, that is, by

$$\text{for all sentences } A \text{ in } L: Det(A) \text{ iff for all } F \in Adm, F \models A, \quad (22)$$

the only way for ' Det ' to be vague would be for ' Adm ' to be vague, too. And since ' Adm ' was characterized as the class of classical interpretations F that conform to the existing metasemantic constraints on the interpretation of L , ' Adm ' is vague only if the metalinguistic term 'conforms to the existing metasemantic constraints on the interpretation of L ' is vague. *So does that term have a factually uniquely determined intended interpretation?*²³

If yes, ' Adm ' would have a factually determined unique intended interpretation, and the same would hold for (8)–(10), and for ' Det ' as defined in (22); hence, Ramsey semantics would steer clear of higher-order vagueness. None of this would affect any of the considerations from previous sections: for the semantic determinacy of

²³ The question is not whether the existing metasemantic constraints themselves are vague, which would be yet another form of metaphysical vagueness: a vagueness of metasemantic facts or metasemantic laws. The question is rather whether the way in which the metalanguage of L describes these constraints is vague.

the metalinguistic term ‘*Adm*’ would still allow for the interpretation of the *object* language *L* to be semantically *indeterminate* in the sense that *Adm* includes more than one interpretation mapping *F*. If so, the relevant metalinguistic epsilon term ‘ $\varepsilon F(F \in Adm)$ ’ of Ramsey semantics will still be semantically indeterminate in virtue of it being indeterminate which member of *Adm* gets ‘chosen’ by the term (which is to be distinguished from ‘*Adm*’ itself being indeterminate). In terms of our previous analogy: even when the equation ‘ $z^2 = x + y$ ’ has a uniquely determined reading over the real numbers, this still permits the existence of more than one real-valued solution for ‘*z*’ given $x = 3$ and $y = 1$.

But what if the answer to the question above is *no*? In that case, more than just one interpretation can be assigned to ‘conforms to the existing metasemantic constraints on the interpretation of *L*’ without invalidating any of the existing *metametase* semantic constraints on *its* interpretation. If so, ‘*Adm*’ is semantically indeterminate, the same is presumably true for (8)–(10), and Ramsey semantics will have to cope with higher-order vagueness. In our analogy again: ‘ $z^2 = x + y$ ’ might tacitly include some additional parameters that need to be set appropriately before the equation can be applied, which is why ‘ $z^2 = x + y$ ’ actually represents a whole *family* of equations each of which comes with its own set of solutions.

So which answer is it? This depends on the metasemantic properties of the object language in question.

For instance, consider the purely mathematical language *L*’ from Example 2 in Section 1, even when strictly speaking the example did not concern vagueness: it is quite possible that the corresponding class *Adm*’ can be characterized completely in purely class-theoretic terms, e.g., as the class of *all* interpretations that satisfy the second-order Dedekind-Peano axioms, and that any potentially remaining indeterminacy concerning the class-theoretic membership predicate ‘ \in ’ can be safely ignored when dealing with the countably infinite matters of arithmetic (rather than, say, large-cardinal issues). If so, whether for systematic reasons or for ‘all practical purposes’, there should not be much room left for higher-order indeterminacy. In contrast, if *Adm*’ were e.g., characterized as the class of all those interpretations satisfying the second-order Dedekind-Peano axioms *that are additionally easily definable set-theoretically*, then ‘easily’ would be likely to be vague, and hence there should be higher-order vagueness. Ramsey semantics should be prepared for such possibilities, too.

More urgently, as far as the formalization of some fragment of *natural* language with vague terms is concerned, such as language *L* from Example 1, it is not even clear how one could tell which of the two answers from above applies, and there might be principal reasons for why we *could not* tell which of them applies. Therefore, once again, Ramsey semantics should at least be prepared for higher-order vagueness.

And it is not hard to see it is. For the Ramsification ‘scheme’ may simply be iterated at higher levels in the Tarskian hierarchy: a *metametalinguistic* epsilon term is used to express how ‘the’ intended interpretation of ‘*Adm*’ is picked from a class *Adm*₂ of interpretations of the *metalinguage* of *L*; a *metametametalinguistic* epsilon term is used to express how ‘the’ intended interpretation of ‘*Adm*₂’ is picked from a class *Adm*₃ of interpretations of the *metametalinguage* of *L*; and so forth. Or in other words: *I* is a member of *Adm*, which in turn is a member of *Adm*₂, which in turn is a member of *Adm*₃, and so on. Picture such choices being carried out ‘simultaneously’ at all levels of the Tarskian hierarchy, such that at each level there might be *metameta...*-semantic facts constraining the ‘process’, and where the outcomes of that ‘process’ might not

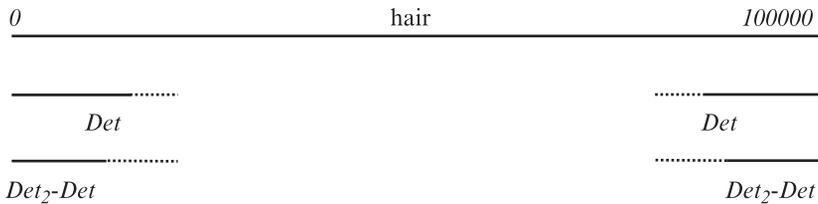


Fig. 1. Determinacy pattern for B (baldness).

be determined uniquely at any level. As [47] (p. 230) point out, it is at least possible that ‘As we ascend the hierarchy of metalanguage, we find vagueness all the way up’—though in the present case, in which such a hierarchy is erected on top of a first-order language L with ordinary vague terms and a fixed first-order universe of discourse, one might also expect Adm_α to become a singleton set from some ordinal level α and to remain so at all levels $\beta > \alpha$. (But what that ‘stabilization ordinal’ α would be like might itself be indeterminate.)

In any case, on each language level, a corresponding determinacy predicate may be introduced again in the expected manner; e.g., for all sentences A of the *metalanguage* of L :

$$Det_2(A) \text{ iff for all } F_2 \in Adm_2, F_2 \models A.$$

Previously, ‘ $Det(A)$ ’ expressed metalinguistically that the sentence A of L comes out true in whatever way a classical interpretation F satisfies all existing metasemantic constraints on the interpretation of L . Similarly, one may now employ ‘ $Det_2(‘Det(A)’)$ ’ to express *metametalinguistically* that ‘ $Det(A)$ ’ comes out true in whatever way a classical interpretation F_2 satisfies all existing *metametase*semantic constraints on the interpretation of the *metalanguage* of L , including the *metalinguistic* term ‘ Adm ’. Combining determinacy predicates and truth predicates, one may express multiply iterated determinacy claims at one and the same language level and prove in Ramsey semantics how they interact. E.g., at the *metametalevel* level, one can prove that the determinacy of the determinacy of $B(n)$ implies the determinacy of $B(n)$, but one cannot in general prove the converse. (See the Appendix for the details.) Thus, the expected determinacy pattern emerges for B —the determinacy area for B is a superset of the determinacy–determinacy area for B , and each area exhibits indeterminate boundaries (Figure 1).

If, on each language level, the Ramsey semanticist can rely on sufficiently many *metameta...-semantic* findings concerning the respective extensions of ‘ Adm_i ’—whether regarding these findings as likely or firmly accepting them as true or even adding them as axioms to the respective *metameta...-theories*—the resulting theoretical package should allow her to draw non-trivial conclusions on higher-order vagueness. If so, at each level, the Ramsification of classical semantics will be informative, and the Ramsification at the level will be consistent if classical semantics is; no ‘infinite regress’ in any bad sense of the term will emerge. No strong assumptions will be required other than postulating the existence of an admissible interpretation at each level of the Tarskian hierarchy. Nor will there be any novel phenomena over and above those

covered by previous sections. Higher-order vagueness does not constitute a greater challenge to Ramsey semantics than first-order vagueness.²⁴

§6. Interpretational change and continuity. Let me finally return to the mathematical and scientific Examples 2 and 3 from Section 1: by now, it should be clear what their Ramsey semantics will have to look like. This will prompt more general considerations on diachronic and synchronic referential/extensional continuity of terms from different languages and how this is compatible with Ramsey semantics.

Example 2 (reconsidered): Let L' again be a second-order formalization of the language of arithmetic. Let us assume the class Adm' of its admissible interpretations is identical to the class of all classical interpretations that satisfy the second-order Dedekind-Peano axioms (and which are thus pairwise isomorphic):

$$Adm' = \{F : F \models PA_2\}.$$

What Ramsey semantics for L' adds to this is:

$\exists F'(F' \in Adm')$. (In fact, this would not have to be added, since it follows set-theoretically from the equality above.)

$$I' =_{df} \varepsilon F'(F' \in Adm').$$

For all sentences A in L' : A is true(-in- L') iff $I' \models A$.

The resulting semantics takes the structuralist conception of arithmetic seriously according to which only the structure of the interpretation of the arithmetical symbols is determined factually. And it combines this with a unique intended (though not *factually* uniquely determined) set $Uni(I')$ of natural numbers and with a classical concept of truth for the language of arithmetic that supports classical logic as used by number theorists. Acknowledging semantic indeterminacy in that way does not spell any trouble whatsoever for the Ramsey semantics of the language of second-order arithmetic.

On the contrary, the semantics has some noteworthy consequences: the epsilon calculus and the definitions of 'I'' and 'true' above just by themselves entail (recall Section 2) that

the second-order Dedekind-Peano axioms are true iff $\exists F'(F' \in Adm')$,

which, since Adm' is the class of *all* classical interpretations that satisfy the second-order Dedekind-Peano axioms, can be reformulated as:

the second-order Dedekind-Peano axioms are true iff they are satisfiable.

²⁴ In their argument against supervaluationism, Fodor and Lepore [26] worry that admissible interpretations are bound to invalidate certain conceptual truths, such as 'Someone with hair of number and configuration so-and-so is a borderline case of baldness'. As explained in this section, such a statement belongs to the *metalanguage* of L , since 'borderline' is a metalinguistic semantic predicate, and I have assumed the object language L not to include any semantic terms itself. Requiring such a conceptual truth to be satisfied is therefore a *metametase*semantic constraint that ought to be respected by all admissible interpretations of the *metalanguage* of L . The methods from this section and from the Appendix demonstrate how such constraints can be implemented in Ramsey semantics.

In more traditional terminology: the structuralist Ramsey semantics for L' yields that the truth of the second-order Dedekind-Peano axioms is tied analytically to their satisfiability—which resembles in some way (though only in *some* way) how Hilbert [29] had famously characterized mathematical truth in his exchange with Frege.

Example 3 (reconsidered): Let L'' formalize again the language of Newtonian mechanics. The class Adm'' of admissible interpretation of L'' is likely to include at least two members, where one member F of Adm'' interprets ‘mass’ as relativistic mass, while another member F' of Adm'' interprets it as proper mass. Let us assume that these are the only members of Adm'' :

$$Adm'' = \{F, F'\}.$$

Ramsey semantics for L'' postulates:

$\exists F''(F'' \in Adm'')$. (Once again, this would not actually have to be stated, since it follows set-theoretically from the equality above.)

$$I'' =_{df} \epsilon F''(F'' \in Adm'').$$

For all sentences A in L'' : A is true(-in- L'') iff $I'' \models A$.

In this way, Ramsey semantics applies the classical concept of truth to the language of Newtonian mechanics and interprets the logic of Newtonian arguments and proofs to be classical, while granting that, to the best of our present physical knowledge, it is not possible to ascribe the Newtonian term ‘mass’ a unique factually determined intended reference. In the terminology of Field [23], Newtonian ‘mass’ only *refers partially*, which Field himself makes precise in a supervaluationist manner, while the Ramsey semantics from above does so by Ramsifying classical semantics.

For instance, Ramsey semantics for L'' derives from the postulates above that

$$I''(m) = F(m) \text{ or } I''(m) = F'(m),$$

without deriving either of the disjuncts (recall Section 2), which seems like the right prediction to make.

Example 3 concerned *revolutionary* language change in the following sense: one is giving a semantics for the language of a scientific theory that belongs to an earlier scientific paradigm (cf. [38]), and one does so by using terms from the language of a scientific theory that belongs to a contemporary scientific paradigm. More generally, Ramsey semantics offers semanticists the resources to study interpretational change and diachronic continuity through all stages of scientific development, including *normal* ones in which the central theoretical terms remain to be used in similar ways. In either case, scientific realists would expect referential/extensional continuity between empirically successful successor theories, that is, the references/extensions of theoretical terms in successful theories should be preserved in the transition to their successful successor theories (see [55]).

Carnap’s [14, 15] proposal to view theoretical terms as definable by epsilon terms (which he employed object-linguistically while Ramsey semantics uses them metalinguistically) was meant to support similar continuity considerations, but without the presupposition that a unique intended interpretation has been determined factually.

Instead, he meant to capture the open-ended, increasing, and yet partial specification of theoretical terms in scientific progress (as [56] had done before):

this definition [gives] just so much specification as we can give, and not more. We do not want to give more, because the meaning should be left unspecified in some respect, because otherwise the physicist could not—as he wants to—add tomorrow more and more postulates, and even more and more correspondence postulates, and thereby make the meaning of the same term more specific than [it is] today. So, it seems to me that the ε -operator is just exactly the tailor-made tool that we needed, in order to give an explicit definition, that, in spite of being explicit, does not determine the meaning completely, but just to that extent that it is needed. ([14], pp. 171–172)

Of course, present-day scientific realists would not buy into Carnap's overly descriptivist conception of scientific theoretical terms here. But so long as the linguistic metasemantic facts underlying the metasemantics of theoretical terms involve at least *some* theoretical component—as acknowledged, e.g., by combined causal-theoretical accounts of theoretical terms (see [53], Chapter 12)—and hence some scientific theory is at least partially constitutive of these terms, it is plausible that the interpretation of such terms may remain partial in virtue of the theory being deductively incomplete or vague or both. What is more, the causal or ostensive components of the metasemantics of theoretical terms may themselves give rise to semantic indeterminacy: reference-fixing phrases such as 'whatever causes phenomenon so-and-so shall be called...' may be semantically indeterminate by leaving 'causal distance' and the relevant type of causality open; and an act of ostension may be directed simultaneously towards a plurality of objects, kinds, and physical structures, such that neither intentions nor interests may suffice to rule out all but one potential target. For the same reason, it should be possible to extend the metasemantic constraints on the interpretation of a theoretical term by extending or 'precisifying' the theory that partially characterizes it, or by diambiguating the term's reference-fixing description, or by excluding relevant causal pathways as semantically unintended: in all of these cases, the class of admissible references/extensions of theoretical terms may be expected to change.

If so, Ramsey semantics (as, to some extent, supervaluationist semantics) may capture the respective semantic changes as follows: let L_1 be the language of a scientific theory at a time, let L_2 be the language of its immediate successor theory, and let *Terms* be a set of theoretical terms that belong simultaneously to both languages. Let Adm_1 be the class of all restrictions of L_1 -admissible interpretations to *Terms* (while leaving the domain of the interpretations the same), such that only the members of *Terms* are interpreted by the interpretation functions F_1 in Adm_1 ; and let Adm_2 be the corresponding class of all interpretation functions F_2 that result from restricting L_2 -admissible interpretations to *Terms*.

Then one can define:

The L_2 -interpretation of the members of *Terms* is a (proper) *specification* of their L_1 -interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_2 \subseteq Adm_1$, that is, Adm_2 is a (proper) subclass of Adm_1 .

Specification is what Carnap described in the quote from before: the interpretation of the members of *Terms* become more specific in the transition from L_1 to L_2 . In the language of supervaluationism, one might also speak of increased ‘precisification’ or ‘sharpening’.

But specification is just one kind of interpretational change next to others:

The L_2 -interpretation of the members of *Terms* is a (proper) *diversification* of their L_1 -interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_1 \subseteq Adm_2$, that is, Adm_2 is a (proper) superclass of Adm_1 .

The L_2 -interpretation of the members of *Terms* is a (proper) *compatible modification* of their L_1 -interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_1 \cap Adm_2 \neq \emptyset$, that is, Adm_1 (differs from and) has non-empty overlap with Adm_2 .

The L_2 -interpretation of the members of *Terms* is a *complete revision* of their L_1 -interpretation just in case $Adm_1 \cap Adm_2 = \emptyset$, that is, Adm_1 does not overlap with Adm_2 .²⁵

Proper diversification is the ‘opposite’ of proper specification: it adds new ways of interpretation and thereby makes interpretation *less* specific.

(Proper) specification and (proper) diversification of interpretation are both special cases of (proper) compatible modification (assuming the relevant sets are non-empty), but compatible modification is broader: it also covers cases in which the previous metasemantic constraints on the interpretation of the members of *Terms* are compatible with newly introduced ones ($Adm_1 \cap Adm_2 \neq \emptyset$) while the interpretation of the terms does not become more or less specific.

Finally, in cases of complete revision, the new metasemantic constraints rule out the previous ones completely (that is, $Adm_1 \cap Adm_2 = \emptyset$). In such cases, Ramsey semantics also *rules out* perfect referential/extensional continuity between L_1 and L_2 with respect to all theoretical terms in *Terms*: for the logical laws of the epsilon calculus imply in that case that

$$\varepsilon F_1 (F_1 \in Adm_1) \neq \varepsilon F_2 (F_2 \in Adm_2),$$

that is, there must be some P in *Terms*, such that $(\varepsilon F_1 (F_1 \in Adm_1))(P) \neq (\varepsilon F_2 (F_2 \in Adm_2))(P)$.

However, in the case of compatible modification ($Adm_1 \cap Adm_2 \neq \emptyset$), Ramsey semantics *does allow* one to regard all terms in *Terms* as preserving their references/extensions in the transition from L_1 to L_2 : for it is easy to see that the postulates of Ramsey semantics may always be expanded consistently in that case by

$$\varepsilon F_1 (F_1 \in Adm_1) = \varepsilon F_2 (F_2 \in Adm_2),$$

that is, by

$$I_1 = I_2.$$

²⁵ More fine-grained classifications might also take into account how the domain(s) of interpretation may change in the transition from L_1 to L_2 . But I will put this to one side here for the sake of simplicity.

In such case, let us call ' $I_1 = I_2$ ' the statement of *perfect interpretational (referential/extensional) continuity* (holding between L_1 and L_2 and with respect to *Terms*).

Cases of compatible interpretational modification include the *extreme* case of improper specification/diversification in which Adm_1 is identical to Adm_2 and where they are *singleton* classes, that is, $Adm_1 = \{I_1\} = \{I_2\} = Adm_2$: in that extreme case, Ramsey semantics even *entails* perfect interpretational continuity, by the logic of epsilon terms. Scientific realists, who are normally wedded to classical semantics, either mean that extreme case when they are speaking of referential/extensional continuity between scientific successor languages, or they mean (more realistically) that Adm_1 and Adm_2 are singletons while their members are merely similar or approximately identical: $Adm_1 = \{I_1\}$, $Adm_2 = \{I_2\}$, and $I_1 \approx I_2$. Call ' $I_1 \approx I_2$ ' a statement of *approximate interpretational (referential/extensional) continuity* (again between L_1 and L_2 and with respect to *Terms*). Cases of approximate continuity that are not cases of perfect continuity would be cases of complete (but still approximate) revision.²⁶

But Ramsey semantics also permits perfect and approximate continuity in cases of compatible modification in which Adm_1 and Adm_2 are *not* both singleton classes, that is, when the members of *Terms* are semantically indeterminate either *qua* L_1 -terms or *qua* L_2 -terms or both. For instance, consider the case of specification again ($Adm_2 \subseteq Adm_1$), that is, when *more* metasemantic constraints on the interpretation of the members of *Terms* get introduced in the transition from L_1 to L_2 : if so, the metasemantic facts determine that every admissible L_2 -interpretation on *Terms* is also an L_1 -interpretation, and hence every L_1 -determinate statement A that is composed solely of members of *Terms* is also L_2 -determinate. (That is: if $Det_{L_1}(A)$ then $Det_{L_2}(A)$.) But so long as Adm_2 (and hence Adm_1) is not a singleton class, there will not be any facts of the matter—no existing metasemantic constraints—that will require the interpretations of L_1 and L_2 to perfectly or approximately coincide on *Terms*. That is, given such Adm_1 and Adm_2 , Ramsey semantics will *not by itself entail* that

$$I_1 = \varepsilon F_1 (F_1 \in Adm_1) = / \approx \varepsilon F_2 (F_2 \in Adm_2) = I_2.$$

However, one may still consistently *extend* Ramsey semantics for L_1 and L_2 by the perfect/approximate continuity statement ' $I_1 = / \approx I_2$ ' in that case: if one does so, this will not serve the purpose of *describing* existing metasemantic facts but rather correspond to a 'free semantic posit' by which one expresses one's *choice* of talking on the metalevel *as if* the facts had determined the members of *Terms* to precisely or approximately preserve their references/extensions in the transition from L_1 to L_2 .²⁷ *Semantically*, any such interpretational continuity between theoretical terms from consecutive theories would still look like ordinary semantic realism.

²⁶ Semantically, the main difference between approximate and perfect continuity is that the former may be accompanied by changes of truth values of atomic statements and, more importantly, of complex law-like statements; not so for perfect continuity (assuming the respective universes of discourse remain invariant).

²⁷ There would also be the option of enforcing ' $I_1 = I_2$ ' by introducing new metasemantic constraints by which a new class Adm_2' of admissible interpretations of L_2 would be determined so that $Adm_2' = \{I_1\}$, where $I_1 = \varepsilon F_1 (F_1 \in Adm_1)$, and $I_2 \in Adm_2'$, which would entail that $I_1 = I_2$. However, that set Adm_2' would be guaranteed to be non-empty in every logically possible case, even if ' $\exists F_1 (F_1 \in Adm_1)$ ' were false, in which case ' $\varepsilon F_1 (F_1 \in Adm_1)$ ' would denote an *arbitrary* member of the domain. And thus there would

But not so *metasemantically*: since the continuity would not be grounded in facts, the continuity would remain merely verbal. (One might view ‘ $\varepsilon F_1 (F_1 \in Adm_1) = / \approx \varepsilon F_2 (F_2 \in Adm_2)$ ’ as describing a ‘brute semantic fact’ in the sense of [6, 34], or as a true sentence that is not describing any fact at all.)

I will have to postpone further investigation of that case to a different occasion, but *if* the semantic indeterminacy of scientific theoretical terms happens to be a common phenomenon at all—to which Ramsey semantics is not committed, but for which it is prepared—scientific realists should be regularly forced to seek refuge to such *posited* referential/extensional continuities between scientific terms from successor theories. (Alternatively, they might weaken the notion of referential/extensional continuity in the face of semantic indeterminacy, such that what is meant by it would merely be $Adm_2 \subseteq Adm_1$ or even just $Adm_1 \cap Adm_2 \neq \emptyset$.)²⁸

Instead of continuing the study of *diachronic* interpretational continuity between theoretical terms in *subsequent* theories, let me turn now to an example of *synchronic* interpretational continuity between theoretical terms from theories held true *at the same time*.

Reconsider the second-order language L' of arithmetic of Example 2 with its class Adm' of admissible interpretations satisfying the second-order Dedekind-Peano axioms, and compare it to the second-order language L^* of *real analysis* with its class Adm^* of admissible interpretations that satisfy Dedekind’s second-order axioms of the real numbers. As in the case of number theory, the second-order theory of real numbers is categorical, that is, it pins down the *structure* of the real numbers uniquely. At the same time, infinitely many (pairwise isomorphic) set-theoretic systems instantiate that structure by satisfying the axioms: let us be set-theoretic structuralists again about the real numbers and assume that the class Adm^* consists of all such systems. Ramsey semantics for L^* will thus consist in:

$$\begin{aligned} Adm^* &= \{F : F \models RA_2\}. \\ \exists F^* (F^* \in Adm^*) &\text{ (which follows set-theoretically from the previous equation)}. \\ I^* &=_{df} \varepsilon F^* (F^* \in Adm^*). \\ \text{For all sentences } A \text{ in } L^*: A \text{ is true(-in-} L^*) &\text{ iff } I^* \models A. \end{aligned}$$

be no guarantee that I_2 would still be a member of the original class Adm_2 of admissible interpretations of L_2 , undermining interpretational continuity.

²⁸ In ways like that, Ramsey semantics provides the conceptual resources to describe how ‘conceptual engineering’ [9, 18] may be expressed (meta-)semantically: by specifying, diversifying, modifying, or completely revising semantic interpretation. On the metasemantic/pragmatic side, the theory would have to be complemented by an account of speech acts by which the relevant interpretational changes could be brought about. E.g., the most straightforward instances of Carnapian explication [12] correspond semantically to the specification of the interpretation of a predicate, and one way of effecting such specifications is by *assertion*. For asserting A may have *two* effects on the target subject: the traditional one of belief revision, as in ‘update your beliefs, by restricting your set of live possibilities to those that satisfy A ’; and a novel one of interpretation specification, along the lines of ‘update the interpretation of your terms, by restricting the class of admissible interpretations to those that satisfy A ’. The first type of effect may prompt epistemic progress, whereas the second one may lead to semantic progress. Either way, the aim of asserting A would remain the truth of A , where truth is defined by Ramsey semantics as in Section 2.

Resources similar to those of the present section are of course also available to supervaluationists. However, Ramsey semantics is much closer to the classical semantics that scientific realists normally presuppose (see Section 3), and there is no supervaluationist counterpart to epsilon-term formulations of interpretational continuity.

The intended domain $Uni(I^*)$ of real numbers includes as a special subset the set $\{I^*(0), I^*(0+1), I^*((0+1)+1), \dots\}$ of real 'natural' numbers. However, combining Ramsey semantics for arithmetic with that of analysis still leaves open how these real-valued 'natural' numbers relate to the 'actual' natural numbers in $Uni(I')$.²⁹

Suppose that one intends to correct this by *stipulatively identifying the former with the latter*: then Ramsey semantics allows one to express this stipulation by adding

$$I^*(0) = I'(0), I^*(0+1) = I'(s(0)), I^*((0+1)+1) = I'(s(s(0))), \dots$$

as semantic posits to one's semantic metatheory, in which, e.g., ' $I^*(0) = I'(0)$ ' is short for

$$(\varepsilon F^* (F^* \in Adm^*)) (0) = (\varepsilon F' (F' \in Adm')) (0),$$

and the like. Once again, these are statements of perfect *interpretational continuity* (here between L^* and L'), but this time concerning languages that are used at the same point in time. (*Approximate* continuity would not make much sense in a purely mathematical context.)

Just as in the diachronic case before, continuities established in that manner would not be due to metasemantic facts existing prior to identification but rather result from free semantic choices by which some factual gaps are 'covered' semantically. While there is no proper *mathematical* reason for identifying real-valued $0, 1, 2, \dots$ with natural $0, 1, 2, \dots$, as the existence of an isomorphism between them suffices for all theoretical purposes, it may still be practically convenient for mathematicians to talk *as if* the mathematical facts had engendered the identification, and the continuity statement from above captures that semantically.

We find that Ramsey semantics allows for diachronic and synchronic interpretational continuity between languages even in cases of semantic indeterminacy for which there is no direct counterpart in classical semantics. According to classical semantics, every instance of interpretational continuity must already have existed 'from the start': a theoretical term in the 'old' theory must have happened to refer or apply to the same or similar phenomena as the same term does in the successor theory; the extension of 'real natural number' must have coincided with the extension of '(actual) natural number'; and so forth. While one may completely revise the classical interpretation of terms and sentences, and while two classical interpretations may be more similar to each other than to another one, there is no way of literally making a classical interpretation 'more specific', or of 'identifying' classical interpretations that had not been identical beforehand.³⁰ Ramsey semantics, in contrast, makes room for the existence of linguistic acts by which the (meta-)semantics of scientific and mathematical terms and sentences can be altered without affecting interpretational continuity; and in some cases, continuity may even be established by such acts in the first place.

For the same reason, it is generally difficult for classical semantics to make sense of the project of *rational reconstruction* [10] as applied to language and semantics:

²⁹ Mathematical structuralists are themselves divided over this question; but, e.g., Resnik [57] defends the view that there is no fact of the matter whether natural number 2 is identical to real number 2.

³⁰ Some of the criticisms of abstract semantics in Wilson [80] may be understood as criticisms of precisely these features of classical semantics as applied to terms from applied mathematics and science.

the normative project of improving language and interpretation for some purposes by means of clarification, precisification, systematization, simplification, correction, and more.

For example: how should one understand the claim that *number talk* in mathematics can be rationally reconstructed in different ways for different purposes (as suggested by [11, 13, 16])? Such as, including contemporary reconstructions: (set-)structuralistically, logicistically, formalistically, category-theoretically, homotopy-type-theoretically, and so on. In classical semantics, the whole project would seem rather pointless: e.g., the numeral ‘1’ that is used by number theorists either refers to a Hilbertian sequence of stroke symbols or it does not. If it does, there is no need for rationally reconstructing ‘1’-talk formalistically; if it does not, rationally reconstructing ‘1’-talk formalistically would be a mistake. *Tertium non datur*.

Ramsey semanticists, in contrast, could argue that ‘1’ might be semantically indeterminate and hence can be further specified in different ways: by excluding all interpretations except for those that are admissible on (set-)structuralist grounds, or except for those that satisfy all logicist desiderata, or except for those that are formalistically admissible; etc. Each such rational reconstruction may come with its own interpretational continuity statement and remains compatible with classical logic, truth, and mathematics, as can be shown in Ramsey semantics.

§7. Conclusions and extensions. I have argued that Ramsey semantics combines the best of two worlds: Ramsifying classical semantics preserves the syntax, semantic rules, concept of truth, truth values, and logic of classical semantics; but it also avoids the classical presupposition of a unique factually determined intended interpretation. Instead, it follows supervaluationist semantics in merely presupposing a non-empty class of admissible interpretations. The resulting semantics circumvents the Sorites paradox, scales up to higher-order vagueness (if there is such), permits semantically indeterminate theoretical terms in mathematics and science, allows for such terms to retain their interpretation across theories, and makes sense of the rational reconstruction of language and interpretation. When there is no semantic indeterminacy, Ramsey semantics collapses into classical semantics; but when there is, it will be handled more classically by Ramsey semantics than by supervaluationist semantics.

Ramsey semantics can and should be developed into various directions. One was mentioned already at the beginning of Section 1: the Ramsification of classical *intensional* semantics and its application in pragmatics. For instance: classical semanticists postulate what is communicated by the assertion of $B(n)$ to be ‘the’ intension of $B(n)$, that is, ‘the’ class of possible worlds in which it is true. Accordingly, intensional Ramsey semantics will postulate the existence of an admissible intensional interpretation F , that is, an assignment of references and intensions to the descriptive terms of the object language that fits the metasemantic facts, such that F defines truth, and..., and asserting $B(n)$ conveys the intension of $B(n)$ as given by F . The aim of such an assertion will still be the truth of $B(n)$, but truth will be understood as given existentially, as in Section 2. Metalinguistic intensional epsilon terms will be used to implement this idea formally, and what they ‘pick’ from the space Adm of admissible intensional interpretations will also depend on the context of assertion. The resulting Ramsey account of assertion and assertability may be expected to resemble that of supervaluationist semantics, but without changing truth into super-truth.

A different kind of extension concerns Ramsey semantics for a *type-free* truth predicate that avoids semantic paradoxes, such as the Liar paradox, while maintaining classical logic and truth. In that extended semantics, ‘true’ and ‘determinate’ would be type-free predicates of sentences, the extension of ‘true’ would be given by *some* admissible interpretation, while the extension of ‘determinate’ would coincide with the set of sentences satisfied in all admissible interpretations. And each admissible interpretation would itself interpret the same predicates ‘true’ and ‘determinate’ again (which is the type-free aspect). The relevant literature that comes closest to this are McGee [46] and Cantini [8], though neither of them treats type-free truth as given by Ramsification. Ramsey semantics might provide suitable variants of their theories with new philosophical support. At the same time, a type-free version of Ramsey semantics might improve the hierarchical account of higher-order vagueness in Section 5 and in the Appendix in much the same way in which Kripke’s [36] type-free theory of truth improved Tarski’s classical typed theory. But all that is left for future work.

The new semantics may also have philosophical applications beyond the philosophy of language proper. Let me conclude this paper by mentioning one of them: partially reconciling (versions of) semantic *anti-realism about mathematics* with *classical logic and truth*.

Consider intuitionism about mathematics: clearly, Ramsey semantics will *not* be suitable to orthodox intuitionists who insist that ‘existence’ means constructibility, ‘truth’ means provability, and a disjunction is provable only if one of its disjuncts is. However, there might also be more liberal intuitionists who are driven by very different considerations: they might worry about the existence of mathematical statements A , such that the existing metasemantic constraints on the interpretation of A would neither be sufficient to determine A to be true nor sufficient to determine A to be false—no facts would fill the gap. If one assumes the facts in question to be matters of mental construction, as intuitionists normally do, the worry should be quite obvious. But even someone who merely thinks that mathematical facts supervene on mental, linguistic, physical,... facts, might well question whether these facts will be able to settle the status of, say, ‘ $x \in y$ ’, for all sets x and y at arbitrary ordinal heights in the cumulative hierarchy of sets. And that worry also becomes relevant to complex mathematical statements, if one accepts that truth conditions are compositional. Indeed, if compositionality is implemented by the semantic rules of intuitionistic semantics, the truth of $A \vee \neg A$ would be in doubt. The rationale for these semantic rules is to *track determinacy by truth*, much as supervaluationist semantics does, but *compositionally*, unlike supervaluationism: for these rules may be understood as compositionally employing (in present terminology) a determinacy predicate or operator in the clauses for atomic formulas, negation formulas, conditionals, and universal quantifications (as can be seen from Kripke semantics for intuitionistic logic or from Gödel’s translation of intuitionistic logic into the modal system S4³¹).

It is to intuitionists of such more liberal stripe that Ramsey semantics may provide a profitable offer, since it accepts both the possibility of metasemantic gaps and the compositionality of truth, and *yet delivers the logical truth of the excluded middle*. The way it manages to do so is by excluding determinacy from the semantic rules for truth (see Section 2), and by postulating instead, at the outset, one non-constructive

³¹ Though one should stress that the axiom scheme 4 of S4 is actually rather questionable for *determinacy*.

existence statement: a metalinguistic Ramsey sentence. If liberal intuitionists could make their peace with that Ramsey sentence by interpreting it as expressing some kind of ‘regulative ideal’ (cf. [60], p. 209) that is meant to systematize semantics in a practically beneficial manner, they might be able to embrace the logical truth of classical logic and the truth of classical mathematics without alarming factual commitments.³²

Appendix: Iterating Ramsey semantics throughout the Tarskian hierarchy. The metalinguistic Ramsey semantics of the first-order extensional object language L from Example 1 of Section 1 has been described in Section 2. I am now going to introduce Ramsey semantics for the metalanguage L_2 of L in which Ramsey semantics for L has been formulated. In an analogous manner, Ramsey semantics may be introduced at yet higher levels of the Tarskian hierarchy. The main technical challenge, on which I did not comment in Section 5, consists in the interpretation of the metalinguistic epsilon terms that figure prominently in L_2 and in the formal interaction between determinacy and truth predicates.

For simplicity, let us assume that L_2 is a first-order extensional language again; e.g., the metalinguistic expression ‘ $F \in Adm$ ’ in L_2 is really a first-order formula of the form ‘ $Adm(F)$ ’ in which ‘ F ’ is a first-order variable and ‘ Adm ’ is a predicate. Classical interpretations F_2 of L_2 are defined as described in Section 1, except that now also the epsilon terms of L_2 require interpretation: here one may simply follow the standard choice semantics of epsilon terms (see [39]) by defining each F_2 to come equipped with some choice function $Ch(F_2)$ by which epsilon terms in L_2 are interpreted. E.g., $Ch(F_2)$ applied to a non-empty subclass X of $Dom(F_2)$ yields some member of X , and $F_2(‘\varepsilon F(F \in Adm)’)$ is defined to be equal to $Ch(F_2)(F_2(‘Adm’))$. Two interpretations F_2 and F_2' may well differ only in their respective choice functions. ‘ \models ’ for satisfaction is defined as in (1) from Section 1 but now relative to interpretations F_2 of L_2 , and logical consequence is still defined by (2) from Section 1.

Next, one assumes all existing metametasemantic constraints on the interpretation of L_2 to be summed up by a class Adm_2 of classical (L_2 -)admissible interpretations F_2 of L_2 . The universe of each F_2 is going to be a set that includes all classical interpretations F of L for which $Uni(F) = U$ (which was the intended set universe of L). For simplicity again, let us assume all interpretations in Adm_2 to have one and the same set-domain U_2 . Each (L_2 -)admissible F_2 assigns over U_2 , amongst others, an (L_2 -)admissible interpretation to the metalinguistic expression ‘conforms to the existing metasemantic constraints on the interpretation of L ’, hence also to ‘ Adm ’, and thereby, as in (22), to ‘ Det ’. Amongst the metametasemantic constraints on the interpretation of ‘ Adm ’, there will be conceptual constraints, such as: for each $F_2 \in Adm_2$, $F_2(‘Adm’)$ is a subset of U_2 and indeed a non-empty set of classical interpretations F of L . And all $F_2 \in Adm_2$ interpret the metalinguistic terms ‘ I ’ and ‘true(-in- L_1)’ so that the postulates of Ramsey semantics for L from Section 2 are made true: e.g., $F_2(‘I’) = F_2(‘\varepsilon F(F \in Adm)’)$.

Once again, Ramsey semantics for the metalanguage L_2 of L commits itself to Adm_2 being non-empty, ‘the’ intended interpretation I_2 of L_2 being given by a

³² It is well known that extending intuitionistic logic by the logical laws of the epsilon calculus with extensionality (as used in Section 2) yields full classical logic (see [3]). If intuitionists were to accept the epsilon calculus on similarly pragmatic grounds as a kind of ‘regulative ideal’, they should be able to embrace classical logic, too.

metametalinguistic epsilon term, and truth for L_2 being defined as classical satisfaction by I_2 ; that is,

$$\begin{aligned} &\exists F_2 (F_2 \in Adm_2), \\ &I_2 =_{df} \varepsilon F_2 (F_2 \in Adm_2), \\ &\text{for all sentences } A \text{ in } L_2: A \text{ is true}(-in-L_2) \text{ iff } I_2 \models A. \end{aligned}$$

The epsilon term ' $\varepsilon F_2 (F_2 \in Adm_2)$ ' that defines ' I_2 ' thus 'picks' an interpretation in Adm_2 . In that way, the same epsilon term also 'picks' the set I_2 (' Adm ') of admissible interpretation of L that serves as 'the' intended interpretation of the term ' Adm ' that was used in the metalanguage of L when Ramsey semantics was stated in previous sections (and from which I got 'picked' by a metalinguistic epsilon term). 'The' intended interpretation of the metalinguistic predicate ' Det ' is thereby defined, too, and similarly for ' I ' and 'true(-in- L_1)'.

For instance, by the definition of ' I_2 ' and the previous assumptions, it holds that

$$\begin{aligned} I_2('I') &= I_2(\varepsilon F(F \in Adm)) = Ch(I_2)(I_2('Adm')) \\ &= Ch(\varepsilon F_2(F_2 \in Adm_2))((\varepsilon F_2(F_2 \in Adm_2))('Adm')), \end{aligned}$$

where the second and third occurrence of ' ε ' denotes the epsilon operator of the metalanguage L_3 of L_2 , that is, the *metametalinguistic* language of L . L_3 is the language in which Ramsey semantics for L_2 is formulated; but I will put the details of *its* syntax to one side here.

Last but not least, one may introduce a new *metametalinguistic* determinacy(-in- L_2) predicate ' Det_2 ' into L_3 , such that ' Det_2 ' applies to sentences of L_2 in the expected manner (see Section 5). Similarly, one may introduce a *binary* metametalinguistic determinacy(-in- L_2) predicate into L_3 that applies to all open formulas $A[x]$ of L_2 with precisely one free variable x , and to all objects d in U_2 . I will use the predicate ' Det_2 ' again for that purpose:

$$\begin{aligned} &\text{for all } A[x] \text{ in } L_2, \text{ for all } d \in U_2 : \\ &Det_2(A[x], d) \text{ iff for all } F_2 \in Adm_2 : F_2, s \frac{d}{x} \models A[x], \end{aligned}$$

where s is an arbitrary variable assignment over U_2 . ' $Det_2(A[x], d)$ ' is read as: $A[x]$ is determinate of d .

For instance, one may now express the semantic indeterminacy of the metalinguistic term ' $\varepsilon F(F \in Adm)$ ' in L_2 by means of

$$\text{it is not the case that there is a } d \in U_2, \text{ such that } Det_2(\varepsilon F(F \in Adm) = x', d).$$

And that indeterminacy of ' $\varepsilon F(F \in Adm)$ ' can indeed be derived in the Ramsey semantics for L_2 whenever one can derive that Adm_2 includes at least two

interpretations F_2 and F_2' of L_2 that assign distinct interpretations to $\varepsilon F(F \in Adm)$, that is, when³³

$$\begin{aligned} F_2(\varepsilon F(F \in Adm)) &= Ch(F_2)(F_2('Adm')) \neq Ch(F_2')(F_2'('Adm')) \\ &= F_2'(\varepsilon F(F \in Adm)). \end{aligned}$$

As another example, consider the sentence $B(0)$ of L : $'Det(B(0))'$ is a sentence of L_2 that is short for

$$\text{for all } F \in Adm, F \models B(0).$$

Similarly, $'Det_2(Det(B(0)))'$ is a sentence of L_3 that is short for

$$\text{for all } F_2 \in Adm_2, F_2 \models Det('B(0)').$$

Taken together, and using the semantic rules by which \models is defined, $'Det_2(Det('B(0)))'$ expresses: for each interpretation $F_2 \in Adm_2$, such that F_2 assigns an interpretation $F_2('Adm')$ to $'Adm'$, and for each interpretation $F \in F_2('Adm')$, the sentence $B(0)$ is true relative to F .

For instance, if for all $F_2 \in Adm_2$ and all $F \in F_2('Adm')$ it holds that $0 \in F(B)$, it is also going to be the case that $Det_2('Det('B(0))')$, which in turn implies (by the definitions sketched before) that $Det_2('True-in-L('B(0))')$ and $True-in-L_2('Det('B(0))')$, and hence also that $True-in-L_2('True-in-L('B(0))')$. In plain words, and using operator-talk: having no hair at all on one's head is not just a case of being bald ($True-in-L_2('True-in-L('B(0))')$) but also one of being determinately bald ($Det_2('True-in-L('B(0))')$ and $True-in-L_2('Det('B(0))')$) and of being determinately determinately bald ($Det_2('Det('B(0))')$).

More generally, from the axioms and assumptions above one can derive every instance of the four material conditional claims

$$\begin{aligned} &\rightarrow Det_2('True-in-L('A')') \rightarrow \\ Det_2('Det('A')') & \quad \quad \quad True-in-L_2('True-in-L('A')') \quad , \\ &\rightarrow True-in-L_2('Det('A')') \rightarrow \end{aligned}$$

in which $'A'$ may be replaced by an arbitrary sentence of L . The converses of these claims are not necessarily derivable, nor is there any guarantee for all instances of $'Det_2('True-in-L('A')') \rightarrow True-in-L_2('Det('A')')$ or its converses to be derivable. (But if one assumes $'Det_2('Det('A')')$, then both conditionals can be derived from that assumption.)

By unpacking the two truth predicates and the determinacy predicate $'Det'$ in the four conditionals above, one may reformulate the conditionals as statements involving epsilon terms. In the case of semantic indeterminacy, these epsilon terms cannot be replaced *salva veritate* by terms with a unique factually determined intended interpretation. And the same holds for $'Det_2'$, if $'conforms to the existing metametasemantic constraints on the interpretation of L_2' and hence $'Adm_2'$ turn$

³³ One can show that no Evans [22]-style argument against statements of the form 'it is indeterminate that $a = b$ ' with a sentential (in-)determinacy operator can be run against statements of the form $\neg \exists d Det_2(\varepsilon F(F \in Adm) = x, d)$ in which the predicate Det_2 is applied to an identity statement with an indefinite description under quotation marks. This said, the semantics of identity statements opens up highly important questions both in Ramsey semantics and in supervaluationist semantics: for the latter, see Varzi [73].

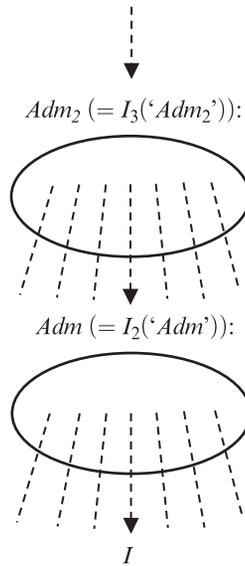


Fig. 2. Ramsey semantics throughout the Tarskian hierarchy.

out to be indeterminate, too (which may or may not be the case). In order to be prepared for *that* possibility, Ramsification may be repeated on the next level (on which L_3 would be interpreted), and so forth, throughout the Tarskian hierarchy—if necessary including levels of transfinite ordinals α . In short: classical semantics ought to be Ramsified at any ordinal level of metameta...metalanguage of L if one wants to stay on the safe side.³⁴ Figure 2 shows what the resulting structure of intended interpretations I, I_2, I_3, \dots (of, respectively, L, L_2, L_3, \dots) and corresponding non-empty classes Adm, Adm_2, Adm_3, \dots looks like, where each intended interpretation I, I_2, I_3, \dots gets ‘picked’ by an epsilon term from its corresponding set Adm, Adm_2, Adm_3, \dots of admissible interpretations (the ‘picking’ being visualized by arrows), and where in turn each such set of admissible interpretations coincides with $I_2('Adm'), I_3('Adm_2'), I_4('Adm_3'), \dots$, respectively.

On each level, hypotheses concerning the corresponding class of admissible interpretation may be added to the respective metameta...-theory. E.g., as far as the ‘ B ’ in the baldness case is concerned, the following kind of axiom would be plausible for each $n \geq 2$ (where $F_1 = F$):

for all $F_n \in Adm_n$, for all $F_{n-1} \in F_n('Adm_{n-1}')$, ..., for all $F \in F_2('Adm')$: $0 \in F(B)$.

But generally one should not expect all plausible hypotheses concerning (L_n) -admissible interpretations in Adm_n to be added as *axioms* to one’s semantic metameta...-theory at level $n+1$: after all, assumptions concerning (L_n) -admissible

³⁴ With sufficient syntactic resources, and the height of the ordinal α being sufficiently constrained, it will also be possible to express on level α that a sentence A is true at every level $\gamma < \alpha$, as well as that a sentence B is determinate at every level $\gamma < \alpha$. It will not be possible to express at any level α that A is true at every level whatsoever, including α , for the usual Tarskian reasons. Such restrictions could be avoided by turning Ramsey semantics into a type-free theory of truth and determinacy, on which I have commented briefly in Section 7.

interpretations do not belong to semantics proper (recall Section 1), and one might not be confident enough of them to ‘dignify’ them with the status of *provability* in one’s metameta...-theory.

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BIBLIOGRAPHY

- [1] Ackermann, W. (1924). Begründung des ‘tertium non datur’ mittels der Hilbertschen Theorie der Widerspruchsfreiheit. *Mathematische Annalen*, **93**, 1–36.
- [2] Alston, W. P. (1967). Vagueness. In Edwards, P., editor. *The Encyclopedia of Philosophy*, Vol. 8. New York: Macmillan, pp. 218–221.
- [3] Bell, J. L. (1993). Hilbert’s ε -operator and classical logic. *Journal of Philosophical Logic*, **22**, 1–18.
- [4] Benacerraf, P. (1965). What numbers could not be. Reprinted in Benacerraf, P., and Putnam, H., editors. *Philosophy of Mathematics*, second ed. Cambridge: Cambridge University Press, pp. 272–294.
- [5] Bandom, R. (1996). The significance of complex numbers for Frege’s philosophy of mathematics. *Proceedings of the Aristotelian Society*, **96**, 293–315.
- [6] Breckenridge, W., & Magidor, O. (2012). Arbitrary reference. *Philosophical Studies*, **158**, 377–400.
- [7] Button, T., & Walsh, S. (2018). *Philosophy and Model Theory*. Oxford: Oxford University Press.
- [8] Cantini, A. (1996). *Logical Frameworks for Truth and Abstraction: An Axiomatic Study*, Studies in Logic and the Foundations of Mathematics, vol. 135, Amsterdam: Elsevier.
- [9] Cappelen, H. (2018). *Fixing Language: An Essay on Conceptual Engineering*. Oxford: Oxford University Press.
- [10] Carnap, R. (1928). *Der Logische Aufbau der Welt*. Berlin: Weltkreis.
- [11] ———. (1934). *Logische Syntax der Sprache*. Vienna: Springer, translated as *The Logical Syntax of Language*. London: Routledge, 1937.
- [12] ———. (1950a). *Logical Foundations of Probability*. Chicago: University of Chicago Press.
- [13] ———. (1950b). Empiricism, semantics, and ontology. *Meaning and Necessity*. second edition. Chicago: University of Chicago Press, pp. 205–221.
- [14] ———. (1959). Theoretical concepts in science (edited by Stathis Psillos). *Studies in History and Philosophy of Science*, **31**, 151–172.

- [15] ———. (1961). On the use of Hilbert's Epsilon-operator in scientific theories. In Bar-Hillel, Y., editor. *Essays on the Foundations of Mathematics*. Jerusalem: Magnes Press, pp. 156–164.
- [16] ———. (1963). Intellectual autobiography. In Schilpp, P., editor. *The Philosophy of Rudolf Carnap*. LaSalle, IL: Open Court, pp. 3–84.
- [17] ———. (1966). *Philosophical Foundations of Physics: An Introduction to the Philosophy of Science*. New York: Basic Books.
- [18] Carus, A. (2007). *Carnap and Twentieth-Century Thought. Explication as Enlightenment*. Cambridge: Cambridge University Press.
- [19] Cobreros, P. (2008). Supervaluationism and logical consequence: a third way. *Studia Logica*, **90**(3), 291–312.
- [20] ———. (2011). Paraconsistent Vagueness: a positive argument. *Synthese*, **183**(2), 211–227.
- [21] Dedekind, R. (1888). *Was sind und was sollen die Zahlen?* Brunswick: Vieweg. English translation contained in: *Essays on the Theory of Numbers*. Chicago: Open Court, 1901.
- [22] Evans, G. (1978). Can there be vague objects? *Analysis*, **38**(4), 208.
- [23] Field, H. (1973). Theory change and the indeterminacy of reference. *The Journal of Philosophy*, **70**(14), 462–481.
- [24] ———. (2008). *Saving Truth from Paradox*. Oxford: Oxford University Press.
- [25] Fine, K. (1975). Vagueness, truth and logic. *Synthese*, **30**(3), 265–300.
- [26] Fodor, J., & Lepore, E. (1996). What can't be evaluated can't be evaluated, and it can't be supervalued either. *Journal of Philosophy*, **93**, 516–536.
- [27] Hamkins, J. D. (2012). The set-theoretic multiverse. *Review of Symbolic Logic*, **5**(3), 416–449.
- [28] Hellman, G., & Shapiro, S. (2019). *Mathematical Structuralism*. Cambridge: Cambridge University Press.
- [29] Hilbert, D. (1899/1980). Letter to Frege, 29.12.1899. In Gabriel, G., and McGuinness, B., editors. *Gottlob Frege: Philosophical and Mathematical Correspondence*. Chicago: University Chicago Press, pp. 41–48.
- [30] Hilbert, D., & Bernays, P. (1934). *Grundlagen der Mathematik*, Vol. 1. Berlin: Springer.
- [31] ———. (1939). *Grundlagen der Mathematik*, Vol. 2. Berlin: Springer.
- [32] Hyde, D. (1997). From heaps and gaps to heaps and gluts. *Mind*, **106**, 641–660.
- [33] Hyde, D., & Colyvan, M. (2008). Paraconsistent Vagueness: why not? *Australasian Journal of Logic*, **6**, 107–121.
- [34] Kearns, S., & Magidor, O. (2012). Semantic sovereignty. *Philosophy and Phenomenological Research*, **85**, 322–350.
- [35] Keefe, R. (2000). *Theories of Vagueness*. Cambridge: Cambridge University Press.
- [36] Kripke, S. (1975). Outline of a theory of truth. *The Journal of Philosophy*, **72**(19), 690–716.
- [37] ———. (1980). *Naming and Necessity*. Cambridge, MA: Harvard University Press.
- [38] Kuhn, T. S. (1962). *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press.
- [39] Leisenring, A. (1969). *Mathematical Logic and Hilbert's Epsilon-Symbol*. London: MacDonald.

- [40] Leitgeb, H. (2007). Struktur und Symbol. In Schmidinger, H., and Sedmak, C., editors. *Der Mensch - ein "animalsymbolicum"?*, Topologien des Menschlichen IV. Darmstadt: Wissenschaftliche Buchgesellschaft, pp. 131–147.
- [41] ———. (2021). On non-eliminative structuralism. Unlabeled graphs as a case study (Part B). *Philosophia Mathematica*, 29(1), 64–87.
- [42] Lewis, D. 1970. How to define theoretical terms. *The Journal of Philosophy*, 67(13), 427–446.
- [43] ———. (1972). Psychophysical and theoretical identifications. *Australasian Journal of Philosophy*, 50(3), 249–258.
- [44] ———. (1986). *On the Plurality of Worlds*. Oxford: Oxford University Press.
- [45] Lynch, M. (2000). Alethic pluralism and the functionalist theory of truth. *Acta Analytica*, 15(24), 195–214.
- [46] McGee, V. (1991). *Truth, Vagueness, and Paradox: An Essay on the Logic of Truth*. Indianapolis: Hackett Publishing.
- [47] McGee, V., & McLaughlin, B. (1994). Distinctions without a difference. *The Southern Journal of Philosophy*, 23, 203–251.
- [48] ———. (1997). Review of *Vagueness*. *Linguistics and Philosophy*, 21, 221–235.
- [49] ———. (2004). Logical commitment and semantic indeterminacy: a reply to Williamson. *Linguistics and Philosophy*, 27, 123–136.
- [50] Pagin, P. (2010). Vagueness and central gaps. In Dietz, R., and Moruzzi, S., editors. *Cuts and Clouds*. Oxford: Oxford University Press, 254–272.
- [51] Partee, B. H. (2011). Formal semantics: origins, issues, early impact. *Logic and Communication*, 6, 1–52.
- [52] Pettigrew, R. (2008). Platonism and Aristotelianism in mathematics. *Philosophia Mathematica*, 16, 310–332.
- [53] Psillos, S. (1999). *Scientific Realism. How Science Tracks Truth*. London and New York: Routledge.
- [54] Putnam, H. (1975a). The meaning of ‘meaning’. *Minnesota Studies in the Philosophy of Science*, 7, 131–193.
- [55] ———. (1975b). What is realism? *Proceedings of the Aristotelian Society*, 76, 177–194.
- [56] Ramsey, F. P. (1929/1978). Theories. In Mellor, D. H., editor. *Foundations: Essays in Philosophy, Logic, Mathematics and Economics*, London: Routledge & Kegan Paul, pp. 101–125.
- [57] Resnik, M. (1997). *Mathematics as a Science of Patterns*. Oxford: Oxford University Press.
- [58] Sahlin, N.-E. (1990). *The Philosophy of F. P. Ramsey*. Cambridge: Cambridge University Press.
- [59] Schiemer, G., & Gratzl, N. (2016). The Epsilon-reconstruction of theories and scientific structuralism. *Erkenntnis*, 81(2), 407–432.
- [60] Shapiro, S. (1997). *Philosophy of Mathematics: Structure and Ontology*. New York: Oxford University Press.
- [61] ———. (2006). Structure and identity. In MacBride, F., editor. *Identity and Modality*. Oxford: Clarendon, pp. 109–145.
- [62] ———. (2008a). *Vagueness in Context*. Oxford: The Clarendon Press.
- [63] ———. (2008b). Identity, indiscernibility, and *Ante Rem* structuralism: the tale of *i* and *-i*. *Philosophia Mathematica*, 16, 285–309.

- [64] ———. (2012). An ‘*i*’ for an *i*: singular terms, uniqueness, and reference. *The Review of Symbolic Logic*, **5**(3), 380–415.
- [65] Smith, N. J. J. (2008). *Vagueness and Degrees of Truth*. Oxford: Oxford University Press.
- [66] Sneed, J. (1971). *The Logical Structure of Mathematical Physics*. Dordrecht: D. Reidel.
- [67] Sorensen, R. (2001). *Vagueness and Contradiction*. Oxford: Oxford University Press.
- [68] Suppes, P. (1957). *Introduction to Logic*. New York: Van Nostrand Reinhold Company.
- [69] ———. (1967). What is a scientific theory? In Morgenbesser, S., editor. *Philosophy of Science Today*. New York: Basic Books, pp. 55–67.
- [70] Tarski, A. (1933). *Pojęcie prawdy w językach nauk dedukcyjnych*, Warsaw: Nakładem Towarzystwa Naukowego Warszawskiego.
- [71] Van Fraassen, B. C. (1966). Singular terms, truth-value gaps, and free logic. *The Journal of Philosophy*, **63**(17), 481–495.
- [72] Varzi, A. (2007). Supervaluationism and its logics. *Mind*, **116**(463), 633–676.
- [73] ———. (2020). Indeterminate identities, supervaluationism, and quantifiers. *Analytic Philosophy*, **61**(3), 218–235.
- [74] Weatherson, B. (2010). Vagueness as indeterminacy. In Dietz, R., and Moruzzi, S., editors. *Cuts and Clouds: Vagueness, Its Nature and Its Logic*. Oxford: Oxford University Press, pp. 77–90.
- [75] Williamson, T. (1992). Vagueness and ignorance. *Proceedings of the Aristotelian Society*, **66**, 145–177.
- [76] ———. (1994a). *Vagueness*. London: Routledge.
- [77] ———. (1994b). Definiteness and knowability. *The Southern Journal of Philosophy*, **33**, 171–191.
- [78] ———. (2003). Everything. *Philosophical Perspectives*, **17**, 415–465.
- [79] ———. (2004). Reply to McGee and McLaughlin. *Linguistics and Philosophy*, **27**, 113–122.
- [80] Wilson, M. (2006). *Wandering Significance. An Essay in Conceptual Behavior*. Oxford: Clarendon Press.
- [81] Woods, J. (2014). Logical indefinites. *Logique et Analyse*, **227**, 277–307.
- [82] Worrall, J., & Zahar, E. (2001). Appendix IV: Ramseyfication and structural realism. In Zahar, E., editor. *Poincaré’s Philosophy: From Conventionalism to Phenomenology*. Chicago: Open Court, pp. 236–251.
- [83] Wright, C. (1975). On the coherence of vague predicates. *Synthese*, **30**(3–4), 325–365.
- [84] ———. (2010). The illusion of higher-order vagueness. In Dietz, R., and Moruzzi, S., editors. *Cuts and Clouds: Vagueness, Its Nature and Its Logic*. Oxford: Oxford University Press, 523–549.
- [85] ———. (2010). Truth, Ramsification, and the pluralist’s revenge. *Australasian Journal of Philosophy*, **88**(2), 265–283.

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