COMPUTING PARALLACTIC REFRACTION FOR STELLAR TRIANGULATION

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1. INTRODUCTION

The parallactic refraction correction should be applied to the photographic observations of the light signals carried either by artificial satellites or by meteorological balloons. The correction to the right ascension and declination are made with the equations

$$\left.\begin{aligned} \Delta\alpha &= \frac{1}{15}\frac{\cos\phi\ \sinh}{\cos\delta\ \sin z}\Delta\zeta \\ \Delta\delta &= \frac{\sin\phi\ \cos\delta - \cos\phi\ \sin\delta\ \cosh}{\sin z}\Delta\zeta \end{aligned}\right\} \tag{1.1}$$

where ϕ is the latitude of the observation station, z the zenith distance, α the right ascension, δ the declination and h the hour angle of the light signal. Parallactic refraction is denoted by $\Delta\zeta$.

Numerous formulae have been derived for computing the parallactic refraction term $\Delta\zeta$. The form of the formula depends on the purpose for which the parallactic refraction is needed. In the case of distant light signals, such as artificial satellites or natural moon, the formula for $\Delta\zeta$ is very simple. In the case of balloon observations the formula for computing the term $\Delta\zeta$ is more complicated and depends on the atmospheric model used. Oterma (1960) and others have derived formulae for both cases. Her formula in the case of a distant light signal is

$$\Delta\zeta = 0.0012568\ \frac{1-s}{s}\ \zeta_\infty \tag{1.2}$$

and in the case of balloon-borne light signals

$$\Delta\zeta = \left(\frac{p}{760}\ \eta + \frac{t}{10}\ \kappa\right)\ \zeta_\infty \tag{1.3}$$

where the whole astronomic refraction

$$\begin{aligned}\zeta_\infty = \frac{1}{10}\tan z_0'\ (601''\!.7052 - 66''\!.968c + 20''\!.971c^2 - \\ - 10''\!.704c^3 + 7''\!.655c^4 - 7''\!.091\frac{c^5}{1-c})\end{aligned} \tag{1.4}$$

applies to standard conditions at which the temperature is 0 ^{o}C and the air pressure 760 mmHg, and the parameter c has the form:

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$$c = \left(\frac{1}{10}\sec z_0'\right)^2 \tag{1.5}$$

The air pressure p is taken in mmHg and the air temperature in $^{\circ}$C and both are measured at the station. The terms

$$\left.\begin{aligned} \eta &= \frac{\Delta\zeta_0}{\zeta_\infty} \\ \kappa &= \frac{\Delta\zeta_{10} - \Delta\zeta_0}{\zeta_\infty} \end{aligned}\right\} \tag{1.6}$$

where $\Delta\zeta_{10}$ and $\Delta\zeta_0$ are values of the parallactic refraction corresponding to the temperatures 10 $^{\circ}$C and 0 $^{\circ}$C; they are given in tables compiled by Oterma (1960) for given s and apparent zenith distance z_0'. The parameter s depends on the height H of the light signal from the ground level according to

$$s = \frac{H}{r_o + H} \tag{1.7}$$

where r_o is the mean curvature radius of the earth (6368.8 km used). Oterma's tables are based on the assumption that the quantity μ-1, in which μ is the index of refraction, is directly proportional to the air density ρ and that the vertical temperature decreases linearly from t_o on the ground to t_1 of the tropopause and then remains constant to relatively great heights. Oterma states that

$$t = t_o - ks \quad (k = \text{constant}), \quad 0 \leqq s \leqq s_1,$$

$$t = t_1, \quad s \geqq s_1,$$

where t_o = 0 $^{\circ}$C, t_1 = -50.687 $^{\circ}$C and s_1 = 0.0014 corresponding to H = 8.929 km for the tropopause, and the air pressure on the ground is taken to be 760 mmHg.

The astronomical refraction formula (1.4) is given in a series in powers of the secant of the apparent zenith distance. Such a series can be used only up to a certain zenith distance. In Oterma's formula the limit is around 85°. The value of the formula (1.4) in this case differs insignificantly from the numerical solution of the original refraction integral formula used by Oterma (1960).

Investigations in which the curvature of the light ray is of principal concern suggest that formula (1.3) should be used instead of formula (1.2) if the height of the light signal is below 100 kilometres.

2. APPLICATION OF OTERMA'S THEORY TO FINNISH STELLAR TRIANGULATION

The above refraction model prepared by Oterma has been applied to Finnish Stellar Triangulation. In this work balloon-borne flash lights were used, and because the balloon always remains below a height of 40 km formula (1.3) should be used. In the first computation (Kakkuri 1973) the ratios η and κ as functions of given s and z_0' were taken from Oterma's tables and nomographs and the astronomic refraction calculated from formula (1.4). An electronic computer program was later prepared for computing the parameters η and κ with the help of numerical integration. The refraction integral has been written in the form

$$\zeta = \int_0^{\omega} \frac{\beta\, d\omega}{1-\beta\omega} \frac{\sin z_o'}{\sqrt{\cos^2 z_o' + \psi}} \tag{2.1}$$

for obtaining rapid converging. When following Oterma's model the variables and constants in formula (2.1) are:

$$\left.\begin{aligned}
&\mu_0 - 1 = (\bar{\mu}_0 - 1)\frac{p_o}{\bar{p}_o(1+\alpha t_o)}\\
&\beta = \frac{\mu_0 - 1}{\mu_0}\\
&\omega = 1 - (1 - Bs)^{(A/B)-1}, \text{ when } 0 \leqq s \leqq s_1\\
&\omega = 1 - (1 - \omega_1)e^{A_1 s_1}\, e^{-A_1 s}, \text{ when } s \geqq s_1\\
&\psi = \left(\frac{1 - \beta\omega}{1 - s}\right)^2 - 1
\end{aligned}\right\} \tag{2.2}$$

where $\bar{\mu}_0$ is the index of refraction on the ground at standard conditions $\bar{t}_o = 0\ ^oC$ and $\bar{p}_o = 760$ mmHg and

$$\left.\begin{aligned}
&A = \frac{796.8}{1+\alpha t_o},\\
&A_1 = \frac{796.8}{1+\alpha t_1},\\
&B = \frac{\alpha k}{1+\alpha t_o},\\
&\alpha k = \frac{\alpha t_o - \alpha t_1}{s_1},\\
&\alpha = 0.003668,\\
&s_1 = 0.0014.
\end{aligned}\right\} \tag{2.3}$$

The above parameters can naturally be defined according to other atmospheric models.

The parallactic refraction $\Delta\zeta$ can be calculated from the true zenith distance z and the height parameter s using the following algorithm:

$$z_o' = z - \zeta_\infty \tag{2.4}$$

where ζ_∞ is obtained from formula (2.1) integrating between the limits 0 and 1 or from formula (1.4).

Using some additional notations we have

$$z' = z_o' + \zeta \tag{2.5}$$

where ζ is the integral (2.1) in which the upper integration limit ω comes from formulae (2.2) as a function of s,

$$\sin i = \frac{1-s}{1-\beta\omega} \sin z_o' \tag{2.6}$$

$$\frac{H}{r} \sin z' = \sin i - (1-s)\sin z' \tag{2.7}$$

and

$$\sin(z' - \bar{z}) = \frac{H}{r} \sin z' \frac{\sin\bar{z}}{\sin(z'-i)} \tag{2.8}$$

where $z' - \bar{z}$ is obtained by iteration.

Finally we have the parallactic refraction

$$\Delta\zeta = \zeta_\infty - \zeta + z' - \bar{z} \tag{2.9}$$

The results obtained are encouraging. One test is as follows:

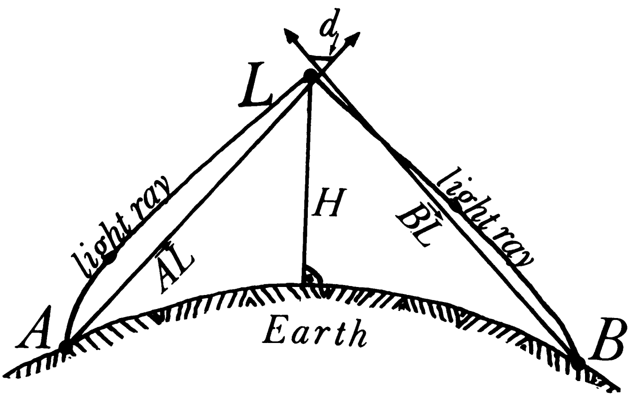

Fig. 1. A and B are the stations, H is the height of the signal L, $\vec{AL}$ and $\vec{BL}$ are uncorrected direction vectors and d is shortest segment between them.

A light signal was photographed from stations A and B, both of which were geographically known. In order to make the parallactic refraction correction to the direction vectors $\vec{AL}$ and $\vec{BL}$ determined from the background stars, the height of the light signal and hence the parameter s should be known (Fig. 1). This is done using an iterative process. In the first iteration step the height of the light signal is approximated with that of the shortest segment (d) between the uncorrected direction vectors $\vec{AL}$ and $\vec{BL}$. This height approximation is used for computing the parallactic refraction correction, which is then applied to the uncorrected direction vectors. The corrected direction vectors obtained redetermine the height, and a new value is then calculated for the parallactic refraction using recalculated height, etc. The iteration is continued until the shortest segment becomes constant. This is generally obtained after three repetitions. If the geographical stations' coordinates are correctly known and if there are no systematic errors in timing, or in modelling the parallactic refraction, etc., the segment d between the direction vectors, which have been finally corrected, should be zero in the limits of random errors. The accuracy of the parallactic refraction correction can then be estimated.

The analyses show that parallactic refraction can be calculated using Oterma's theory with an accuracy of about ± 2 sec of arc for zenith distances from 70° to 75°. The accuracy of this order is also obtained for zenith distances from 75° to 80°, but from 80° to 85° the segment seems to become greater and correlated to the zenith distance, which may be an indication of unsatisfactory modelling of the atmosphere.

3. REFERENCES

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DISCUSSION

T.J. Kukkamäki: Thank you and prof. Oterma for this presentation. I hope that nobody has anything against it, neither prof. Oterma herself.