A. F. Cook
Harvard-Smithsonian Center for Astrophysics
(Cambridge, Massachusetts)

ABSTRACT

Observations of the Rings of Saturn from the Pioneer spacecraft, discovery of the Ring of Jupiter, ground based polarimetry of the Rings of Saturn and some theoretical studies may be combined to markedly advance our understanding of the Rings of Jupiter, Saturn and Uranus. In particular, narrow rings can be self-gravitatingly stable inside Roche's limit and outside another closer limit. They can be created from a satellite which evolves across its Roche limit either by inward tidal drift or by growth of the planet by accretion.

These considerations suggest that Neptune may well be surrounded by one or more narrow rings like those of Uranus.

INTRODUCTION

Two very similar and recent review articles have been written on the subject of rings around Saturn and Uranus by Goldreich (1979) and Goldreich and Tremaine (1979). The present review has been prepared to treat the many very recent developments from the two Voyager flights by Jupiter and the Pioneer passage by Saturn. It also happens that a theoretical proposal by Dermott and Murray (1979) provides an additional key building block which enables us to assemble a plausible system of models for the generation of the present structures of the various rings. In addition there is evidence that the Earth may have had a temporary ring of small particles at the end of the Eocene era (O'Keefe, in preparation).

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 401–416.

SATURN'S RINGS

Gehrels (private communication on behalf of the Pioneer Photopolarimeter Team) reports that there is no material inside the sharply bounded inner edge of Ring C, i.e., Ring D (Guérin, 1973) apparently does not exist; there is structure in Ring C; the outer edge of Ring C is separated by a division from Ring B (originally alleged by Dawes, 1851, and argued for by Lyot and Dollfus); there is structure in the inner part of Ring B consisting of scattered bright spots, some appearing to trace a minor division; Cassini's Division is partially filled with material except for a narrow gap near its centre; the innermost part of Ring A is densely populated; outside this is a thinner region followed by three minor divisions, the last being designated as Encke's Gap; beyond Ring A there is a gap followed by a narrow ring called Ring F and finally farther out at 2.53 ± 0.01 equatorial radii of Saturn lies the orbit of the newly discovered satellite, 1979 S 1 (IAU Circ. 3471, 1979), The equatorial radius of Saturn was measured at 60,000 ± 500 km.

The revised definitions of "division" and "gap" used by the Pioneer Team are that a division may appear to contain material but a gap shall be either a sharp minimum within a division as found within Cassini's Division or an empty region like Encke's Gap.

It appears to this reviewer that the definition of a division should be made more precise by requiring the presence of visible material but that of a gap should be as they have proposed, either a narrow sharp minimum in concentration of material or a region devoid of visible material. In that case the space between Ring A and their Ring F would be called the Pioneer Gap.

This reviewer would prefer that the disordered scattering of letter designations of rings be stopped at once. To that end the following counter-proposal is offered: Instead of Ring F, use the designation, A_1; for the region between the Pioneer Gap and Encke's Gap, use the designation, A_2; for the region between Encke's Gap and Cassini's Division, use the designation, A_3, and finally let us not use the names of Dawes, Lyot or Dollfus for the division between Rings B and C in honour of observers who could not have been expected to separate visual contrast effects from reality. Let us use the term Pioneer Division instead. This scheme is particularly appropriate as both the division between Rings B and C and the gap between Rings A_1 and A_2 can truly be said to have been discovered from Pioneer 11 and thus it is fitting to have both a Pioneer Division and a Pioneer Gap.

The random faint speckles on the inner part of the dark face of Ring B may be explained, in a ring which is a monolayer, by embedded particles much larger than average which will be spun up by the shear field to retrograde rotation at half their orbital angular speed. Along with this spinning there will be lanes opened inward and preceding, and also outward and following. The large particles need not be roughly spherical, they may be temporarily formed pancakes held together by adhesion of surfaces of smaller particles (verbal suggestion by T. Gold). Further evidence that the Rings are a monolayer is provided by the polarisation measurements of Johnson et al. (1979) which show no component in the plane of the Rings as would be expected from extensive multiple scattering between particles.

We all look forward to the publication of the report on Saturn by the Imaging Photopolarimeter Team (Gehrels et al. 1980). Correlation of the structure seen in reflection from Ring B from ground based photographs with that found by the team in transmission from their images of the Rings should be very interesting.

Ingersoll et al. (1980) report that the infra-red radiometer on Pioneer 11 observed at 45 μm and measured the brightness temperature of the sunlit north side of the Rings at 70K which dropped in Saturn's shadow to 67K - 60K whilst the dark south side showed only about 55K (upper limit, 60K) which is the expected equilibrium temperature in Saturn's emitted and reflected radiation field. The Saturnocentric declination of the Sun was +2.83°. The difference between the two faces implies that the dark faces have not been in sunlight for many times the duration of passage through the shadow. The latter is approximately 2 Arcsin (1/2.13) = 56°, whence we estimate for the arc travelled that several revolutions may well be involved. Such a consistency in orientation supports the concept that Ring B is a monolayer. In this connection it should be noted that the maximum optical thickness for a monolayer of spheres of equal radii undergoing shear with only grazing collisions is $-\ln(1 - \pi/4) = 1.539$. This value is consistent with the observations by the photopolarimeter.

The structure of Cassini's Division, seen by the photopolarimeter showing a gap in the middle (Cassini's Gap in the new terminology?) with some population everywhere else, appears not to fit Goldreich and Tremaine's (1978) model of a sweeping spiral density wave driven by the gravitational field of Mimas.

The detailed structure found must be correlated with the satellites which we all confidently expect to be discovered during the current apparition of Saturn.

URANUS' RINGS

Goldreich (1979) suggested that there is a satellite embedded in each Ring of Uranus. This has been amplified by Dermott and Murray (1979) to call for the satellite's size to reach beyond the boundary between satellite solutions of the restricted three body problem into the regions of horseshoe solutions and of circulating solutions, i.e., to overflow its Roche lobe, or zero velocity surface. In that case released particles leave from the inward preceding and outward following sectors to perform half circulations and accrete upon the inward following and outward preceding sectors respectively. The rate of differential progression of the lines of apsides of the released particles versus that of the satellite will be much slower than the differential orbital speeds so that the oblateness of the planet will not have time to play a significant role even if the satellite's orbit is eccentric. Dermott and Murray have confirmed this behaviour by numerical integrations.

However, the postulated model of Goldreich as further developed by Dermott and Murray implies that the satellite has drifted (perhaps by tidal action) within its Roche limit and in that case it should progressively disintegrate at successive passages through periapsis after which further drift will cause it to disintegrate around more and more of its orbit and perhaps all the way around in the end. Two cases may be conveniently considered with the help of a plot of zero velocity curves by Hagihara (1961, fig. 5, p. 149).

In the first case we assume that the satellite had no tensile strength so that it changed shape until it just filled its zero velocity surface, or Roche lobe, at periapsis. It should then have begun to lose material past its first Lagrange point into circulating interior paths and also into horseshoe paths in the inward preceding direction. This process should then have continued until the loop filled. Direct local interaction near the first Lagrange point on each circulation should have kept the resulting filament in an orbit synchronized in eccentricity and direction of the apse with that of the satellite.

All that is then required to generate a narrow Uranian ring is that its self gravitation should keep it condensed so that it could not dissipate. The process could then have continued until the satellite disappeared. We shall show that the condensed narrow ring is probably a stable state inside Roche's limit and outside another closer limit.

Let us consider two co-planar rings of equal mass, m, in circular equatorial orbits at mean distance, $\bar{r}$, from a planet of mass, M, the orbits being separated by a distance, Δr. The total potential energy of the rings is then

$$V = -GMm\left[(\bar{r} - \Delta r/2)^{-1} + (\bar{r} + \Delta r/2)^{-1}\right] - \left[Gm^2/(2\pi)\right]\int_0^{2\pi}\left\{\left[\bar{r} + \Delta r/2 - (\bar{r} - \Delta r/2)\cos\lambda\right]^2 + \left[(\bar{r} - \Delta r/2)\sin\lambda\right]^2\right\}^{-1/2} d\lambda \tag{1}$$

We introduce the auxiliary parameters

$$\gamma \equiv m/M, \ \alpha \equiv (\bar{r} - \Delta r/2)/(\bar{r} + \Delta r/2), \tag{2}$$

to find

$$V = -\left[GM^2\gamma(1 + \alpha)/(2\bar{r}\alpha)\right]\left\{1 + \alpha + \alpha\ (\gamma/\pi)\int_0^{\pi}\left[(1 - \alpha\cos\lambda)^2 + (\alpha\sin\lambda)^2\right]^{-1/2} d\lambda\right\} \tag{3}$$

Since α approaches unity from below, we introduce

$$\varepsilon \equiv 1 - \alpha, \ \psi \equiv \lambda/2 \tag{4}$$

and the auxiliary parameter

$$\delta \equiv \varepsilon/\left[2\ (1 - \varepsilon)\right]^{1/2}, \tag{5}$$

to find

$$V = -(2GM^2\gamma/\bar{r})\left(1 + \delta^2\right)^{1/2}\left\{\left(1 + \delta^2\right)^{1/2} + \left[\gamma/(2\pi)\right]\int_0^{\pi/2}\left(\sin^2\psi + \delta^2\right)^{1/2} d\psi\right\} \tag{6}$$

We note that

$$\lim_{\delta \to o} \int_o^{\pi/2} \left(\sin^2\psi + \delta^2\right)^{-1/2} d\psi = \lim_{\delta \to o} \left[\int_o^{\psi_1} \left(\psi^2 + \delta^2\right)^{-1/2} d\psi \right.$$

$$\delta << \psi_1 << \pi/2$$

$$\left. + \int_{\psi_1}^{\pi/2} \operatorname{cosec}\psi d\psi \right] \tag{7}$$

$$= \ln(2\psi_1/\delta) + \ln(2/\psi_1) = \ln(4/\delta),$$

whence it follows that

$$\lim_{\delta \to o} V = -(2GM^2\gamma/\bar{r}) \left\{ 1 + \delta^2 + \left[\gamma/(2\pi)\right] \ln(4/\delta) \right\} \tag{8}$$

From (5), (4) and (2) we find that

$$\lim_{\Delta r \to o} \delta = \sqrt{2}\Delta r/(2\bar{r}), \tag{9}$$

whence the potential energy (8) may be written in terms of Δr as follows,

$$\lim_{\Delta r \to o} V = -(2GM^2\gamma/\bar{r}) \left\{ 1 + (1/2)\ (\Delta r/\bar{r})^2 + [\gamma/(4\pi)] \ln [32(\bar{r}/\Delta r)^2] \right\} \tag{10}$$

We expect dissipation to decrease the potential energy and thus we examine the partial derivative

$$\lim_{\Delta r \to o} \partial V/\partial(\Delta r) = -(2GM^2\gamma/\bar{r}^2) \left\{ \Delta r/\bar{r} - [\gamma/(2\pi)]\ (\bar{r}/\Delta r) \right\} \quad . \tag{11}$$

The first term on the right exhibits the decrease in total potential energy associated with spreading in the gravitational field of the planet and the second term exhibits the decrease associated with contraction under self-gravitation. These will balance at

$$\Delta r/\bar{r} = \varepsilon_L \equiv [\gamma/(2\pi)]^{1/2} \tag{12}$$

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For $\Delta r/\bar{r} < \varepsilon_L$ the self-gravitation will dominate and pull the two rings together under dissipation whilst for $\Delta r/\bar{r} > \varepsilon_L$ the planetary field will dominate and spread them apart.

Let a denote the radius of an assumed circular cross-section of each ring and $\bar{\rho}$ its average density. Then the mass of each ring is

$$m = 2\pi\bar{r}\ \pi\bar{\rho}a^2 \tag{13}$$

and our mass-ratio, γ, is given by (2),

$$\gamma = 2\pi^2(\bar{\rho}/M)\ \bar{r}a^2 \tag{14}$$

If we put $\Delta r = 2a$, the distance of the two rings if they just touch, we find from (11)

$$\bar{r} = \left[4\,M/(\pi\bar{\rho})\right]^{1/3} \tag{15}$$

Let us express the mean density, $\bar{\rho}$, of each ring in terms of the density, ρ_p, of the solid particles. To this end we assume that each occupies a circumscribed cube so that

$$\bar{\rho} = (\pi/6)\ \rho_p, \tag{16}$$

whence we have

$$\bar{r} = \left[(24/\pi)\ M/(\pi\rho_p)\right]^{1/3} = 1.97\left[M/(\pi\rho_p)\right]^{1/3} \tag{17}$$

Smoluchowski (1979) has recently presented several other characteristic distances in the form of an expression like the above but with different coefficients:

1. Outer limit beyond which all encountered particles can accrete upon a satellite: coefficient 2.62.

2. Roche's limit: coefficient 2.21.

3. Inner limit within which only molecules can accrete: coefficient 1.31.

4. Inner limit for break-up by tidal fracture at an assumed tensile strength of 1 bar (Aggarwal and Oberbeck, 1974) in circular orbit: coefficient 1.25.

5. Inner limit for break-up by tidal fracture at an assumed tensile strength of 1 bar (Aggarwal and Oberbeck, 1974) in radial orbit: coefficient 1.08.

It is evident that we have found a process that could have been effective well inside Roche's limit provided that the parent satellite had a low tensile strength.

We turn to the second case for consideration in which the tensile strength does not allow full relaxation to fill the Roche lobe. In that case we anticipate that extrusion beyond that lobe will occur near the plane through the satellite's centre and normal to the mean radius vector from the planet, assuming co-rotation of the satellite. Injections of material will then occur only into horseshoe paths from an inward preceding region and an outward following region, the material returning to the surface upon an inward following region and an outward preceding region. This process will leave an unpopulated region between an inner and an outer ring. The reader will recognise the model of Goldreich (1979) as amplified by Dermott and Murray (1979).

As already described above, each ring should remain condensed and retain the orbital eccentricity and apse of the satellite until the satellite becomes too small to control the system. Thereafter the rings themselves will interact near the satellite. So long as the satellite controls the developing ring it will have its narrowest radial extent at the satellite and its widest extent on the opposite side of the planet. Once the satellite loses control the differential rate of progression of the apse across the ring will take effect.

This process may be thought of as starting from two tangent ellipses. If this tangency occurs at apoapsis, then as the ellipses progress at different speeds they will separate and only close when the periapses line up again. At these aligned epochs interaction may be expected to slow down the outer ring into a more eccentric orbit and speed up the inner ring into a less eccentric orbit. In this case the self-resonant state of Goldreich and Tremaine (1979b) cannot be reached and the two rings would ultimately coalesce.

If we start with tangency at periapsis, then as the ellipses progress at different speeds, disconnection will occur until the periapses again line up. At these times the eccentricity of the outer ring will decrease and that of the inner ring will increase until the self-resonant steady-state of Goldreich and Tremaine (1979b) is reached. This state is characterised by a common rate of revolution of the apse throughout the ring at the rate for a single particle falling in the middle of the ring. The rate is determined by the gravitational field of the oblate planet combined with that of the ring.

If tangency occurs after apoapsis and before periapsis, the differential rate of progression of the apses will disconnect the rings until reconnection occurs on the other side between periapsis and apoapsis. In that case and also if we begin from that configuration of tangency after periapsis and before apoapsis, we anticipate an intense interaction steadily altering both orbits and propagating back toward the two periapses. It is reasonable to anticipate that this one swing toward the two periapses will also lead to the steady-state self-resonant rigidly precessing ring discussed by Goldreich and Tremaine (1979b) provided that the starting eccentricity of the parent satellite is sufficiently large.

Near the end of the life of the satellite an interactive region of collisions between particles should develop before and behind the satellite each time the satellite passes through periapsis and continue until passage through apoapsis. After the satellite has passed apoapsis, the preceding and following wake will open up, partially filling in the space between the streams and the process will repeat again the next time around.

It would then appear that, as the satellite loses control, a region of interaction on the outward half of the orbit from periapsis to apoapsis would develop and then contract toward periapsis to reach the self-resonant steady and rigidly precessing state of Goldreich and Tremaine (1979b). The sharp boundaries on the ε-Ring are then to be understood as a pair of self-gravitatingly condensed rings. This removes the need to postulate a pair of satellites constraining the Ring by resonances from within and without as proposed by Goldreich and Tremaine (1979b).

Of course, if the satellite is in circular orbit, the two rings would coalesce as the satellite vanished. These points need further detailed study, probably by means of numerical modelling.

The Voyager encounters with Saturn will present an opportunity to look for this type of structure in the outer part of the Rings with the purpose of finding its inner limit from which the density of the particles might be obtained, at least in the vicinity of that limit.

JUPITER'S RING

Owen et al. (1979) have presented a description of the new Ring about Jupiter discovered during the encounter of Voyager 1 with Jupiter (Smith et al., 1979a) and extensively observed during the later encounter of Voyager 2. The essential points of the Voyager 2 encounter were that the Jovicentric declination of the Sun was -0.03°, i.e., two-thirds of the solar diameter was south of the equatorial and ring plane and one-third was north of it. The brightest and rather narrow portion of the Ring was all that could be seen in back-scatter and exhibited a radius at the outer edge (measured when the spacecraft was in the plane) of 126,380 ± 140 km (1.772 ± 0.002 equatorial radii of

Jupiter). As seen during eclipse of the spacecraft by the planet (or eclipse of the Sun by Jupiter as seen from the spacecraft) at Jovicentric declination of the spacecraft of -2.0°, the inner edge of the brightest part was at 125,580 ± 140 km (1.761 ± 0.002 radii) and the inner edge of a less bright shelf lay at 120,400 ± 200 km (1.688 ± 0.003 radii). The Ring extended as a relatively faint disk all the way down to the edge of Jupiter's shadow and presumably does so to its atmosphere.

The Ring has a characteristically orange color viewed in narrow forward scatter. From its change in brightness with scattering angle the radius of the particles was estimated at about 4 μm.

An attempt to explain the 6000 km width from the outer edge to the shelf has been made by Burns (1979) in terms of forced eccentricities of orbits of such small particles under solar radiation pressure. The relevant formula was evidently taken from an earlier review paper (Burns, 1977) in which the oblateness of the planet was neglected and the Sun was seen to circle about the elliptic orbit in the planet's orbital period. In fact, it will do so in the much shorter time occasioned by the progression of the apse. The relevant formula (Burns, 1977, p. 149) is

$$e^2 - e_o^2 = (9/2)\ \beta^2 (a/a_p)(M_\odot/M_p)(1 - e_o^2)(1 - \cos n_p t), \tag{18}$$

where e_o is the minimum orbital eccentricity, e its current value, β is the ratio of solar radiation pressure to gravity, a is the semi-axis-major of the particle's orbit about the planet, a_p that of the planet about the Sun, $M_\odot$ the mass of the Sun, M_p that of the planet, n_p is the mean motion of the planet and time, t, is reckoned from an epoch of minimum orbital eccentricity, necessarily at western elongation. If account is taken of the oblateness of the planet, we must replace n_p by

$$d\tilde{\omega}/dt - n_p$$

where $d\tilde{\omega}/dt$ is the rate of revolution of the apse given by (Brouwer and Clemence, 1961, pp. 55, 56 and 70)

$$\mu = k^2(M_p/M_\odot), \tag{19}$$

$$a^3 n^2 = \mu \left[1 + J(R/a)^2 + (1/2)\ K(R/a)^4\right], \tag{20}$$

$$d\tilde{\omega}/dt = n \left\{ J(R/a)^2 - \left[(J/2)(R/a)^2\right]^2 + K(R/a)^4 \right\}, \tag{21}$$

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where n is the mean motion of the particle, k is the Gaussian constant of gravitation, R is the equatorial radius of the planet and J and K are the normalised second and fourth moments of mass of the planet about its polar axis. The constants are taken from Brouwer and Clemence (1961) and Smoluchowski (1976), and e_o, the proper eccentricity, is taken as zero to find

$$0 \leq e \leq 0.105\beta,$$

where we have re-scaled the amplitude by a factor $[n_p/(d\tilde{\omega}/dt - n_p)]^{1/2}$ to allow for the much shorter period of the pumping by the radiation pressure.

We anticipate that minimum eccentricity (zero) will occur at western elongation of the perijove so that at that position the orbit will extend in both directions by the amount, a. By the time that the perijove has moved to superior conjunction, the two elongations of the orbit will extend approximately to mean distance, a, for small eccentricity, and there will have been a shallow minimum in apparent eastern extent and a small maximum in apparent western extent in between the western elongation and superior conjunction. A similar pattern will have applied if we extrapolate backward from eastern elongation to inferior conjunction of the perijove.

In the other half circle the large shifts in the edges of the orbit appear, at eastern elongation of the perijove the edges of the projected orbit extend $a(1 - e)$ to the east and $a(1 + e)$ to the west. Thus the apparent width of the Ring as a whole caused by this effect should be ae with the two mid-points of the bright ring lying at $a(1 - e/2)$ and $a(1 + e/2)$ respectively. The width of the Ring implies that $0.105\beta = 0.0062$, $\beta = 0.055$ which is compatible with the 4 μm radii of the particles estimated from the diffraction lobe in the scattering function.

We have not followed Burns (1979) in identifying the wide 6000 km width (inner edge of shelf to outer edge) as caused by this effect but here instead tested the narrow 800 km width as the result of radiation pressure. Burns' hypothesis would yield $\beta = 0.46$, entirely too large for these particles. It would appear that he used the mean motion of the planet instead of that of the apse less that of the planet. As indicated above, the effect is reduced in proportion to the square root of the ratio of these angular velocities.

A search for asymmetry in the extent of the Ring should be made in future observations from space. According to the above discussion it should equal the width of the brightest narrow part of the Ring, i.e., the Ring should be about 400 km out of centre toward the west.

The new satellite, 1979 J 1, discovered during Voyager 2's passage by Jupiter (Smith _et al._, 1979b) is not the source of the small particles if we follow the spirit of the foregoing discussion of the Rings of Uranus. Instead we must postulate the existence of a circular ring at about 1.766 equatorial radii of Jupiter and less than 10 km wide embedded in the system. Such an embedded ring of particles on the scale of one or more kilometres would not have been seen against the background of small particles in the Voyager pictures. The small particles would then be injected by meteoroidal bombardment and spiral inward under magnetospheric plasma and radiative drag. The inner edge of the 5200 km wide shelf may involve resonances with one or more satellites like 1979 J 1.

A PRE-HISTORIC RING AROUND THE EARTH?

J. A. O'Keefe (in preparation) points out that a drop of 20K in winter temperatures occurred 34 million years ago in both hemispheres of the Earth and persisted for 1 to 2 million years. The most straightforward explanation is a temporary ring around the Earth. The alternative that a tremendous torque of unknown origin re-oriented the Earth's axis to increase the obliquity of the ecliptic from 5° to 10° before to 25° to 30° afterward has no appeal. The alternative that the Arctic Sea was all but cut off in circulation from the North Atlantic Ocean and that Antarctica arrived beneath the South Pole under continental drift at that time isn't much better as the subsequent simultaneous warming should only have occurred in the northern hemisphere by an improved connection of the Arctic Sea to the North Atlantic Ocean which would have had no counterpart in the southern hemisphere. This cooling of winters was the terminal event of the Eocene Era.

In an attempt to account for such a ring, O'Keefe points out that the largest strewn field of tektites and microtektites is also 34 million years old. It extends from the Atlantic coast of the United States and the Caribbean Sea across North America and the Pacific Ocean onto the Indian Ocean over an arc of about 180° and may reach farther in both directions. There is also no putative terrestrial impact site or astrobleme associated with this field. In any case such an impact would have left the Earth's upper atmosphere filled with dust for 2 or 3 years, produced depressed temperatures year-round for that interval after which conditions would have returned to those prevailing previously. O'Keefe, therefore, suggests a lunar origin for this event. He prefers a volcanic origin to an impact origin as the latter requires a crater with an enormous ray structure like that of Tycho and such a crater does not appear in the proper place on the Moon nor indeed anywhere other than at Tycho itself. Impacts which did not produce very long rays may be presumed to have either ejected vapourised material or nothing from the Moon.

O'Keefe places the lunar volcano between 30° and 150° East selenographic longitude and 15° North and South selenographic latitude. For departure velocities extrapolated back to the lunar surface of 2.5 to 3.0 km s^{-1}, 20% should reach the Earth's atmosphere. Slower ones fall back onto the Moon, faster ones escape into heliocentric orbit.

He estimates the mass in the strewn field at $10^9 - 10^{10}$ metric tonnes whence there should have been $5 \times 10^9 - 5 \times 10^{10}$ tonnes in geocentric orbit. He adopts 2.5×10^{10} tonnes which corresponds to 10 km^3 of rock. He distributes the rocks in a ring from 1.5 to 2.5 equatorial radii of the Earth in particles 50 μm in radius to establish an optical thickness of 0.3. Such a cloud would be flattened rapidly by collisions amongst the particles. The radius of the particles was chosen to keep them above the Earth's atmosphere for 1 to 2 million years in the presence of the Poynting-Robertson effect and magnetospheric plasma sweeping.

It is important to note that the discovery of an astrobleme at either end of the strewn field would only invalidate association of the tektites with the ring. It would not invalidate the arguments for a lunar volcanic origin of the particles in the ring. It would imply that we have yet to identify or recognise the strewn field of lunar particles associated with the creation of the ring.

NEPTUNE

It is evident from the foregoing discussions that there is nothing to prevent the presence of narrow rings of the Uranian type about Neptune.

SUMMARY

We have arrived at a relatively satisfactory understanding of the manner in which narrow rings might be formed. Uniform rings have long been regarded as formed in place during the formation of the planet. The profiles of the divisions and gaps in Saturn's Rings cannot really be studied from the point of view of arriving at explanations for them until several required observations are in hand: These are (1) a complete search from satellites like 1966 S 1, 1966 S 2, ..., 1979 S 1 during the current observing season, (2) completion of measurement of fine detail in the Rings from the Pioneer 11 observations and (3) measurements of the much finer detail which should be seen from the two Voyager spacecraft.

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REFERENCES

Aggarwall, H.R., and Oberbeck, V.R.: 1974, Astrophys. J. 191, p. 577.
Brouwer, D., and Clemence, G.M.: 1961, in "Planets and Satellites" G.P. Kuiper and B.M. Middlehurst, eds., U. of Chicago, p. 31.
Burns, J.A.: 1977, in "Planetary Satellites", J.A. Burns, ed., U. of Arizona, Tucson, p. 113.
Burns, J.A.: 1979, Late paper presented at the meeting of the Division of Planetary Sciences, Amer. Astron. Soc., St. Louis, 26 October.
Dawes, W.R.: 1851, Mon. Not. Roy. Astron. Soc. 11, p. 52.
Dermott, S.F., and Murray, C.D.: 1979, Bull. Amer. Astron. Soc. 11, p. 596.
Gehrels, T., Baker, L.R., Beshore, E., Bleman, C., Burke, J.J., Castillo, N.D., DaCosta, B., Degewij, J., Doose, L.R., Fountain, J.W., Gotobed, J., Kenknight, C.E., Kingston, R., McLaughlin, G., McMillan, R., Murphy, R., Smith, P.H., Stoll, C.P., Strickland, R.N., Tomasko, M.G., Wijesinghe, M.P., Coffeen, D.L., and Esposito, L.: 1980, Science, 207, p. 434.
Goldreich, P.: 1979, in "Dynamics of the Solar System", R.L. Duncombe, ed., Reidel, Dordrecht-Holland, p. 191.
Goldreich, P., and Tremaine, S.: 1978, Icarus 34, p. 320.
Goldreich, P., and Tremaine, S.: 1979a, in "Instabilities of Dynamical Systems", V.G. Szebehely, ed., Reidel, Dordrecht-Holland, p. 129.
Goldreich, P., and Tremaine, S.: 1979b, Nature 277, p. 97.
Goldreich, P., and Tremaine, S.: 1979c, Astron. J. 84, p. 1638.
Guérin, P.: 1973, Icarus 19, p. 202.
Hagihara, Y.: 1961, in "Planets and Satellites", G.P. Kuiper and B.M. Middlehurst, eds., U. of Chicago, p. 95.
Ingersoll, A.P., Orton, G.S., Neugebaurer, G., Münch, G., and Chase, S.C.: 1980, Science, 207, p. 439.
Johnson, P.E., Kemp, J.C., King, R., Parker, T.E., and Barbour, M.S.: 1979, submitted to Nature.
Nicholson, P.D., Persson, S.E., Mathews, K., Goldreich, P., and Neugebauer, G.: 1978, Astron. J. 83, p. 1240.
O'Keefe, J.A.: 1980, submitted to Nature.
Owen, T.C., Danielson, G.E., Cook, A.F., Hansen, C., Hall, V.L., and Duxbury, T.C.: 1979, Nature 281, p. 442.
Smith, B.A., Soderblom, L.A., Ingersoll, A.P., Collins, S.A., Shoemaker, E.M., Hunt, G.E., Masursky, H., Carr, M.H., Davies, M.E., Cook, A.F., Boyce, J., Danielson, G.E., Owen, T.C., Sagan, C., Beebe, R.F., Veverka, J., Strom, R.G., McCauley, J.F., Morrison, D., Briggs, G.A., and Suomi, V.E.: 1979a, Science 204, p. 951.
Smith, B.A. et al. (as above): 1979b, Science, 206, p. 927.
Smoluchowski, R.: 1976, in "Jupiter", T. Gehrels, ed., U. of Arizona, Tucson, p. 3.
Smoluchowski, R.: 1979, Nature 280, p. 377.

Post scriptum: The spread between the coefficient in (17) and Roche's limit is too small to accommodate the observed range in distances of the Rings of Uranus unless the density of the particles increases inward. The density which satisfies (17) just inside the κ-Ring is 2.9 g cm^{-3} and that which puts Roche's limit just outside the ε-Ring is 2.3 g cm^{-3} which suggests that the density of the ε-Ring is somewhat lower than 2.3 g cm^{-3} and that of the κ-Ring is somewhat more than 2.9 g cm^{-3}.

If we use 2.0 g cm^{-3} as an estimate for the ε-Ring along with Goldreich and Tremaine's (1979b) mean surface density of 25 g cm^{-2} and an optical thickness of unity we find 15 cm for the radius of the particles. This size applies only away from the edges of the Ring. It need not, and indeed, cannot apply at the edges. The condensed Rings and the edges of the ε-Ring can only involve two adjacent rings of large particles of radius about one quarter the widths of the narrow Rings, i.e., the dominant masses must be provided by kilometre sized particles. The dominant particles in terms of cross section may be of the much smaller size just found.

Goldreich and Tremaine (1979c) have just published a new treatment of the procession of the ε-Ring of Uranus. They do not consider the case in which the shape of the profile is conserved with respect to true anomaly even though this is strongly suggested by the observed profiles (Nicholson et al. 1978, figs. 5 and 6, p. 1246) and is an expected consequence of dissipation by collisions amongst particles in the ring. Their solutions involve radial oscillations of the epicyclic frequency. It seems strange to this reviewer to consider such hypothetical oscillations when observations imply that they probably do not occur and they are not expected in the presence of dissipation by collisions. The oscillations are induced in their solutions by adoption of four starting distributions of mass at true anomaly ±90°. It would seem much more to the point to assume that the shape of the profile is conserved.

DISCUSSION

Lokanadham: What are the main differences in the character of Jupiter's rings as compared with Saturn's rings?
Cook: The Jovian Ring appears to resemble Rings C and D of Saturn if we accept the model of the latter two in terms of small particles.

Keller: Recently I saw some pictures released by JPL in which captions claimed that there is structure in the ring, even gaps. You don't seem to confirm this.
Cook: The combination of drift of the orientation of the spacecraft and of oscillation of the scan platform between two limit switches can generate spurious structure from the structure which we regard as certain and report in this paper. A final conclusion must await a full deconvolution for the pattern of drift.

Grün: Why do you believe that Jupiter's ring is made out of 4 μm sized particles?
Cook: The gradient of brightness of the Ring decreases in the direction moving away from the Sun in a particular image and the brightness diminishes toward shorter wavelengths. A fit of the diffraction lobe of published scattering curves leads to our estimate of 4 μm for the characteristic radius of the particles.

Grün: What is the current view on the particle sizes of the rings of Saturn and Uranus?
Cook: These lie in the centimetre to metre range of radii, at least for Rings A and B around Saturn and also for Ring G of Uranus. For Saturn the evidence is the absorption of the radiation of the planet at centimetre wavelengths, the radar return from the rings at decimetre wavelengths, and the drop in emission as we proceed from μm through mm wavelengths, combined with the identification of solid H_2O absorption in spectra of the rings. For Ring G of Uranus I rely on the dynamical analysis by Goldreich and Tremaine.