3-INTEGRAL MODELS FOR GLOBULAR CLUSTERS

Robert Lupton
Princeton University Observatory
Princeton, NJ 08544

This work was done in collaboration with Jim Gunn.

In order to represent rotating clusters with an anisotropic velocity dispersion we took

$$f = C(e^{-\beta E}-1)e^{-\beta\gamma I3}e^{-\beta\tilde{\Omega}J3} \tag{1}$$

and approximated the third integral I_3 by the total angular momentum, J^2.

The term in J_3 leads to solid body rotation in the core, with the rotational velocity dropping to zero at the tidal radius. This rotation leads to flattened isopotentials which means that J^2 is not an integral of the motion. This approximation is discussed below. We further extended equation 1 by introducing a number of mass classes, with masses m_j.

MAKING THE MODELS

Introducing dimensionless variables with substitutions

$$M = \frac{\sum\limits_i M_j \rho_{jo}}{\sum\limits_i \rho_{jo}}, \quad W = \beta\bar{M}\psi, \quad v_o^2 = \frac{1}{\beta M}, \quad u^2 = \frac{v^2}{v_o^2},$$

$$\mu_j = \frac{M_j}{\bar{M}}, \quad r_c^2 = \frac{9v_o^2}{4\pi G\rho_o}, \quad \xi = r/r_c, \quad \Omega = \frac{3}{(4\pi G\rho_o)^{1/2}}\tilde{\Omega}, \tag{2}$$

$$\alpha_j = \frac{\rho_{jo}}{\rho_j}, \quad \sigma = \frac{\rho}{\rho_{jo}},$$

We have

$$f_j = C_j(e^{-1/2\mu_j u^2+\mu_j W}-1)e^{-1/2(\xi/\xi_t)^2\mu_j(u^2-u_r^2)}e^{\xi\sin\theta\Omega\mu_j u_\phi} \tag{3}$$

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 327–330.

The C_j are found by the condition that

$$\rho_{jo} = \int f_j(\underline{x}=0)d^3x$$

To find the potential W, which is needed to calculate the density and velocity moments, we must solve Poisson's equation which takes the form

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}(\xi^2\frac{\partial W}{\partial\xi}) + \frac{1}{\xi^2\sin^2\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial W}{\partial\theta})=-9\sigma \tag{4}$$

As the angular part of this equation is simply the legendre operator, we expand W and σ in legendre polynomials in $\cos\theta$, and end up with a set of ordinary differential equations for the coefficients $W^{(i)}$ and $\sigma^{(i)}$, i=0,2,4,...which may be solved using Green's functions.

In practice, we truncate at $P_4(\cos\theta)$, evaluate the density and the velocity moments, weight by the mass-to-light ratios of the different mass classes, and project onto the sky.

Some of these models for M13 and M92 are given in the talk by Lupton and Gunn at this conference. For M13, both anisotropy and rotation play an important role.

USE OF J^2 AS AN INTEGRAL

We have proceeded along two paths to justify our use of J^2 as an approximate integral. Firstly, by the use of an Eddington potential to approximate our true potential, and secondly by integrating f around orbits in our model potentials.

EDDINGTON POTENTIALS

For the class of potentials given by

$$\psi(r,\theta)=\zeta(r)+\frac{\eta(r)}{r^2} \tag{5}$$

$I_3=J^2-2\eta(r)$ is an exact third integral. As may be seen from figure 1, our potential approximates this form in the outer parts, with

$$\eta(r)=\lambda m_j{}^2r_c{}^2v_o{}^2P_2(\cos\theta) \tag{6}$$

and

$$W=W^{(0)} + \frac{\lambda P_2}{\xi^2} \tag{7}$$

To the extent that this is correct, the only difference between our (J^2) models and a model using the true third integral I_3 is a factor

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$$k=e^{\beta\gamma(J^2-I_3)}=e^{\lambda\mu_j P_2/\xi_t^2} \qquad (8)$$

For the case shown, $\lambda\approx0.8$ and the μ_i = 1.75, 1.17 and 0.58. These lead to values of k = 1.057, 1.038 and 1.019 at θ=0. Due to mass segregation the first mass class is negligible this far out ($\xi>\xi_t$) in these models.

It is clear that the use of J^2 leads to errors of a few percent that are proportional to P_2.

DIRECT INTEGRATION

We have integrated orbits in our model potential and looked at the variation of f, results are shown in figures 2 and 3. For the orbit in figure 2 the period is about 80 time units, and the ap- and peri-clusticon distances are about 3 and 27 core radii. J^2 was constant at about 3%, and f to about 2%. For the orbit in figure 3, the period was around 2 units, and it spent all of its time in the region from 0.8 - 1 core radii. J^2 varied by about 7%, although f varies by only 0.1% due to the factor $e^{(\xi/\xi_t)^2}$=1.02.

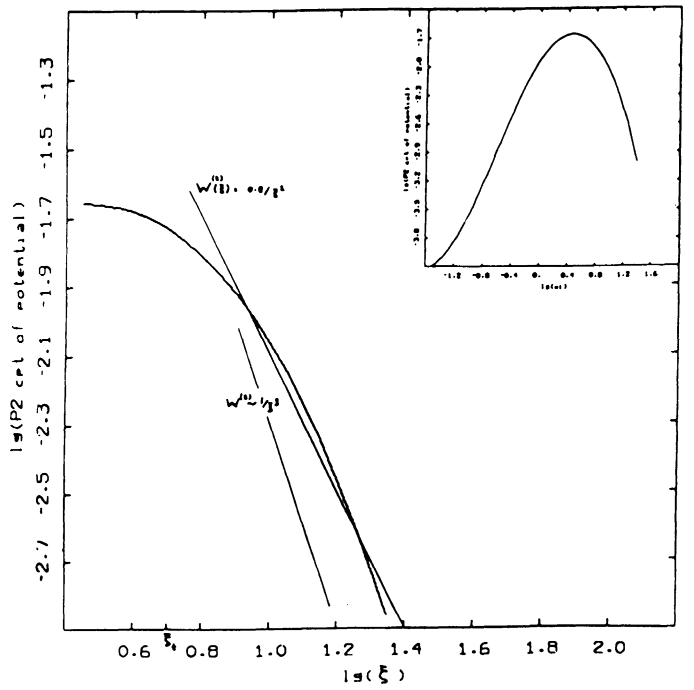

Figure 1. The P_2 component of the potential for a cluster model as a function of dimensionless radius. The insert is for the whole model, the main body of the figure shows only the outer parts. The two straight lines show two power law fits, $1/\xi^2$ and $1/\xi^3$. The $1/\xi^2$ is an Eddington potential.

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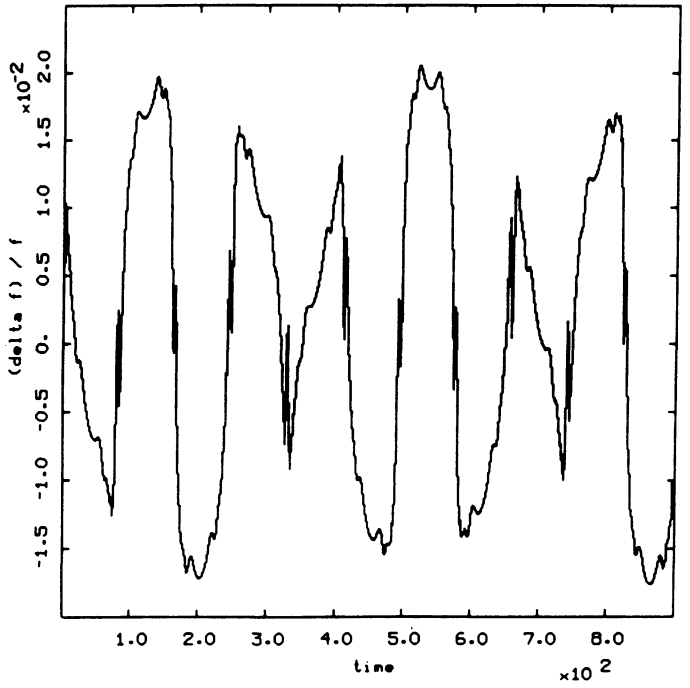

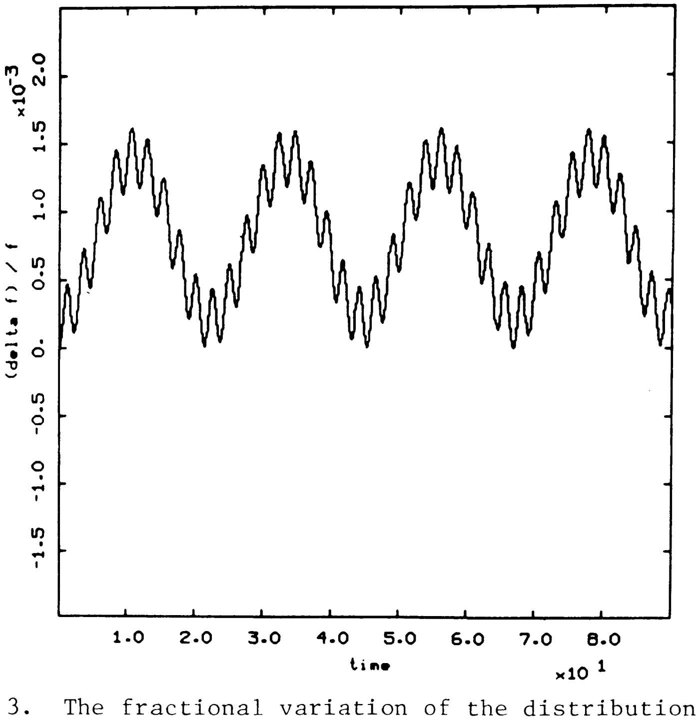

Figures 2 & 3. The fractional variation of the distribution function f as a function of time for two orbits, as described in the text.

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