SYNCHROTRON RADIATION AND SOLAR RADIO OUTBURSTS AT MICROWAVE FREQUENCIES

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The characteristics and the generation mechanisms of solar radio outbursts seem to be distinct at the frequencies above and below a certain cross-over frequency somewhere between 300 and 600 Mc/s (see Table I).

TABLE I

CHARACTERISTICS OF OUTBURSTS BELOW AND ABOVE CROSS-OVER FREQUENCY

	Below	Above
Power flux	Decreases with increasing frequency	Increases with increasing frequency
Equivalent temperature	10^8 to 10^{10} °K	10^7 to 10^9 °K
Superposed fluctuations	Large and frequent	Small
Polarization	Almost randomly polarized	Circular plus random. The sense of polarization reverses somewhere in 2000-4000 Mc/s
Harmonics	Sometimes observed	None ?
Duration	One to thirty minutes	Several minutes to two hours
Decay time	Shorter	Longer
Frequency drift	Higher to lower frequencies	Almost simultaneous

The average power spectra for the outbursts are shown in Fig. 1.

It is probable that the characteristics of outbursts *above* this cross-over frequency fall into the same category as type IV and are caused by the synchrotron radiation from electrons with $\beta = v/c = 0.2$ to 0.9 or more.

1. EMISSIVITY

According to Schwinger [1], the total-power spectrum emitted per second by an electron in uniform circular motion with an angular frequency of ω_1 is given by

$$P(\omega) = \sum_{n=1}^{\infty} \delta(\omega - n\omega_1)\, P_n\,, \qquad (1)$$

where

$$P_n = \frac{\omega_0^2 e^2}{c}\, p_n\,,$$

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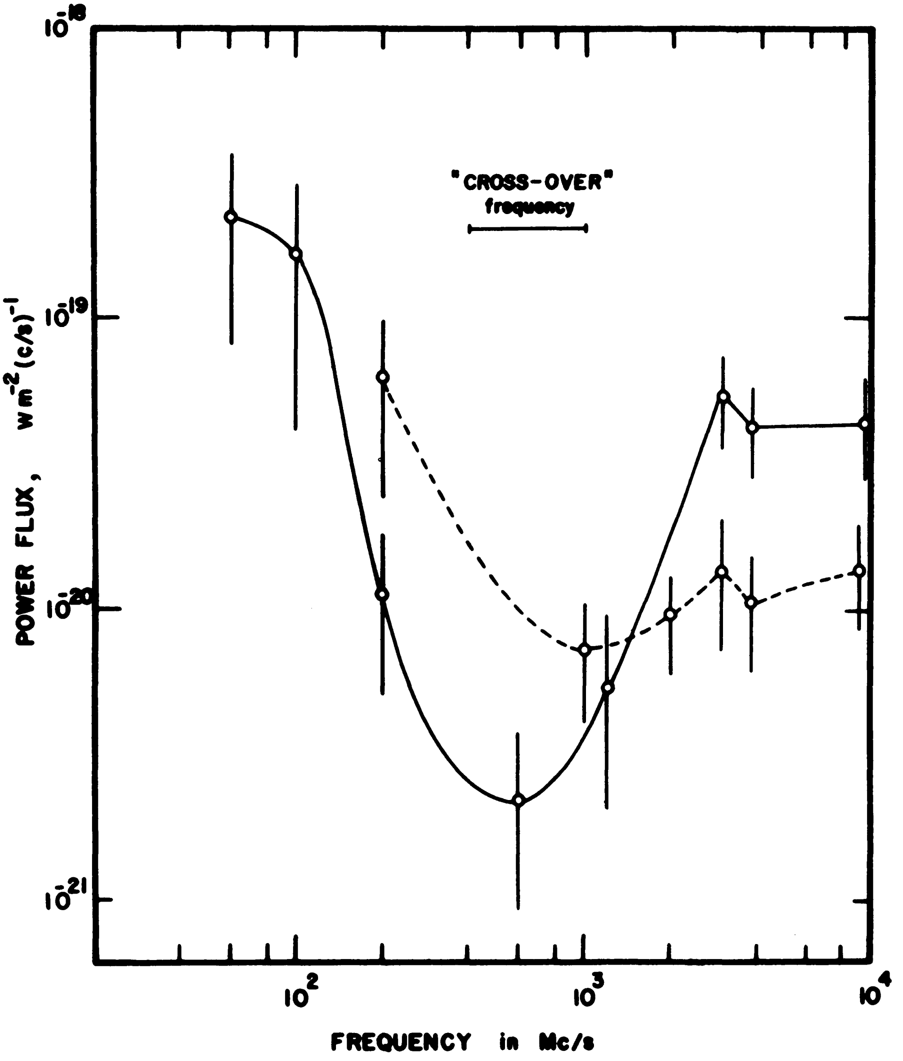


FIG. 1. Average power spectra for the outbursts. Solid curve: 1950–1953 (20 cases); dashed curve: 1957 March–June (15 cases).

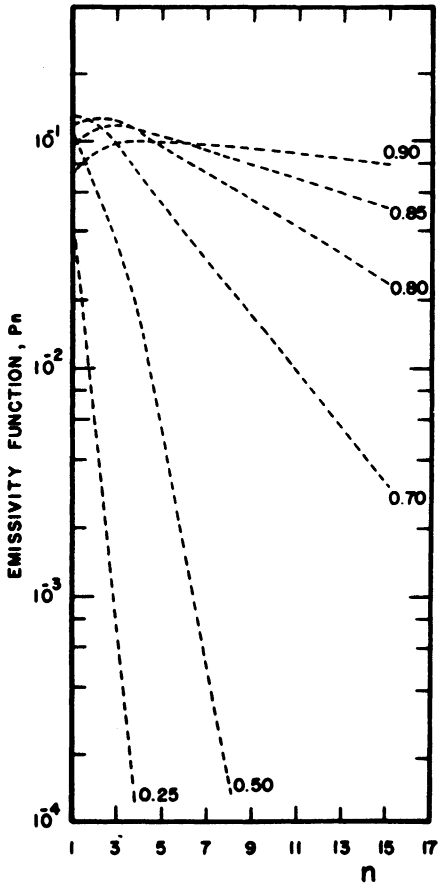
with

$$p_n = \frac{1-\beta^2}{\beta} n\left[\beta^2\{J_{2n}(2n\beta) - J_{2n+1}(2n\beta)\} - (1-\beta^2)\int_0^{2n\beta} J_{2n}(x)\, dx\right], \qquad (2)$$

and

$$\omega_1 = \frac{eH}{m_0 c}\sqrt{1-\beta^2} = \omega_0\sqrt{1-\beta^2} < \omega_0 .$$

Equation (1) shows that $P(\omega)$ is a line spectrum, but it is smeared out owing to the distribution of the velocities of electrons, since ω_1 is a function of β.



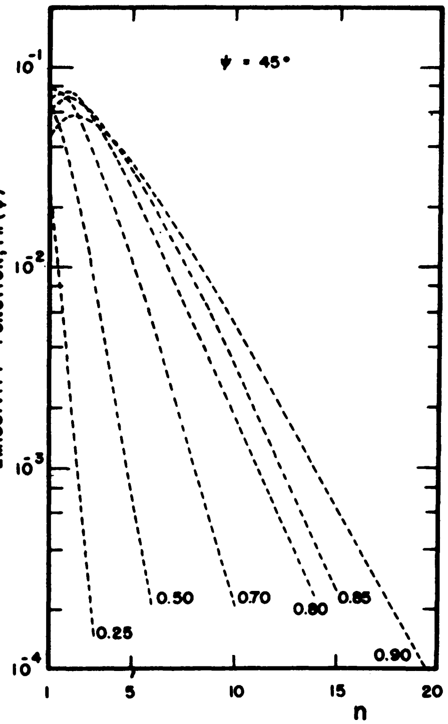


FIG. 3. The envelope of the emissivity function, $p_n(\psi)$, for $\psi = 45$ degrees.

FIG. 2. The envelope of average emissivity function, p_n, plotted against the number of harmonics n. The numbers on the curves indicate the value of β.

In Fig. 2, p_n is plotted against the number of harmonics, n. This figure shows that the emissivity for harmonics depends strongly on β.

The emission is nonisotropic, and the power emitted in the nth harmonic into a unit solid angle at an angle ψ with the orbital plane is given by

$$P_n(\psi) = \frac{1}{2\pi} \frac{\omega_0^2 e^2}{c} p_n(\psi) , \tag{3}$$

with

$$p_n(\psi) = \beta^2(1-\beta^2)n^2\left[\frac{1}{4}\{J_{n-1}(n\beta\cos\psi) - J_{n+1}(n\beta\cos\psi)\}^2 + \frac{\tan^2\psi}{\beta^2}\{J_n(n\beta\cos\psi)\}^2\right].$$

One example, when $\psi = 45$ degrees, is shown in Fig. 3.

2. ABSORPTION COEFFICIENT

Absorption coefficients κ_f are computed by the thermodynamical considera- tion

$$\frac{\eta_f}{\kappa_f} = \frac{2kT}{\lambda^2}, \tag{4}$$

and

$$\frac{3}{2}kT = \text{kinetic energy of an electron} = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right), \tag{5}$$

where η_f indicates volume emissivity.

The volume emissivity for the nth harmonic η_n is given by, cf. (2),

$$\eta_n = N_f \times P_n, \tag{6}$$

where N_f indicates the number of high-energy electrons that contribute to the radiation within a unit *frequency* bandwidth around f. The absorption coefficients for two cases were computed:

$$\text{Case 1:} \quad N_K \propto K^{-1}, \tag{7}$$

$$\text{Case 2:} \quad N_K \propto K^{-2}, \tag{8}$$

where K indicates the kinetic energy of an electron, and N_K indicates the number of electrons within a unit *energy* interval. Absorption coefficients for two cases, when

$$N = \int N_K\, dK = 1.0\,\text{cm}^{-3} \text{ and } 10\,\text{cm}^{-3},$$

were computed. Some of them are shown in Fig. 4. The frequencies are determined by n and β:

$$f = f_0\, n\sqrt{1-\beta^2}\,. \tag{9}$$

3. POWER SPECTRUM

The brightness temperature is given by

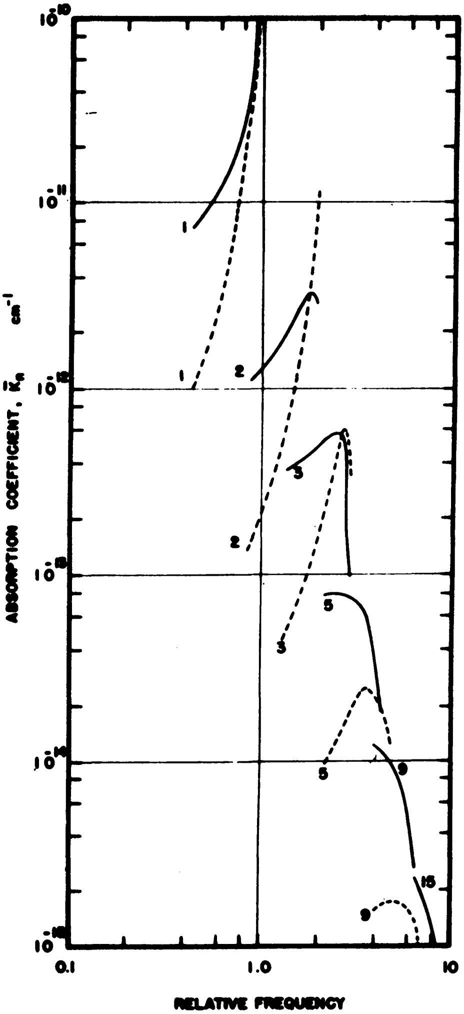
$$T_f = \int_0^l \sum_{n=1}^{\infty} T_n\, \kappa_n\, e^{-\kappa t}\, dx\,, \tag{10}$$

where $\kappa = \sum \kappa_n$, and T_n is given by equation (5). One example for Case 1, putting $N = 1\,\text{cm}^{-3}$, $f_0 = 10^3$ Mc/s, $2r = 10^5$ km, and $l = 10^5$ km is shown in Fig. 5. Here, a certain reducing factor is multiplied into the flux with frequencies below f_0, since the extraordinary component is perfectly absorbed by the resonance absorption due to the ambient thermal electrons. This reducing factor will be mentioned later.

4. POLARIZATION

The electric field radiated from an electron is given by

$$\mathbf{E} = \frac{e}{c^2 s^3}\left[\mathbf{r}\left[\mathbf{r} + \frac{\mathbf{v}}{c}r,\ \dot{\mathbf{v}}\right]\right], \tag{11}$$



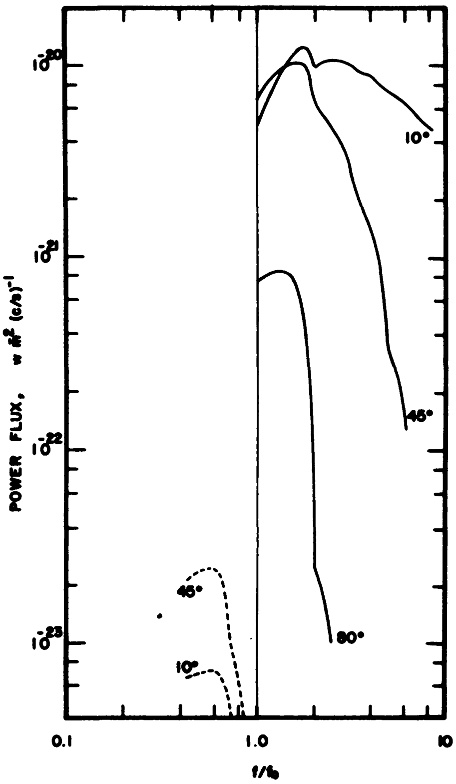


FIG. 5. Power spectra in case 1. The numbers on the curves indicate the angle ψ. $N = 1\ \text{cm}^{-3}$, $f_0 = 10^3$ Mc/s, $2r = 10^5$ km, and $l = 10^5$ km.

FIG. 4. Average absorption coefficient, K_n, plotted against the frequency, $f/f_0 = n\sqrt{1-\beta^2}$. The numbers on the curves indicate the number of harmonics, n. Solid curves: case 1; dashed curves: case 2.

with

$$s = r + \frac{(\mathbf{v}\,\mathbf{r})}{c}\,.$$

The waveform of the radiated electric field can be computed from this equation. The waveform contains harmonics as mentioned before, when v is large. The harmonic components can be computed by Fourier analysis of this wave form. Suppose the orthogonal components of the electric vector for the nth harmonic are $A_{n,x}$ and $A_{n,y}$, the shape of the polarization ellipse is given by

$$R_n = \frac{A_{n,x}}{A_{n,y}},$$

which reduces to

$$R_n = -\sin\psi \frac{J_n(n\beta\cos\psi)}{\beta\cos\psi\, J_n'(n\beta\cos\psi)}, \tag{12}$$

where the negative sign means the sense of rotation of the extraordinary wave when $\beta \to 0$, $R_1 = -\sin\psi$, where ψ is the angle between the orbital plane and the electron and the observer.

The sense of polarization is extraordinary for emission with frequencies above f_0. Because of strong resonance absorption of the extraordinary component by the thermal electrons about their gyro-frequency f_0, the sense of polarization of the escaping radiation below f_0 becomes ordinary. Consequently, the sense of polarization is reversed at the gyro-frequency of the thermal electrons. This is consistent with the observed reversal of sense of polarization of outbursts at somewhere between 2000 and 3750 Mc/s [2].

The escaping fraction of the power below f_0 is given by

$$\frac{W_{or}}{W} = \frac{(R_1 + \sin\psi)^2}{(1 + R_1^2)(1 + \sin^2\psi)}.$$

These factors are small (of the order of $10^{-2} \sim 10^{-4}$) for the emission from electrons in a circular orbit, but when the orbit is helical the factors are about a hundred times higher.

5. LIFETIME

The lifetime for synchrotron radiation is determined by three factors:

(i) *Radiation damping*

Radiation loss can be computed by the emissivity as in (2). It becomes

$$\tau_r \alpha f_0^{-2} \quad (\text{or } H^{-2}). \tag{14}$$

The lifetimes are plotted against β (or f_1/f_0) as dashed curves in Fig. 6, for $f_0 = 500$, 1000, and 2000 Mc/s.

(ii) *Collision with thermal electrons*

Suppose a high-energy electron makes a collision with a thermal electron, a comparatively large fraction of kinetic energy is shared with the thermal electron. Therefore, the lifetime for collision is determined by the mean time during which the relativistic electron makes a collision with a thermal electron. It is given by

$$\tau_c = 2.2 \times 10^{13} \frac{(1 - \sqrt{1 - \beta^2})^2}{N_e \times \beta(1 - \beta^2)} \text{ seconds}. \tag{15}$$

The lifetimes for collision are given as solid lines in Fig. 6, for $N = 10, 10^8$, 10^9, and $10^{10}\ \text{cm}^{-3}$.

(iii) *Duration of supply of high-energy electrons*

It is not likely that the supply of high-energy electrons continues as long

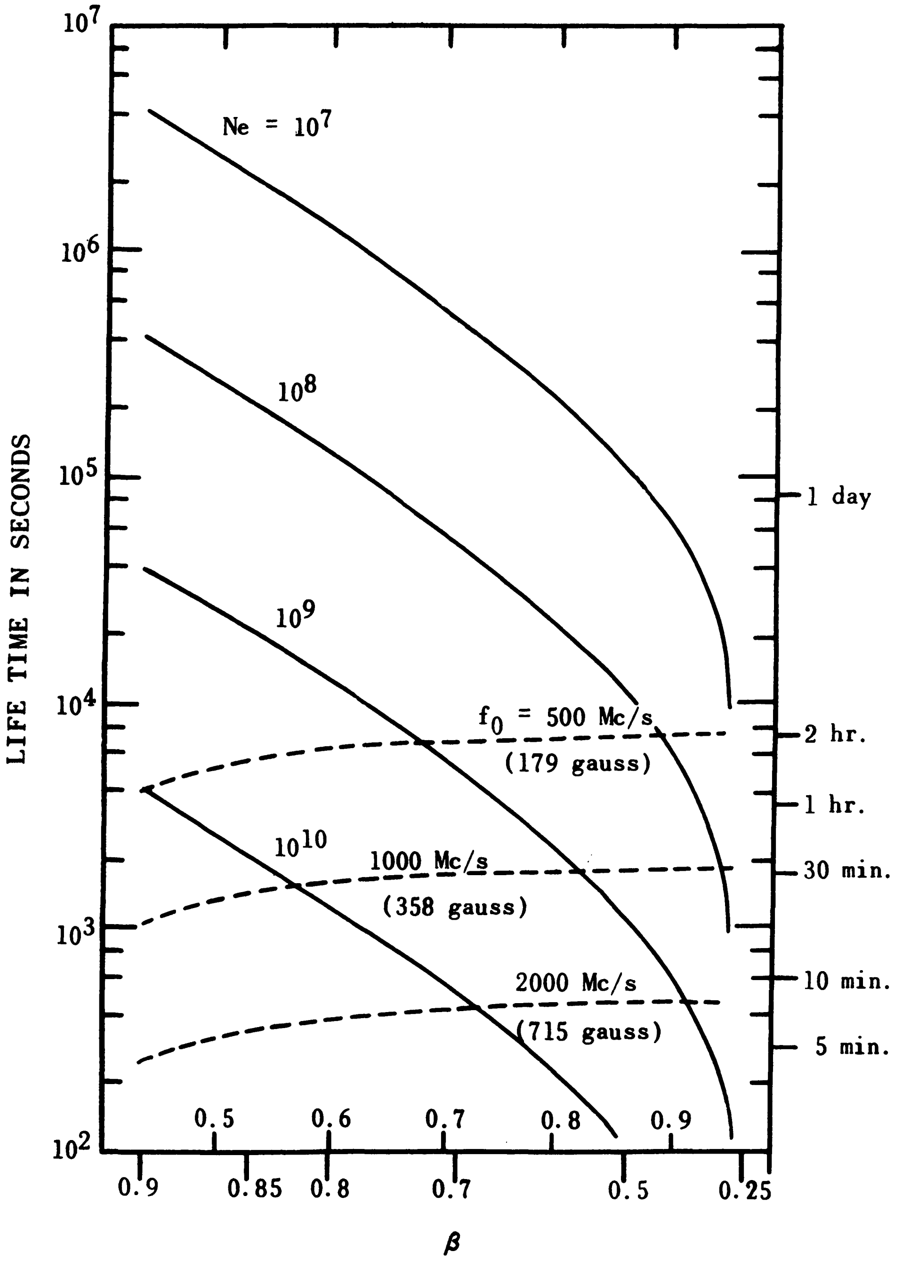


FIG. 6. Lifetime of the synchrotron radiation plotted against β (or fundamental frequency f_1/f_0). Solid curves: the lifetime due to the collision; dashed curves: the lifetime due to the radiation damping; N_e = the number density of ambient thermal electrons; f_0 = gyro frequency of the thermal electrons.

as one hour, compared with the duration of flares. When a radio source is located in the corona where $N < 10^9\,\mathrm{cm}^{-3}$, the lifetime of outbursts caused by synchrotron radiation is mainly determined by the radiation damping (see Fig. 6). When the location of the radio source is in the chromosphere or when the magnetic field is small, the lifetime is mainly determined by collisions or by the duration of the supply of high-energy electrons. On the other hand, outbursts last from several minutes to one hour. They are of the same order as the theoretically expected lifetimes.

6. CONCLUSION

The type IV bursts observed by the French group and also by the dynamic spectrometers were probably exceptional cases, when the location of the radio sources was very high in the corona. Since the magnetic field was weak, the frequency was low and the lifetime was long. But generally type IV bursts seem to be limited to the higher frequencies.

As a conclusion, the outbursts in the microwave region are type IV and are caused by the synchrotron radiation from the electrons with medium energies ($\beta = 0.9 - 0.2$). A dynamic spectrum of a typical outburst is shown in Fig. 7.

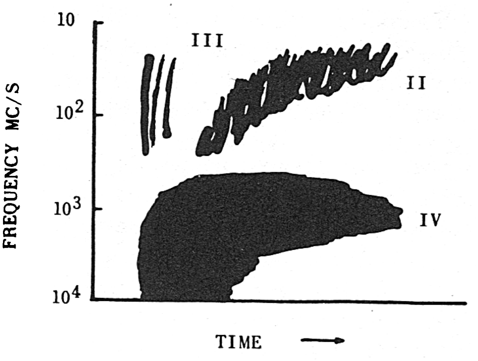


FIG. 7. A schematic dynamic spectrum of an outburst.

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[1] Schwinger, J. *Phys. Rev.* **75**, 1912, 1949.
[2] Tanaka, H., and Kakinuma, T. Meeting of Astronomical Society of Japan. 1957.

Discussion

Kundu: Are your temperatures for frequencies higher than 600 Mc/s based upon measurements? Our measurements of temperatures at 3 cm and 21 cm made at Nançay show temperatures generally of the order of 10^6 to 10^7 °K. I agree with the power spectra given for the centimeter and decimeter regions and I would like to point out that our measurements show that the dimension of bursts increases with wavelength (at least for measurements at 3 cm and 21 cm), which means that temperatures should generally decrease with wavelength (in the centimeter and decimeter regions). Finally, I feel that your conclusion that *all* centimeter and decimeter bursts are of type IV has yet to be verified by observations, because we have observed temperatures as low as 10^5 °K at 3 cm and only 60 per cent of the 3 cm bursts have been found to be polarized.

Takakura: We have some exceptionally strong bursts in the centimeter region. Therefore, we chose the upper limit of apparent temperature of 10^8 °K in the centimeter region. If the magnetic field is turbulent enough, we can expect the random polarization, even if the mechanism of emission is synchrotron.

Field: Is the relation between emission and absorption used by Takakura the correct one? Since the particles do not have a Maxwellian distribution, one cannot use the conventional Kirchhoff law. Twiss has shown in a paper in the *S.E.R.L. Technical Journal* that the correct relation between emission and absorption follows from the assumed (non-Maxwellian) particle energy distribution.

Takakura: The electrons with different energy emit at different frequencies. Therefore, there is no coupling through the radiation between the electrons with different energy. In this case, we can apply Kirchhoff's law for each energy of electrons. Even if Kirchhoff's law cannot apply, the difference may be a factor of two or three.