## THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of The Mathematical Gazette, Mathematics in School, Primary Mathematics (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is 259 London Road, Leicester LE2 3BE, UK (telephone 0116221 0013). The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to Teaching Committee should be addressed to Mr Drew Foster. The Association's Library is housed in the University Library, Leicester.
Views expressed in the Mathematical Gazette by authors or advertisers are not necessarily those of the Association.

## THE MATHEMATICAL GAZETTE

## Editor:

Dr Gerry Leversha, 15 Maunder Road, Hanwell, London W7 3PN
g.leversha@btinternet.com

Production Editor:
Mr Bill Richardson, Kintail, Longmorn,
Elgin IV30 8RJ
wpr3145@gmail.com

Problem Corner:
Mr Nick Lord,
Tonbridge School,
Tonbridge,
Kent TN9 1JP
njl@tonbridge-school.org
Reviews Editor:
Mr Owen Toller, 4 Caldwell House, 48 Trinity Church Road, London SW13 8EJ
owen.toller@btinternet.com

Potential advertisers e-mail Charlotte Dyason at: charlotted@media-shed.co.uk Material for publication should be sent to the Editor. Books for review should be sent to the Reviews Editor.

Advice to authors of notes and articles.
Study the format of articles in the Gazette. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the Gazette was produced on an Acorn machine using TechWriter and Draw.
10.1017/mag.2021.103

## CONTENTS (continued)

## Notes 105.16 to 105.37 (continued)

Visual proofs of $x^{3}+y^{3}=(x+y)\left(x^{2}-x y+y^{2}\right) \quad$ Manishita Chakraborty ..... 303and $x^{3}-y^{3}=(x-y)\left(x^{2}+x y+y^{2}\right)$
Conditional $2 \times 2$ matrices with three Mehdi Hassani ..... 305
prime elements and given determinant
Families of curves orthogonal to the Zafar Ahmed, ..... 306 lines $y=m x-2 m-m^{3}$ Pallavi S. Telkar
Areas above and below a curve A.F. Beardon ..... 309
A remark on $\lim _{n \rightarrow \infty} \sqrt[p_{n}]{p_{1} p_{2} \ldots p_{n}}=e$ Reza Farhadian ..... 311
Periodic Möbius sequences Clive Johnson ..... 312
A geometric memory Des MacHale ..... 318
A Cinderella theorem in circle geometry Gerry Leversha ..... 323
On the Brocard disc Martin Lukarevski, ..... 327
J. A. Scott
The side-angle duality in geometry: a direct proof Mario Dalcín ..... 329 of sufficiency of a cyclic quadrilateral theorem
A property of bifocal conics, illuminated by David Rose, Li Zhou ..... 331Ptolemy's theorem
The difference between $k$-gonal numbers with Günhan Caglayan ..... 333
$(2 n-1)$ sides and $(n-1)$ sides is $(3 k-4)$-gonal
Reconciling remainders in Maclaurin expansions Nick Lord ..... 334
New error analyses for some old Nick Lord ..... 339 mensuration formulae
A Potter's brief John D. Mahony ..... 343
Teaching Notes
Learning from a mistake Prithwijit De ..... 349
An instructive question Owen Toller ..... 350
Notices
1933 Cumulative Index (A) Bill Richardson ..... 270
1933 Cumulative Index (B) Bill Richardson ..... 348
Graham Hoare: an appreciation Nick Lord, Jenny Ramsden ..... 356
Feedback ..... 354
Problem Corner Nick Lord ..... 358
Student Problems Beth Woollacott ..... 365
Reviews ..... 368

## CONTENTS

## Articles

Passionate souls: Elisabeth of Bohemia Tomoko L. Kitagawa ..... 193
and René Descartes
An unexpected characteristic of tournament Leonard M. Wapner ..... 201 predictive power
A group of Pythagorean triples using Howard Sporn ..... 209
the inradius
The lost boarding pass and other Geoffrey R. Grimmett, ..... 216
David R. Stirzaker
P. Stanley ..... 222
On the frequency of Friday the thirteenth
Parabolic coordinates
Steven J. Kilner, ..... 226David L. Farnsworth
Hole dissections for planar figures Greg N. Frederickson ..... 237
Equality of areas among the ears Mowaffaq Hajja ..... 245
of the Routh triangle
On the structure of polynomial roots R. W. D. Nickalls ..... 253
Integral inequalities in probability Lazhar Bougoffa, ..... 263 theory revisited Panagiotis T. Krasopoulos
Matter for Debate
What makes a good Proof without Words? Gerry Leversha ..... 271
Notes 105.16 to 105.37
Some elementary results in number theory Des MacHale ..... 280
A tale of two sixes Adam McBride ..... 285
Notes on Listener Crossword 4595 by Elap Chris Starr ..... 291
An inverted cone and Fermat's last theorem Angad Singh ..... 298
Revisiting even and odd square-free numbers G. J. O. Jameson ..... 299
Factorial factors revisited John R. Silvester ..... 301
Proof without words Mehdi Hassani ..... 303
© The Mathematical Association 2021
Typeset by Bill Richardson

# CAMBRIDGE UNIVERSITY PRESS 

Printed in the UK by Bell \& Bain Ltd.<br>ISSN 0025-5572

