Dark Matter near the Sun: Simulated Star Counts and the Oort Limit

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ABSTRACT

An ensemble of orbits passing through the solar position have been generated for a specific mass model of the galaxy. These orbits are randomly sampled to form simulated density distributions of tracer stars perpendicular to the galactic disk. The simulated distributions are analyzed in order to determine the sampling errors in a self-consistent derivation of the total amount of matter near the sun (the Oort limit).

The total amount of matter in the vicinity of the Sun can be determined by studying the velocity dispersions and distribution with height above the plane of a population of tracer stars. This problem has a long history (see Oort 1965 and Bahcall 1984a,b for references) and the Oort limits that have been computed over a 50 year period are in fair agreement with each other, ranging from about 0.14 to 0.21 $M_\odot \, pc^{-3}$. Very recently, Bahcall (1984a,b) has improved the theoretical basis for this analysis by using more realistic Galaxy models and by solving the equations self-consistently. He has also estimated the errors in the theoretical analysis from uncertainties in input parameters and from theoretical approximations.

The various techniques and approximations used in translating the observations into a density of tracer stars have not been previously modeled. One wants to have answers to questions like: How many stars must be studied in order to give an accurate determination of the total matter density? How accurate are the estimates based on existing data for F dwarfs and K giants? Does the fact that the tracer stars are located in a cone (and not a cylinder) centered on the Sun affect the answer? We estimate the probable sampling error in the Oort limit by simulating a determination in a model galaxy where the total mass density is known in advance. We have integrated stellar orbits in a model disk galaxy, sampled these orbits in a way that mimics actual observations, formed model density distributions, and then computed the Oort limit using the self-consistent technique of Bahcall (1984a,b). The derived Oort limits were compared with the known value in the input model. We found that samples of about 890 stars have an (unbiased) error of about 10 - 15%. The sampling error arises in two ways; in determining the density profile from star counts, and in measuring the velocity dispersion of the tracer stars. The peculiar shape of the observing region (see below) does not lead to significant errors for parameters similar to those used in constructing the actual observed samples.

In deriving the Oort limit, Bahcall (1984a,b) fit the observed tracer star density distribution with theoretical models, choosing the best-fit model to have the smallest (least-squares) discrepancies. The mass density and scale height of the dark matter were allowed to vary until a best fit was obtained. The tracer stars were usually assumed to be isothermal, although specific departures from isothermality consistent with the observations were considered and found not to be important.

The galaxy model that provided the gravitational potential in our simulation is described in Bahcall, Schmidt, and Soneira (1983). It consists of a single component disk, a central (nuclear) component, a spheroid, and a halo. All four components are needed to reproduce the Galaxy rotation curve, but for the vertical oscillations, only the disk and halo are important. In this model the halo density is about 5% of the disk density at the solar position. The potential was numerically determined and was accessed by the orbit integrator through spline interpolation of tabulated values.

The initial conditions for the orbits were prescribed by random sampling from the disk density distribution and the velocity ellipsoid. First the disk component of the density distribution was sampled to yield random values of z and R. The axisymmetry of the potential did not require a choice of ϑ. The peculiar velocities v_z, v_R, and v_ϑ were chosen from the ellipsoid that was presumed to exist at (z, R). The velocity ellipsoids were constructed to have the same axial ratios as observed at the solar position, but with magnitudes that varied as $\sigma \sim \exp(-R/2h)$. Orbits were computed for those stars that energy and angular momentum conservation permitted, in principle, to reach to the solar position. The radial density gradient in the disk shifted the mean launching position 300 pc inward of the sun to 7.7 kpc.

The sampling procedure for launching tracer stars does not yield an exact solution to the Vlasov equation for our mass model. Numerical integration of individual orbits is necessary in order to form a phase mixed ensemble. Each star was allowed a random number of passages through the disk (with a mean of 5) before its orbit was sampled for use in the simulated star catalogue. Following this initial relaxation period, the stars were allowed about 15 passages through the disk with orbital data being recorded at 100 representative times. An ensemble of 3000 stars formed the catalogue from which random star count distributions were drawn.

We constructed simulated star counts that resemble as much as possible the F star sample described in Hill, Hilditch, and Barnes (1979) (hereafter HHB). The HHB sample consists of two parts. For apparent magnitudes $m > 6$, the F stars were drawn from a North Galactic Pole (NGP) survey (see Upgren 1962, hereafter UP). The plate area of this survey was 396 square degrees. For $m \leq 6$, the Yale *Catalogue of Bright Stars* (Hoffleit 1964) was used. This latter catalogue covers roughly 4π solid angle in the sky and the star count distribution that one obtains from it has an implicit spherical averaging. It can only be used for setting the asymptotic stellar density at the galactic midplane. The star counts from these two catalogues were divided into ½ magnitude bins and the density distribution determined by a simple application of the $m - \log \pi$ method. The total number of stars used in determining the F star density distribution was about 890. The velocity dispersions were determined from a smaller set of about 200 stars.

HHB analyzed the isothermality of their F stars and concluded that for $z > 200$ pc, the F stars are probably not isothermal. Only 15 stars were available for determining the F star velocity dispersion for $200 < z < 300$ pc, and the conclusions regarding non-isothermality are not firm. For the F stars, 200 pc represents about 1.5 density scale heights. We have computed the Oort limits for our simulated stars for truncations of the density profile at 1.5 and at 2.5 density scale heights in order to determine the penalty suffered for not using more of the available star count data. For most of Bahcall's (1984a) calculations, only stars with $z \leq 200$ pc were used.

In our simulated star count distributions we included only as many stars as were available to HHB in defining their F star densities. Stars were randomly selected at random times in their orbit and included in the sample if they would have been included in the *Bright Star Catalogue* or in the UP compilation. Stars were separated according to their apparent magnitudes, assuming that they had the visual absolute magnitude of an F5 star (3.6; see HHB). If $m \leq 6$, then a star in any direction would have appeared in the *Bright Star Catalogue*. Stars with $6 < m \leq 13$ were included in the sample if they lay within a cone of opening angle 11.3° centered on the NGP. The solid angle subtended by 11.3° equals the plate area used in the actual observations, 396 square degrees. The galactic potential we use does not distinguish between up or down relative to the midplane. The UP observations do make this distinction in practice by being confined to the NGP.

In order that the simulated star counts have the same statistical properties as the observed sample of stars, we have randomly sampled in angle about the solar position to give our effective observing volume the desired three dimensional shape. The overall shape is that of a sphere (to $m = 6$, $D = 30$ pc) surmounted by a cone with opening angle 11.3°. The sphere represents the observing volume of the *Bright Star Catalogue*, and the cone the volume used by UP. The observed discontinuity at $m = 6$ is simulated as well as the overall shape of the star count distribution (see Table 3 and Fig. 4 of HHB, Table IV of UP, and Fig.2 below). We did not distinguish between F5 and F8 stars; grouped together, they total about 350 stars for $m \leq 6$ and about 500 stars for $6 < m \leq 13$. The stars were, as in UP and HHB, divided in ½ magnitude bins prior to determining the density distribution.

Poisson noise complicates the determination of the density distribution. For the observed F stars, there are never more than 100 stars within a ½ magnitude bin in the UP sample. If we confine our attention to $z < 200$ pc, this number decreases to about 40. (One could bin the data more coarsely to minimize the counting noise, but any poorer resolution of the distribution would make it unsuitable for deriving the Oort limit). It is both conventional and necessary to smooth the star count distribution in order to derive a density profile. There are a variety of smooth curves that acceptably fit the star count data, leading to similar but not identical density distributions. In Figure 1A we show a typical simulation of the star counts scaled to 1 square degree, and three smooth curves that are representative of the fits that one might attempt in deriving the density distribution. In Figure 1B the corresponding density distributions are shown. The flatness of these profiles at $z = 0$ is a result of the assumption used by HHB that the density is constant withing the volume surveyed by the *Bright Star Catalogue*. These density distributions, and those from other independent simulations, form the basis for our study of the Oort limit.

In Figures 2A, 2B, and 2C we show the isothermal fits to the three representative density profiles shown in Fig. 1B. These fits were constructed in the least squares sense using that part of the density profile with $z \leq 1.5 z_0$. The derived Oort limits for the three density profiles are 0.160, 0.171, and 0.184 $M_\odot\,\mathrm{pc}^{-3}$, where the input Oort limit was 0.176 $M_\odot\,\mathrm{pc}^{-3}$. If the density data up to $z = 2.5 z_0$ are utilized in fitting the density profiles, then the derived limits are respectively 0.161, 0.164, and 0.174 $M_\odot\,\mathrm{pc}^{-3}$. From a series of star count realizations we have a preliminary estimate of the expected variance of about 14% for limits derived from data truncated at $z = 1.5 z_0$, and of about 10% for limits derived from data truncated at $z = 2.5 z_0$. The largest errors scale inversely as the square root of the number of stars included in the sample. This research was supported by NSF grant PHY-82-17352.

References

Bahcall, J. N. 1984a, *Ap. J.*, **276**, 169.
____________ 1984b, submitted to *Ap. J.*
Bahcall, J. N., Schmidt, M., and Soneira, R. M. 1983, *Ap. J.*, **265**, 265.
Hill, G., Hilditch, R. W., and Barnes, J.V. 1979, *M.N.R.A.S.*, **186**, 813.
Hoffleit, D. 1964, *Catalogue of Bright Stars* (New Haven: Yale University Observatory).
Oort, J. H. 1965, in *Galactic Structure*, ed. A. Blaauw and M. Schmidt
(Chicago: University of Chicago Press).
Upgren, A. R. 1962, *A. J.*, **67**, 37.

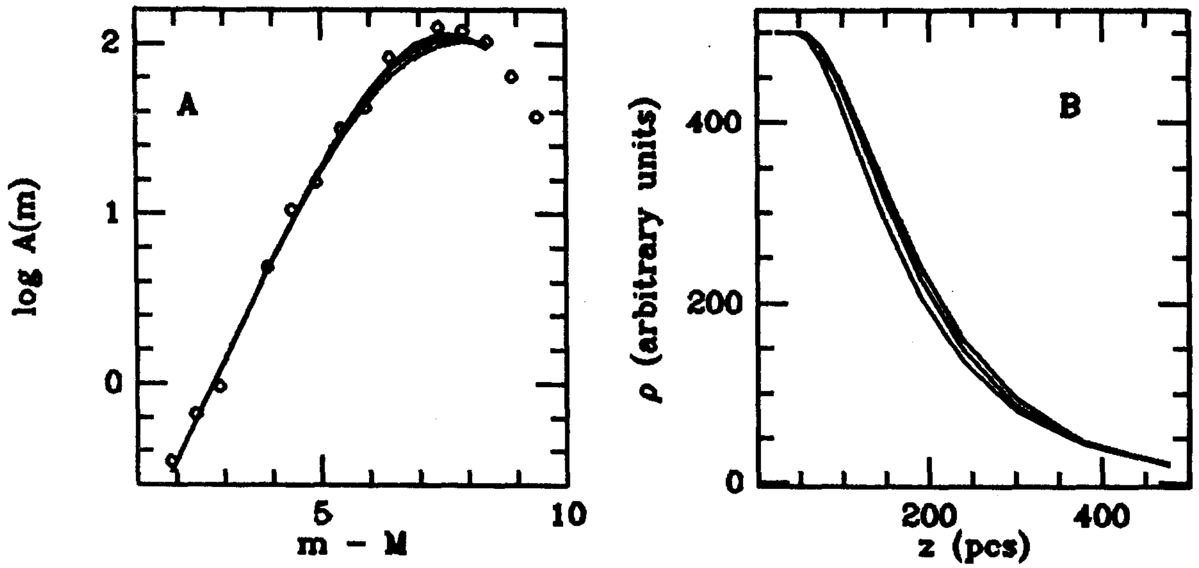

Figure 1: (A) - Star counts per square degree with three trial smooth fits. (B) - Density distributions resulting from the three trial fits in (A).

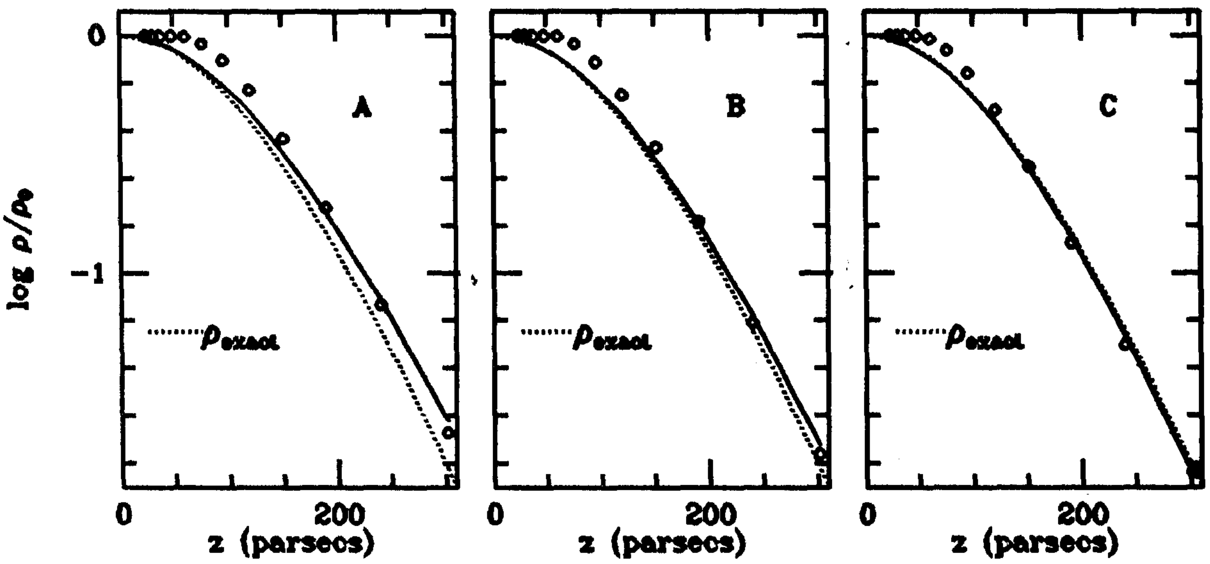

Figure 2: Best isothermal fits for the three density profiles illustrated in Fig. 1B. The open circles define the simulated density distribution, the solid line gives the isothermal fit that minimizes the variance, and the dotted line shows the input density profile.