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BOOK REVIEWS

GILES, J. R., *Convex analysis with application in the differentiation of convex functions* (Research Notes in Mathematics No. 58, Pitman, 1982), £9.95.

Like other terms used to denote areas of Mathematics, "convex analysis" means different things to different people. The core of J. R. Giles's book is a systematic account of the connections between certain properties that Banach spaces (or their duals) may have and Krein–Milman type statements for convex subsets of these spaces. The Krein–Milman statements may be formulated using either extreme, exposed or strongly exposed points. The main Banach space properties in question are (1) the Radon–Nikodym property, defined as: every non-empty, bounded set has "slices" of arbitrarily small diameter, and (2) the Asplund property: every continuous, convex real function on an open, convex subset A is Fréchet-differentiable on a dense G_δ-subset of A.

During the last fifteen years, an extensive theory has been developed by Phelps, Asplund, Stegall, Kenderov and others. For example, the Radon–Nikodym property is equivalent to each bounded, closed, convex set being equal to the closed, convex hull of its strongly exposed points. A space has the Asplund property if and only if its dual has the Radon–Nikodym property. A space has the Radon–Nikodym property if and only if its dual has the "weak-star Asplund property". Many other characterisations of the Asplund property are known, for example, every separable subspace has a separable dual or (an apparent weakening of the definition) every equivalent norm is Fréchet-differentiable on a dense G_δ-subset.

A detailed treatment is given to Ekeland's theorem on lower semi-continuous functions on complete metric spaces. Though clearly not "convex analysis" in any possible sense of the phrase, this theorem has important applications to the subject. It is used here to establish one of the equivalents of the Asplund property, and also to prove the Brønsted–Rockafellar generalisation of the Bishop–Phelps theorem, in which support functionals of convex sets are replaced by "subdifferentials" of convex functions.

Two other topics included "as an epilogue" are the relationship between denting points and the Mazur intersection property, and the problem of convexity of Chebyshev sets.

The author has performed a useful task in assembling and organising this material, in an area in which results have often been rapidly superseded by stronger ones. Proofs are clear and complete, and the original sources are always mentioned. There is a generous scattering of examples and exercises throughout. Prerequisites are minimal, since the first 100 pages are largely devoted to an outline of the relevant background material on convex sets, locally convex spaces and General Topology. The reviewer's only complaint is that the author misses the opportunity to publicise Léger's beautiful proof of the Krein–Milman theorem.

G. J. O. JAMESON

FUHRMANN, PAUL A. *Linear Systems and Operators in Hilbert Space* (McGraw-Hill, 1981), x+325 pp. £19.50.

Must even electrical engineering succumb to the ethos of Bourbaki? Had the illustrious master taken employment in Philips Research Laboratories he might have captured linear systems for pure mathematics as does this book.

In fact there are those in both mathematics and engineering departments who have applied the abstract modern approach to linear algebra (using modules over a principal ideal domain) to linear systems. The axiomatic study of these systems dates from the 1950's. A system transforms a sequence (u_j) of inputs into a sequence (y_j) of outputs: the u_j and y_j are taken to be vectors from spaces u, y respectively. Making the identifications $(u_j) \leftrightarrow \sum u_j \lambda^j$, $(y_j) \leftrightarrow \sum y_j \lambda^j$ we obtain a mapping $U[[\lambda]] \to Y[[\lambda]]$ of the spaces of formal power series over U, Y, and under fairly mild assumptions this mapping is a module homomorphism with respect to the action of the ring of polynomials $C[\lambda]$. Many questions about systems can thus be stated in module-theoretic terms: some convert to familiar questions with classical solutions, while others (notably ones about factorising homomorphisms) appear to be new. The basic theory was worked out in down-to-earth concrete fashion by engineers and has now been elevated into some elegant abstract algebra.

The internal states of some systems are better described by functions than finite vectors, and so there is a need for an infinite-dimensional version of the theory. Here things are very much more complicated. Aside from the introduction of topological considerations, there is the difficulty that $C[\lambda]$ has to be replaced by a ring of analytic functions which is not a principal ideal domain. Fortunately, the necessary generalisations of canonical form theory were being developed at exactly the right time by operator theorists, though in a language so different from the engineers' that the two lines of development did not make contact till around 1975. This must be the first book by an author expert in both operator theory and linear systems, and both camps should welcome it heartily.

The book contains three chapters. The first is a succinct presentation of the algebraic theory. The second takes up over half the book and is devoted to operators on Hilbert space, starting with the definition and covering an immense range of material right up to recent work on canonical models. The third chapter effects a synthesis of the first two, elaborating the theory of linear systems whose states can be represented by elements of a Hilbert space.

The author aims to reach both mathematicians and systems scientists, though he admits that the latter would have to devote "maybe even considerable" time and effort to its study. The "maybe" could safely have been omitted, for Chapter two is substantial fare indeed for all but an operator theorist: it contains more than some full length texts on operator theory, in extremely concise form and without exercises. Surely any engineer who wrestles with, for example, Carleson's Corona Theorem and the Nagy–Foiaş theory of canonical models just to be able to deal with transcendental transfer functions is a pure mathematician *manqué*. However, the book is finely written, well produced and reasonably priced; the first chapter should be accessible to, and rewarding for every mathematician with the slightest hankering for applications, while the rest of the book can be strongly recommended to operator theorists for its novel and enriching viewpoint.

N. J. YOUNG

MATSUMURA, H., *Commutative Algebra*, Second Edition (Benjamin/Cummings, 1980), 313 pp., $19.50.

The first edition of this book was published in 1970, and it provided a stimulating introduction to classical and homological commutative algebra, as well as to Grothendieck's reworking of parts of the classical theory. The lectures on which the book was based were paralleled by a course on algebraic geometry. Fortunately, since 1970, a number of outstanding books have been published in this area, and can now serve as companion works.

As regards the second edition of the book, the chapter on depth has been rewritten, and this, together with a few alterations and additions, is the only change to the original material; a large appendix, covering a diverse range of topics, has also been added. The index has been enlarged.

The new exposition on depth includes a discussion of quasi-regular as well as regular sequences, while a few elegant homological results have been added to shorten and clarify some of the original proofs. It is slightly unfortunate, however, that the new version leaves as implicit matters which formerly were made explicit. For example, the inequality $\operatorname{depth} M \leqq \dim M$, the effect on depth of factoring out a regular sequence, and the connection between graded rings and polynomial rings arising from certain ideals in a Cohen–Macaulay ring, all have to be ferreted out to a greater or lesser extent.

In the main, the appendix covers developments in areas involving the first module of differentials, though a proof of the Eakin–Nagata theorem, following Formanek, is given. Thus