

SECTION III.2

DYNAMICS OF THE DISK

Thursday 2 June, 1440 – 1620

Chairman: C.A. Norman



Norman dons a tie at conference dinner. To his left: Secretary Marijke van der Laan and L. Bronfman

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## DYNAMICAL EVOLUTION OF THE GALACTIC DISK

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### ABSTRACT

After some general remarks on the dynamical evolution of the galactic disk, we review mechanisms which may affect the velocities of disk stars: stochastic heating, deflections, adiabatic cooling or heating. We compare the observed velocities of nearby disk stars with theoretical predictions based on the diffusion of stellar orbits.

### 1. INTRODUCTION

In this paper we discuss some aspects of the dynamical evolution of the axisymmetric part of the stellar galactic disk. We shall not consider here the evolution of modes in the disk, such as bars or spiral density waves.

Direct observational evidence for a dynamical evolution of a galactic disk is provided by the increase of the velocity dispersion  $\sigma(\tau)$  of disk stars with age  $\tau$ , observed for nearby disk stars, and by the age-dependence of the  $z$  distribution of disk stars, observed both for our Galaxy and for external edge-on galaxies. The observable age-dependence at least reflects the unobservable time-dependence of properties of galactic disks.

There is also some indirect observational and theoretical evidence for the dynamical evolution of the galactic disk: (1) Infall of gas: The galactic disk has very probably been formed within the galactic halo and corona by infall of gas. The dynamical effects of such an infall depend strongly on the time-scale of the infall. If the gaseous disk was formed very rapidly, then this would have caused a violent initial phase in the evolution of the disk, while the later dynamical evolution would not be affected. A slow formation of the disk by a rather steady infall, say over  $10^{10}$  years, would produce important effects all the time (Gunn 1982). Estimates on the rate of infall can be obtained from observed high-velocity clouds or from the interpretation of the chemical evolu-

tion of the disk. (2) Star formation alone can produce changes in the stellar and gaseous mass fractions, and such variations may drive a dynamical evolution of the disk, especially in the  $z$  direction. (3) Instabilities in the galactic disk, such as bars, spiral density waves, warps, produce often dynamical effects in the underlying disk: an increase of the random velocity dispersion of disk stars ('heating' of the disk), a redistribution of matter (especially of gas by bars), a change of the distribution of angular momentum in the disk by the non-axisymmetric perturbations. (4) The influence of the environment of the galactic disk on the dynamical evolution is difficult to assess: The mean gravitational field of the luminous halo and the dark corona is probably rather constant in time. If, however, the corona contains a very large number of massive black holes, then the heating of the stellar disk by such penetrating objects would be important (Ostriker 1983). Tidal effects of passing galaxies are of minor importance for the evolution of the inner parts of the disk, probably also for the solar neighbourhood.

## 2. RELAXATION IN THE GALACTIC DISK

The random peculiar velocities of disk stars may have been affected by the following mechanisms: stochastic heating, deflections, and adiabatic cooling or heating.

### 2.1 Stochastic heating

Irregularities in the galactic gravitational field produce a steady increase of the velocity dispersion  $\sigma$  of disk stars with the age  $\tau$  of the stars. Such an increase is well observed for nearby stars (Wielen 1974, 1977).

The basic physical process responsible for the observed stochastic heating of disk stars has not been identified with certainty up-to-now: (1) Encounters between stars are known to be completely inefficient (e.g. Chandrasekhar 1960). (2) Encounters of stars with massive interstellar clouds have first been investigated by Spitzer and Schwarzschild (1951, 1953). However, even the observed giant molecular clouds seem to be insufficient for explaining the observed heating of nearby disk stars (see e.g. Lacey 1983, Villumsen 1983), mainly because these clouds are too rare at the distance of the Sun from the galactic center. (3) If the dark corona of the Galaxy mainly consists of massive black holes, with individual masses of about  $10^6 M_{\odot}$ , this would explain the observed heating of disk stars and especially the observed age-dependence of  $\sigma(\tau)$ , namely  $\sigma(\tau) \propto \sqrt{\tau}$  for old disk stars (Ostriker 1983). (4) One of the most promising mechanisms for the heating of the disk is the fluctuating gravitational field provided by transient instabilities in the disk, such as local Jeans instabilities, wavelets, transient spiral arms (see e.g. Carlberg and Sellwood 1983).

Since the basic physical mechanism for the heating of the disk is not well known at present, the phenomenological description of the heating process by the theory of orbital diffusion seems to be rather adequate (Wielen 1977, Wielen and Fuchs 1983). In this theory, the heating of the disk is basically described by a diffusion process in velocity space, and the diffusion coefficient  $D$  is empirically determined from the observed age-dependence of the velocity dispersion of nearby stars (Wielen 1977). Some results of the theory will be discussed in detail in Section 3.

## 2.2 Deflections

Deflections are random changes in the direction of the velocity vector of a star. Similar to the stochastic heating, deflections may be caused by irregularities in the galactic gravitational field, e.g. due to clouds (Lacey 1983), spiral arms etc.. The overall importance of deflections is that the deflections may transfer energy (and energy changes) between the motions of a star perpendicular and parallel to the galactic plane. Therefore, the axial ratios of the velocity ellipsoid of disk stars may be governed by deflections (Lacey 1983) rather than by the heating mechanism. In Chandrasekhar's notation (1960), the time-scale for heating is the relaxation time  $T_E$ , while the relaxation time  $T_D$  is the time-scale for deflections. If the velocity dispersion of clouds is much smaller than that of the field stars, then we find for encounters of stars with clouds that  $T_D$  is much smaller than  $T_E$ . This means that deflections due to certain irregularities in the galactic gravitational field may be of primary importance for the axial ratio of the velocity ellipsoid even if the heating effect of the same irregularities is nearly negligible. Giant molecular clouds lead to a deflection time-scale  $T_D$  of about  $10^9$  years for nearby stars, slightly longer than the observed heating time-scale, but probably of some importance for the axial ratio of the velocity ellipsoid.

## 2.3 Adiabatic heating or cooling

Slow changes in the regular gravitational field of the galactic disk produce adiabatic changes in the velocities of the stars. This adiabatic heating or cooling of the disk is probably strongest in the  $z$  direction, perpendicular to the galactic plane, because the disk is nearly self-gravitating in  $z$ , but is heavily supported in the radial direction by the combined gravitational field of halo and corona.

We shall discuss here two typical examples of adiabatic changes of a galactic disk which is assumed to be essentially self-gravitating in the  $z$  direction: (1) Adiabatic cooling due to stochastic heating: The stochastic heating increases primarily the velocity dispersion  $\sigma_W$  of the disk stars. The higher velocity dispersion  $\sigma_W$  leads to a larger thickness  $H$  of the disk in the  $z$  direction, thereby lowering the  $z$  force  $K_z$  of the disk. The adiabatic decrease of  $K_z$  causes a corresponding decrease in the  $W$  motion of the disk stars, thus decreasing  $\sigma_W$ . In total, the effect of the stochastic heating in  $\sigma_W$  is partially com-

compensated by the adiabatic cooling (see also Section 3). (2) Adiabatic heating due to infall: Infall increases the surface density  $\mu$  of the disk. Then the thickness  $H$  of the disk decreases for a given velocity dispersion  $\sigma_W$ , and the force  $K_z$  increases both because of the higher mass and the smaller thickness of the disk. The  $W$  motions of the stars increase adiabatically due to the increase in  $K_z$ . Hence, the infall leads finally to an adiabatic heating of the disk stars, i.e. an increase of  $\sigma_W$ .

While the adiabatic heating and cooling discussed above affect primarily the  $z$  motions of the stars, i.e. the velocity dispersion  $\sigma_W$ , deflections may transfer this energy change from the  $z$  direction partially to the motions parallel to the galactic plane, i.e. the velocity dispersions  $\sigma_U$  and  $\sigma_V$  may be changed too.

### 3. DIFFUSION OF STELLAR ORBITS

For an introduction into the theory of the diffusion of stellar orbits, we refer to our earlier papers (Wielen 1977, Wielen and Fuchs 1983). We shall use here the same notations as Wielen and Fuchs (1983). In that paper, we have already presented a solution of the appropriate Fokker-Planck equation for a 'standard case' using the following assumptions: (1) constant isotropic diffusion coefficient  $D_0$ ; (2) no radial variation of the distribution function in phase space,  $f(U, V, W, z, t)$ ; (3) constant star formation rate  $S(t_f)$ ; (4) linear  $K_z$  force,  $\ddot{z} = -\omega_z^2 z$ ; (5) no adiabatic cooling in  $z$ , i.e. a time-independent  $K_z$  force; (6) no infall. Our theoretical results for the standard case are already in good agreement with the observed velocity distribution of nearby disk stars, represented by 317 McCormick K+M dwarfs in Gliese's catalogue (Gliese 1969, Jahreiß 1974). We shall now discuss some modifications of the standard case.

The introduction of a more realistic, non-linear  $K_z$  force, which agrees essentially with that derived by Oort (1965), produces only minor changes in the predicted velocity distribution of disk stars at  $z = 0$  (Fig. 1). The predicted values of the velocity dispersions  $\sigma_U$ ,  $\sigma_V$  and  $\sigma_W$  as functions of the height  $z$  are slightly smaller than for the standard case (Fig. 2). The predicted density  $\rho(z)$ , normalized at  $z = 0$ , has stronger wings at higher  $z$  (Fig. 2).

We shall now take into account the adiabatic cooling in the  $z$  direction. For harmonic oscillations of stars in the  $z$  direction, the adiabatic invariant  $I_z$  is given by

$$I_z = E_z(t)/\omega_z(t) = \text{const.}, \quad (1)$$

where  $E_z$  is the energy of a star in the  $z$  motion and  $\omega_z$  is the frequency of oscillation in  $z$ . For non-linear  $K_z$  forces, the form of the adiabatic invariant is more complicated. For simplicity, we use Eq. 1 even for such non-linear  $K_z$  forces, since the majority of stars still moves in the quasi-linear regime, and we evaluate  $\omega_z$  always at  $z = 0$ .

Figure 1. Velocity distribution of nearby disk stars at  $z=0$ .  
 Observations: histograms.  
 Linear  $K_z$ : solid curves.  
 Non-linear  $K_z$ : dashed curves.

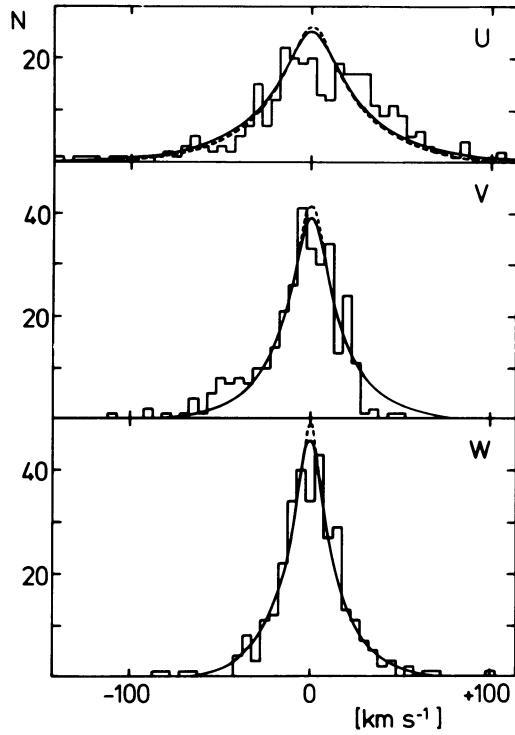


Figure 2. Predicted  $z$ -dependence of the overall velocity dispersions  $\sigma_U$ ,  $\sigma_V$ ,  $\sigma_W$ , and of the overall space density  $\rho$ .  
 Linear  $K_z$ : solid curve.  
 Non-linear  $K_z$ : dashed curve.

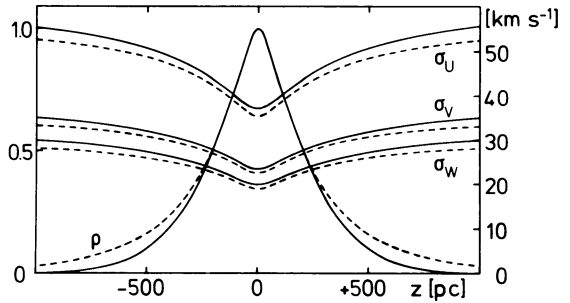
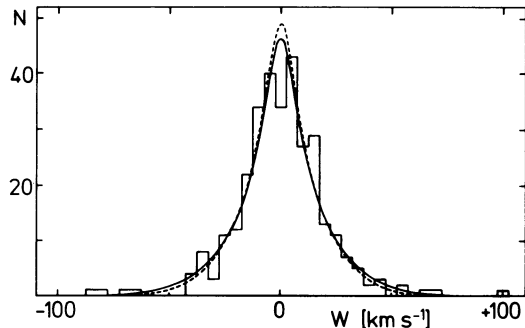


Figure 3. Distribution of  $W$  velocities of nearby disk stars at  $z=0$ .  
 Observations: histogram.  
 Standard case: solid curve.  
 With adiabatic cooling: dashed curve.



The Fokker-Planck equation can be approximately solved for a linear  $K_z$  force with  $\omega_z(t)$  depending slowly on the time  $t$ , by a transformation after which  $\tilde{I}_z$  (Eq. 1) occurs as one of the independent variables. The resulting distribution function  $f$  for a generation of stars formed at time  $t_f$  is essentially the same as obtained for the standard case (Eq. 14 of Wielen and Fuchs). We have only to replace the velocity dispersion  $\sigma_W(\tau)$  by

$$\sigma_W^2(t, t_f) = \omega_z(t) \left( \sigma_{W,0}^2 \omega_z^{-1}(t_f) + \alpha_W D_0 \int_{t_f}^t \omega_z^{-1}(t') dt' \right) \quad (2)$$

with  $\alpha_W = 1/2$ . Differentiating Eq. 2 with respect to  $t$  gives

$$d\sigma_W^2/dt = \alpha_W D_0 + \sigma_W^2 (d\omega_z/dt)/\omega_z \quad (3)$$

The second term on the right-hand side of Eq. 3 can be derived directly from Eq. 1 by noting that the energy in the  $z$  direction, averaged over a stellar generation,  $\langle E_z \rangle$ , is proportional to  $\sigma_W^2$ .

If we assume that the disk is self-gravitating in the  $z$  direction, we can derive  $\omega_z(t)$  from known quantities, by evaluating the Poisson equation at  $z = 0$ :

$$\omega_z^2 = 4\pi G\rho(z=0) \quad (4)$$

For a linear  $K_z$  force and a single generation of stars, the density  $\rho$  at  $z = 0$  can be derived by integrating Eq. 14 of Wielen and Fuchs (1983) over the velocities  $U, V, W$ . The result is

$$\rho(z=0) = (2\pi)^{-1/2} \mu \omega_z / \sigma_W \quad (5)$$

Combining Eqs. (4) and (5), we obtain

$$\omega_z(t) = (8\pi)^{1/2} G\mu / \sigma_W(t) \quad (6)$$

The same result, with a slightly different numerical constant ( $2^{1/2}\pi$ ), is found for a self-gravitating isothermal stellar disk which obeys Eqs. (19) and (20) of Wielen and Fuchs (1983) and

$$\rho(z=0) = \mu / (2H) \quad (7)$$

From Eq. 6, we derive, independent of the value of the numerical constant,

$$(d\omega_z/dt)/\omega_z = -(d\sigma_W/dt)/\sigma_W \quad (8)$$

Inserting Eq. 8 into Eq. 3, we obtain

$$d\sigma_W^2/dt = (2/3)\alpha_W D_0 \quad (9)$$

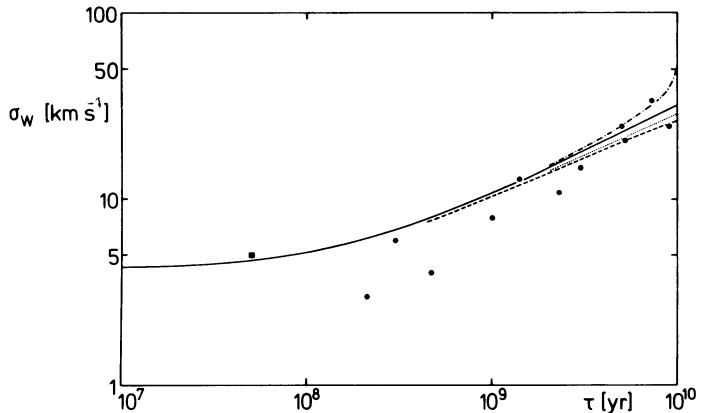
The adiabatic cooling reduces therefore the effect of the diffusion in  $W$  by factor of  $2/3$  for a single generation of stars. Hence we find in this case

$$\sigma_W^2(\tau) = \sigma_{W,0}^2 + (1/3)D_0\tau \quad (10)$$

For a disk made up of many stellar generations, we have to integrate Eq. 5 over all generations, i.e. over  $t_f$  with an appropriate star formation rate  $S(t_f)$ , in order to derive the total density  $\rho(t)$  at  $z = 0$ .



Figure 4. Velocity dispersion  $\sigma_W$ , averaged over  $z$ , as a function of age  $\tau$ . Observations: symbols. Standard case: solid curve. With adiabatic cooling: dashed curve. With infall: dotted (low rate) and dash-dotted (high rate).



Then we have to find  $\omega_z(t)$  from Eq. 4 and finally to use Eq. 2 for obtaining the new velocity dispersion  $\sigma_W$  for each stellar generation. In order to be more realistic, we have also added a gaseous component to the disk. The velocity dispersion of the gas is kept constant. Stars are formed out of the gas according to a chosen star formation rate, thereby decreasing the amount of gas in the disk. The results of such a calculation are shown in Figs. 3 and 4. The adiabatic cooling in  $z$  has only minor effects for both the velocity distribution in  $W$  at  $z = 0$  (Fig. 3) and the age-dependence of the velocity dispersion  $\sigma_W$  (Fig. 4). As long as we do not invoke deflections, the distribution of the velocity components  $U$  and  $V$ , integrated over  $z$ , and the age-dependence of the velocity dispersions  $\sigma_U(\tau)$  and  $\sigma_V(\tau)$ , averaged over  $z$ , are not affected by the adiabatic cooling in  $z$  (nor by a non-linear  $K_z$  force).

We shall now consider the effect of infall. To simulate the infall, we add gas to the disk according to a constant infall rate. We have used either a 'low rate' of about  $2 M_\odot / (10^9 \text{ years pc}^2)$  or a 'high rate' of about  $7 M_\odot / (10^9 \text{ years pc}^2)$  over  $10^{10}$  years. The present total disk surface density  $\mu$  adopted is about  $70 M_\odot / \text{pc}^2$ . Besides the adiabatic heating due to the infall, we have also included the adiabatic cooling caused by the stochastic heating and the depletion of the gas by star formation. For both infall rates, the resulting velocity distribution in  $W$  at  $z = 0$  is so close to the standard case that the difference would be hardly visible in Fig. 3. The age-dependence of the velocity dispersion  $\sigma_W(\tau)$  is also not dramatically affected by infall (Fig. 4). Only in the case of the high infall rate, the heating of the older stars is slightly higher than the observed values.

We conclude that the simple standard case of stellar diffusion gives already a rather good agreement between theory and observations for both the velocity distribution at  $z = 0$  and the age-dependence of the velocity dispersion of nearby disk stars. This indicates that the stochastic heating is the main dynamical mechanism, while other processes like adiabatic cooling and infall are dynamically only of secondary importance.

The isotropic diffusion provides a simple explanation for the observed ratio of the velocity dispersion  $\sigma_W$ , perpendicular to the galactic plane, to the dispersions parallel to the plane,  $\sigma_U$  and  $\sigma_V$  (Wielen 1977). The observed values for the McCormick K+M dwarfs in Gliese's Catalogue, as the most representative sample of nearby stars, averaged over  $z$ , are  $\sigma_U = 47$  km/s,  $\sigma_V = 29$  km/s,  $\sigma_W = 25$  km/s. If we normalize the predicted dispersions always so that  $\sigma_U^2 + \sigma_V^2$  is equal to the observed value, then the following values for  $\sigma_W$  are predicted: (1) Isotropic diffusion without adiabatic cooling (standard case) predicts  $\sigma_W = 25$  km/s, in perfect agreement with the observations. (2) Adiabatic cooling lowers the predicted value slightly to 22 km/s for the case of a constant star-formation rate (not to 20 km/s as would be expected from the factor 2/3 in Eq. 9, which is valid only if all the stars have the same age and if there is no gas). (3) Deflections by clouds (Lacey 1983) would predict  $\sigma_W = 33$  km/s, if other sources of stochastic heating and the adiabatic cooling are neglected. (4) In reality, all three effects, namely stochastic heating, deflections and adiabatic cooling, will jointly determine  $\sigma_W$ . The predicted value of  $\sigma_W/\sqrt{(\sigma_U^2 + \sigma_V^2)}$  would then depend strongly on the direction-dependence of the stochastic heating and of the deflections and on the relative time-scales of the heating and of the deflections. One should remember here that the main sources of stochastic heating and of deflections may be quite different ones. We conclude that there is probably no significant discrepancy between the observed and predicted axial ratio of the velocity ellipsoid of common nearby stars, if all the uncertainties are taken into account.

#### REFERENCES

- Carlberg, R.G., Sellwood, J.A.: 1983, IAU Symposium No. 100, p. 127.  
 Chandrasekhar, S.: 1960, Principles of Stellar Dynamics, Dover Publ., New York.  
 Gliese, W.: 1969, Veröffentl. Astron. Rechen-Inst. Heidelberg No. 22.  
 Gunn, J.E.: 1982, in Astrophysical Cosmology, eds. H.A. Brück, G.V. Coyne, M.S. Longair, Pontificia Academia Scientiarum, Citta del Vaticano, p. 233.  
 Jahreiss, H.: 1974, Diss. Naturwiss. Gesamt-Fak. Univ. Heidelberg.  
 Lacey, C.G.: 1983, Monthly Not. Roy. Astron. Soc. (in press)  
 Oort, J.H.: 1965, Stars and Stellar Systems 5, 455.  
 Ostriker, J.P.: 1983, private communication.  
 Spitzer, L., Schwarzschild, M.: 1951, Astrophys. J. 114, 385.  
 Spitzer, L., Schwarzschild, M.: 1953, Astrophys. J. 118, 106.  
 Villumsen, J.V.: 1983, Astrophys. J. (in press).  
 Wielen, R.: 1974, IAU Highlights of Astronomy 3, 395.  
 Wielen, R., :1977, Astron. Astrophys. 60, 263.  
 Wielen, R., Fuchs, B.: 1983, in Kinematics, Dynamics and Structure of the Milky Way, ed. W.L.H. Shuter, D. Reidel Publ. Co., Dordrecht, p. 81.

## DISCUSSION

F.H. Shu: Toomre's  $Q$  parameter as a stability index is based on an analysis which assumes only a single Schwarzschild distribution. If you have a superposition of Schwarzschild distributions, then the  $Q$  parameter loses its precise original meaning. In particular, a formal value  $>1$  may not guarantee even axisymmetric stability, because the population of stars with low velocity dispersion contribute most importantly to the instability mechanism.

Wielen: I agree completely with you. One has to figure out the meaning of  $Q$  for a sequence of generations of stars.

C.C. Lin: In my review I shall show that there may be a variety of reasons for  $Q$  to exceed 1.

On another matter: how does your rate of growth of velocity dispersions compare with that obtained by Carlberg and Sellwood through numerical modelling? Do they find more rapid evolution? Does their evolution follow the same linear trend?

Wielen: In principle they find similar trends, although their results show a large scatter. Villumsen will discuss the diffusion by molecular clouds, and he also finds agreement.

R.G. Carlberg: The value of  $Q$  will depend on the assumed surface density of the disk. You have used a value of  $70 M_{\odot}/\text{pc}^2$ , while Schmidt in his review (section II.1) gave  $48 M_{\odot}/\text{pc}^2$ .

Wielen: Schmidt uses a value of  $50 M_{\odot}/\text{pc}^2$  in his model. The count of known stars and gas amounts to only  $40 M_{\odot}/\text{pc}^2$ . The  $70 M_{\odot}/\text{pc}^2$  adopted by me agrees with the estimate of the expected surface density in Schmidt's review, which is based on the local determination of  $K_z$  by Oort, after allowance for a contribution of  $10 M_{\odot}/\text{pc}^2$  by the dark corona. This estimate, then, includes "hidden mass". It would imply a value of  $Q$  greater than 2.

L. Blitz: You argued that results for the diffusion calculations do not change very much, as long as the star formation rate is less than a factor  $\sim 5$  greater than it is now. But I wish to point out that in the past the star formation rate in the disk may indeed have been much higher. We know now that most stars, essentially all stars probably, form in molecular clouds. Since stars with mass less than  $1 M_{\odot}$  essentially lock up all that mass for a Hubble time, one can argue that a Hubble time or half that time ago, there may have been 5 or even 10 times as much molecular material in the Galaxy as there is now. If the star formation rate goes as the mass of molecular gas, the effects on the diffusion calculations might be strong. Also, when one looks at the diffusion by molecular clouds in the solar neighbourhood, one should consider not the current surface density of such clouds, but rather a time average over a substantial period.

Wielen: The observed increase of the velocity dispersion with age over the last  $10^8$  years is mainly determined by the present surface density of molecular clouds in the solar neighbourhood. Hence the discrepancy discussed remains at least for the present situation. A variation of the star formation rate does not affect the relation between velocity dispersion and age, but the integrated velocity distribution.

J.P. Ostriker: Wielen has demonstrated that the velocity dispersion increases as  $t^{1/2}$ , in contradiction to expectations based on existing physical theories of diffusion. This led Lacey, Schmidt and me to look at the unlikely possibility of diffusion due to interaction of disk stars with a dark halo comprised of massive black holes. We find that the correct diffusion rate is obtained if the mass of such holes is  $2 \times 10^6 M_{\odot}$ , and the predicted axial ratio of the velocity ellipsoid is also correct. The major byproduct of such a halo would be the existence at the centre of the Galaxy of one or two black holes dragged into that region by dynamical friction.

M. Iye: The existence of massive black holes ( $10^3 M_{\odot} \lesssim M \lesssim 10^6 M_{\odot}$ ) in the halo might be observationally confirmed, if they do exist, by searching for gravitational-lensing effects on images of stars caused by such black holes. One way to do this would be monitoring the movement and light variation of stellar images in globular clusters with a spatial resolution of 0.1 arc second or better for a period of tens of years.

Carlberg: How confident are you as to the power of time in the heating rate of stars?

Wielen: That depends mainly on the ages of the stars used, and I don't think that they can be very wrong. I would say that the linear dependence of  $\sigma^2$  on time is quite established. Even if the diffusion coefficient  $D$  varies with velocity, as e.g. proposed by Spitzer and Schwarzschild ( $D \propto 1/v$ ), one can correct that by assuming the constant to be time-dependent. The irregularities in the gravitational field may indeed have been bigger in the past.