LONG-TERM SOLAR IRRADIANCE VARIABILITY

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Abstract. Measurements of the solar energy throughout the solar spectrum and understanding its variability provide important information about the physical processes and structural changes in the solar interior and in the solar atmosphere. Solar irradiance measurements (both bolometric and at various wavelengths) over the last two decades have demonstrated that the solar radiative output changes with time as an effect of the waxing and waning solar activity. Although the overall pattern of the long-term variations is similar in the entire spectrum and at various wavelengths, being higher during high solar activity conditions, remarkable differences exist between the magnitude and shape of the observed changes. These differences arise from the different physical conditions in the solar atmosphere where the irradiances are emitted. The aim of this paper is to discuss the solar-cycle-related long-term changes in solar total and UV irradiances. The space-borne irradiance observations are compared to ground-based indices of solar magnetic activity, such as the Photometric Sunspot Index, full disk magnetic flux, and the Mt. Wilson Magnetic Plage Strength Index. Considerable part of the research described in this paper was stimulated by the discussions with the late Philippe Delache, who will always remain in the heart and memory of the author of this paper.

1. Introduction

The Sun, a fairly typical star, dominates the physical conditions throughout the solar system due to its influence on planetary atmospheres and the interplanetary medium. The total radiative output of the Sun establishes the Earth's radiative environment and controls its temperature and atmospheric composition. This mostly continuum radiation originates in the

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photosphere and its major part reaches the troposphere and the Earth's surface and oceans. Consequently, small but persistent variations in the solar energy received on the top of the Earth's atmosphere ("solar constant") may be responsible for slow climate changes such as produced the Little Ice Ages, which was accompanied with an unexceptionally low magnetic activity of the Sun, known as the "Maunder Minimum" (Nesme-Ribes *et al.*, 1994). Therefore, the accurate knowledge of the solar radiation received by the Earth and its temporal variations, especially over decades, is critical for an understanding of the role of solar variability in climate change and the climate response to increasing greenhouse gas concentrations.

Although the existence of possible global climate changes based on the changing solar constant had been doubted and debated for a long time, the results of various space experiments for monitoring solar irradiance opened an exciting new era in both atmospheric and solar physics. Various space-borne observations of total solar irradiance over the last two decades established conclusively that total solar irradiance is not constant and the changes on time scales from minutes to the 11-year solar activity cycle are related to changes in the Sun's interior and atmosphere (Willson and Hudson, 1988; Fröhlich et al., 1991). High precision photometric observations of solar-type stars clearly show that year-to-year variations connected with magnetic activity is a widespread phenomenon among such stars (Radick, 1994). As the nearest star, the Sun is the only star where we can observe and identify a variety of structures and processes which lead to irradiance variability on time scales of minutes to decades. On the other hand, the observable characteristics of other stars expand our knowledge on how the Sun works, simply by enlarging the sample to a larger set of conditions.

To establish the possible link between solar variability and climate change, it is necessary to analyze long-term data sets representing solar activity. Fortunately, ever since the earliest telescopic observations, the Sun's variability in the form of sunspots and related magnetic activity has been the subject of careful study. Considerable efforts have been made to develop irradiance models to help identify their physical causes and to provide irradiance estimates when no satellite observations exist. The ultimate goal is to uncover how and why the Sun is changing in order to reconstruct and predict the solar-induced climate changes.

2. Variations Observed in Solar Irradiance

The value of the integrated solar energy flux over the entire solar spectrum, hence total irradiance, arriving at the top of the Earth's atmosphere at 1 AU is called the "solar constant". Continuous observations of total solar irradiance to detect its variability started at the beginning of this century at the



Figure 1. The time series of various space-borne irradiance experiments are presented (from Fröhlich, 1997). Fig. 1 has been provided for this paper by courtesy of the SOHO/VIRGO Science Team.

Smithsonian Institute, first from high altitude mountain stations and later on from balloons and aircraft. These early measurements, however, could not reveal variations in total solar irradiance related to solar effects because of the lack of sufficient radiometric precision and the selective absorption of the terrestrial atmosphere (Fröhlich *et al.*, 1991).

The first continuous and high precision observations of total solar irradiance from space started in the late seventies from various satellite platforms. The time series of the space-borne experiments are plotted in Fig. 1 (from Fröhlich, 1997). The different scale of these measurements is related to the absolute accuracy ($\pm 0.2\%$) of the calibration of the individual instruments (Fröhlich, 1997). Because of the low absolute accuracy of the current total irradiance measurements, it is extremely difficult to maintain long-term homogeneous irradiance data sets, especially when there are interruptions in the observed data. The UARS/ACRIM II data presented in Fig. 1 have been scaled to the SMM/ACRIM I irradiance level by Fröhlich (1997) via the intercomparison of the two ACRIM data sets with the Nimbus-7/ERB and ERBS measurements. The used scaling factor is 1.00123 (Fröhlich, 1997).

Although the absolute accuracy of the total irradiance measurements is limited to about $\pm 0.2\%$, the precision and stability of the instruments is much better, which makes it possible to study the relative variations in total irradiance. As illustrated in Fig. 1, total solar irradiance varies over a wide range of periodicities. The most important discovery of the satellitebased irradiance observations is that total solar irradiance varies by about a small fraction of 1% over the solar cycle, being higher during maximum solar activity conditions (e.g. Willson and Hudson, 1988). This solar-cyclerelated variation of total solar irradiance is attributed in a large part to the changing emission of bright magnetic elements and a third component, the so-called "active network", has been introduced to explain the long-term irradiance changes (Foukal and Lean, 1988).

Since the Sun's irradiance is observed from one direction in space, it is difficult to determine whether the observed irradiance variations represent true luminosity changes which occur in the Sun's radiation in all directions or are simply results of a change in the angular dependence of the radiation field emerging from the photosphere. Kuhn et al. (1988) and Kuhn and Libbrecht (1991) have performed broad-band, two-color photometric measurements of the brightness distribution just inside the limb, which can be used to explain the total irradiance variations over the solar cycle. By integrating the limb brightness measurements, one can compute the solar luminosity. It has been demonstrated that the observed ERB and ACRIM total irradiance and the computed luminosity change in phase and relative amplitude, (e.g., Kuhn et al., 1988) and the active regions faculae alone fail by more than a factor of two in explaining the solar cycle related long-term changes (Kuhn et al., 1988). These brightness observations indicate that the long-term irradiance changes may also be related to variations in the photospheric temperature (Kuhn et al., 1988), although it is not yet clear whether this change can be linked with the bright network component. Note that former comparisons of the Swiss infrared measurements (Müller et al., 1975) with visible data have also indicated that there is a latitudinal dependence of the effective temperature of the Sun which may explain the long-term irradiance observations (Pecker, 1994).

The conclusion we can derive from the current measurements is that the solar radiation flux is anisotropic, a function of latitude, and naturally a function of the time during the migration of solar activity (Pecker, 1994). Note that additional global effects, such as changes in the solar diameter (Delache *et al.*, 1986; Ulrich and Bertello, 1995), large scale convective cells (Ribes *et al.*, 1985), the differential rotation of the Sun's interior and solar dynamo magnetic fields near the bottom of the convective zone (Kuhn, 1996) may also produce variations in total irradiance. These results demonstrate that the long-term changes are really luminosity changes, which may play an important role in global changes of the Earth's climate.

Short-term changes on time scales of days to months are superimposed on the long-term irradiance variations, which are primarily associated with the evolution of active regions via the combined effect of dark sunspots and bright faculae (Chapman, 1987). The most striking events in the short-term irradiance changes are the sunspot-related dips in total irradiance with an amplitude up to 0.3% (Willson *et al.*, 1981). The effect of sunspots on total solar irradiance has been modeled with the "Photometric Sunspot Index" (PSI), which relates the area, position, and contrast of sunspots to a net effect on the radiative output of the observed solar hemisphere and it is corrected for the limb darkening (Hudson *et al.*, 1982). The relationship between the PSI model and total solar irradiance is presented in Fig. 2 for the time interval of 1980 to 1994, where the PSI model has been calculated by Fröhlich *et al.* (1994). The dashed line in Fig. 2a indicates the daily values of total irradiance as measured by UARS/ACRIM II (Willson, 1994), whereas the solid line gives its adjusted value to the SMM/ACRIM I scale (Fröhlich, 1997).

As can be seen from Fig. 2, dips in total irradiance always correspond to the peaks in the PSI model, convincing the skeptics that the darkening effect of sunspots on total irradiance has indeed been detected and that the strong magnetic fields of sunspots can cause negative excursions in the total flux. It is interesting to note that in spite of the irradiance deficit due to sunspots, which are the most pronounced at solar maximum, total solar irradiance varies in phase with the solar cycle (Fröhlich et al., 1991). This indicates that the bright magnetic features overcompensate the effect of sunspots and give the primary source of long-term irradiance variations. The total irradiance corrected for sunspot darkening (thereafter S_c) is shown in Fig. 3a, indicating that the solar cycle variability of total irradiance would be considerably larger without the effect of sunspots. It has been found that the long-term variation of S_c is very similar to that of the solar UV irradiance (Foukal and Lean, 1990; Pap et al., 1991) and about one fifth (Lean, 1989) to one third (London et al., 1989) of the long-term change in total irradiance is related to the variability in the integrated 200 -300 nm UV flux. To compare the variation of the UV irradiance with S_c , the combined Nimbus-7/SBUV1 and NOAA9/SBUV2 Mg II h & k core-towing ratio (Mg c/w) is plotted in Fig. 3b. Although the SBUV experiments suffer from a significant degradation in their diffuser reflectivity, the ratio of the irradiance in the core of the Mg 280 nm line to the irradiance at the neighboring continuum wavelengths can be used as a good index of solar chromospheric variability (Heath and Schlesinger, 1986). The combined Nimbus-7/SBUV1 and NOAA/SBUV2 Mg c/w ratio provides a long-term UV irradiance data set covering two solar cycles, which makes it possible to study the long-term UV irradiance changes in a great detail. As can be seen from Fig. 3., Sc and the Mg c/w ratio vary in parallel over the solar cycle. The formation of the Mg II line is very similar to that of the



Figure 2. The Nimbus-7/ERB, SMM/ACRIM I and UARS/ACRIM II total irradiance data are shown in Fig. 2a. Fig. 2b shows the PSI model calculated by Fröhlich *et al.* (1994).

Ca II K line, and the two time series correlate very well (Donnelly *et al.*, 1994). This demonstrates that the long-term variations in total solar and UV irradiances are primarily caused by the same events, i.e., the changing emission of bright magnetic features, such as plages and the magnetic network (Lean, 1988).

It is interesting to note that the variation of the Mg c/w ratio over solar cycles 21 and 22 is very symmetrical. As can be seen from Fig. 3, the Mg c/w ratio shows about a 3-year long "flat" maximum during solar cycles 21 and 22. In contrast to the similar variation of the Mg c/w over the two solar maxima, there is a substantial difference between the decline of the Mg c/w during solar cycles 21 and 22. As can be seen from Fig. 3, the Mg c/w ratio decreased steadily from 1982 to 1986. Note that the decline of solar UV irradiance at Lyman- α was very similar and both the Mg c/w ratio and



Figure 3. Total solar irradiance corrected for sunspot darkening is presented in Fig. 3a. The combined Nimbus-7/SBUV1 and NOAA9/SBUV2 Mg c/w ratio is shown in Fig. 3b.

Lyman- α irradiances showed a pronounced 300-day periodicity during the descending phase of solar cycle 21 (Pap *et al.*, 1990). In contrast, a sharp decrease was observed in both the Mg c/w ratio and Lyman- α between February and June 1992 (White *et al.*, 1994), which was seen in additional solar indices, e.g. in the PSI function, 10.7 cm radio flux and the full disk magnetic flux.

3. Solar irradiance variability and magnetic activity

It has been assumed that the observed long-term changes in total solar and UV irradiances are related to the evolution of the magnetic fields over the solar cycle (e.g. Harvey, 1994). The absolute value of the full disk integrated magnetic flux measured at the National Solar Observatory at Kitt Peak is plotted in Fig. 4. As can be seen, the long-term variations in the full disk



Figure 4. The absolute values of the full disk magnetic flux measured at NSO/Kitt Peak is shown by the solid line, the dashed line shows the net magnetic flux. (The plot has been provided by Dr. Karen Harvey).

magnetic flux as well as in the UV and total irradiances are very similar, especially during the declining and rising portions of solar cycles 21 and 22. However, substantial differences exist between the magnetic field and irradiance values during the maximum and minimum times of the solar cycle. During the maximum of solar cycle 21, total solar irradiance decreased steadily from at least 1980, while the magnetic field peaked almost two years later, in 1982. In addition, as shown in Fig. 3b, the UV irradiance reached the maximum level of solar cycle 22 in late 1989, whereas the maximum of the magnetic flux occurred only in late 1991.

The relationship between total solar and UV irradiances and the magnetic flux during solar minimum was studied by Pap *et al.* (1996) in detail, using the so-called "dispersion diagrams". Any changes shorter than the solar rotational period were considered as noise and removed from the data by calculating monthly averages. The beginning and the end of the minimum time of solar cycle 21 was established from the distribution of the data in the dispersion diagrams. It was shown that the length of solar minimum was much shorter in the case of total and UV irradiances than in the case of solar indices representing strong magnetic fields, such as the full disk magnetic flux and PSI. These results indicate, as also seen in Figs. 2-4, that total solar and UV irradiances started to increase about 10 months prior to the rise of the magnetic flux (and PSI) at the beginning of the ascending phase of solar cycle 22.

Since the full disk magnetic flux includes the magnetic field of both sunspots and plages, the "Magnetic Plage Strength Index" (MPSI) has been used to study the relation between the changes of the UV irradiance and the magnetic flux related to plages. MPSI has been derived from magnetograms taken at the Mt. Wilson Observatory in the Fe I line at 525.0 nm (Ulrich, 1991). MPSI is defined as the sum of the absolute magnetic fields of all pixels with magnetic strength between 10 and 100 gauss divided by the total number of pixels in the image. Chapman and Boyden (1986) showed that pixels with magnetic fields of 10 to 100 gauss are associated with faculae and plages, whereas pixels above 100 gauss represent sunspots. The time series of the Mg c/w ratio (solid line) and MPSI (dotted line) are presented in the upper panel of Fig. 5 for the time interval of November 1978 to September 1986 (from maximum to minimum of solar cycle 21). The lower panel gives the scatter plot diagram between the two time series, which shows their high correlation (r = 0.95). However, the linear relation between the two time series breaks down at the time of solar minimum.

To study the nonlinear relationship between the Mg c/w ratio and MPSI, a relatively new technique called "Singular Spectrum Analysis" (SSA) has been used. SSA has been developed to study nonlinear and chaotic dynamical systems (Vautard et al., 1992). SSA is based on Principal Component Analysis in the time domain. The examined time series is augmented into a number of shifted time series. The cornerstone of SSA is the spectral, i.e., eigenvalue - eigenvector, decomposition of the lagged covariance matrix which is composed of the covariances determined between the shifted time series. The eigenvalues of the lagged covariance matrix compose the Singular Spectrum, where the eigenvalues are arranged in a monotonically decreasing order. The eigenvalues cut-off at a certain order, forming a "tail" in the spectrum which is considered as the noise floor of the data. The number of eigenvalues above the noise floor represents the degree of freedom of the variability, or in other words, the statistical dimension of the data, which is associated with the number of oscillatory components in the signal. The highest eigenvalues represent the fundamental oscillations in the data and in many cases they are related to the trend.

Fig. 6. shows the Singular Spectra of the Mg c/w ratio and MPSI. As can be seen, the two Singular Spectra are very similar, identifying about 50 oscillatory components in the two time series. The most interesting aspect of SSA is the reconstruction of the examined time series above the noise level and/or part of interest. The various oscillatory components can be reconstructed as a projection to the eigenvectors of the lagged covariance matrix. The Reconstructed Components (RCs) related to the first eigenvalues of the Mg c/w ratio and MPSI give the solar cycle related trends,



Figure 5. The upper panel shows the Mg c/w ratio (solid line) and MPSI (dotted line) for the time interval of November 7, 1978 to August 31, 1994. The lower panel shows the scatter plot between the two time series.

plotted in the upper panel of Fig. 7. The lower panel shows the correlation between the two trends, which clearly indicates the nonlinear relation between the solar cycle related trends in the Mg c/w ratio and MPSI. As

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Figure 6. The Singular Spectra of the Mg c/w ratio (upper panel) and MPSI (lower panel) for solar cycle 21.

Fig. 7 shows, using a quadratic or a cubic fit, the long-term change in UV irradiance can be predicted by MPSI with a correlation coefficient: r = 0.998.



Figure 7. The upper panel shows the 1st RCs of the Mg c/w ratio (heavy solid line) and MPSI (heavy-dashed line) for solar cycle 21. The lower panel shows the correlation between the two RCs.

Finally, the 27-day variability in the Mg c/w ratio and MPSI has been reconstructed as well and the results are presented in Fig. 8. As can be seen, the amplitude of the 27-day variability is the strongest at the time of the



Figure 8. The 3rd and 4th Reconstructed Components of the Mg c/w ratio (heavy line) for solar cycle 21 are presented on the upper panel. These RCs represent the 27-day solar rotational component in the signal. The correlation between these components is shown on the lower panel.

solar maximum in both indices and it decreases towards solar minimum. It is interesting to note that there is about a two year modulation in the rotational variability of both indices. The distribution of the data points on the scatter plot diagram (Fig. 8b) indicates that the linear association between the rotational variability of the Mg c/w ratio and MPSI can be divided into 3 various components, which are related to the different phases of the solar cycle, in this particular case, the maximum, declining portion and minimum of cycle 21.

4. Conclusions

Measuring the solar energy output and understanding its variability are extremely important since they provide information about the physical processes in, below, and above the solar photosphere. It has been demonstrated that both total solar and UV irradiances change over the solar cycle, being higher during maximum activity conditions. It has been shown that the long-term variation of the UV irradiance and total irradiance corrected for sunspot darkening is primarily related to the evolution of magnetic fields over the solar cycle via the changing emission of faculae, plages and the magnetic network (Lean, 1988).

Although there is a reasonably good correlation between the long-term variations of solar irradiance (both bolometric and at UV wavelengths) and solar magnetic flux during the declining and rising portions of solar cycle, there are considerable disrepancies at the time of solar maximum and solar minimum. Pap et al. (1996) pointed out a phase shift between the strong magnetic fields concentrated in sunspots and the full disk magnetic flux and solar irradiance at the time of solar minimum. Comparison of the magnetic field strength of plages with solar UV irradiance demonstrates that the strong linear relation between the plage magnetic flux and UV irradiance breaks down at the time of solar minimum. It remains to be seen whether the discrepancies between the solar magnetic flux (both the full disk and its plage component) and solar irradiance variability are related to small and faint magnetic features, which may not be detected by the current magnetic field observations, or there is a nonlinear coupling between the subphotospheric, photospheric, and chromospheric layers. Further studies on this topic are essential to better understand the underlying physical mechanisms of the long-term irradiance variabilities.

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