CLUSTERS OF GALAXIES

Neta A. Bahcall
Princeton University, Department of Astrophysical Sciences,
Princeton, New Jersey, USA.

Abstract

Clusters and groups of galaxies contain the majority of galaxies in the universe. The rich clusters, while less numerous than the many poor groups, are the densest and largest systems known, and can be easily recognized and studied even at relatively large distances. Their study is important for understanding the formation and evolution of clusters and galaxies, and for a determination of the large-scale structure in the universe.

This talk summarizes the present understanding of the observed cluster sequence (§ I), its dynamical evolution (§ II), correlations of optical and X-ray properties (§ III), structural cluster parameters (§ IV), and the global cluster luminosity functions in the optical and X-ray bands (§§ V, VI).

I. CLUSTER CLASSIFICATION SEQUENCE

Clusters of galaxies can be organized into a one-parameter sequence analogous to galactic Hubble type. The sequence ranges from early to late type clusters, or equivalently, from compact to irregular clusters. The early (compact) are believed to be systems that are dynamically more evolved than the late (irregular) clusters. Many cluster properties (shape, concentration, dominance of bright galaxies, galactic content, density profile, mass segregation and X-ray and radio emission) are correlated with position in the cluster sequence. A summary of the sequence and its related properties, as well as the correlations among the various classification schemes based on these properties, is presented in Table 1. The standard classification schemes are based on the following properties: the morphological appearance of the cluster (Zwicky et al. 1961; Rood and Sastry 1971); the dominance of the brightest galaxies (Bautz-Morgan 1970; Rood and Sastry 1971); and the galactic content of clusters (Morgan 1962; Oemler 1974). (For a detailed discussion see review by Bahcall 1977c.)

Patrick A. Wayman (ed.), Highlights of Astronomy, Vol. 5, 699–714.

TABLE 1

	Cluster Sequence		
	Early		Late
	Compact —	Med. Compact —	Irregular
			
R – S	cD, B	C, L, F	Ir
B – M	I, II	II – III	III
Shape	Symmetrical	Intermediate	Irregular
Central Conc.	High	Moderate	Low
Content	E	S0	Sp
<Sp, S0, E %>	20, 45, 35	30, 55, 15	50, 35, 15
ΔM_{12}	Large ($\gtrsim 1^m$)	Medium	Small ($\lesssim 0.5^m$)
X-Ray (L_X)	High	Intermediate	Low
Radio (L_R)	High	Low	Low
Dyn. Evolution	Higher		Lower

https://doi.org/10.1017/S1539299600004627 Published online by Cambridge University Press

The Rood-Sastry classification scheme is based on the distribution and relative brightness of the ten brightest galaxies in a cluster. It ranges, in steps, from a single dominant central galaxy (cD) or a pair of dominant galaxies (B), to the less-dominant, less-regular, clusters (L, C, F, Ir). (L stands for a "line" of bright galaxies; C for a "core" distribution of bright galaxies; F for a flat distribution; and Ir for an irregular distribution of galaxies.).

The Bautz and Morgan scheme is based on the relative contrast of the brightest galaxy to the other galaxies in each cluster; it ranges from type I to III in order of decreasing contrast. The difference, ΔM_{12}, between the magnitude of the first and second brightest cluster galaxies is about 1 mag in type I clusters, and decreases to < 0.5 mag in type III clusters.

The early-type clusters (see Table 1) are typically R-S types cD and B, and B-M types I and II; they usually show a higher degree of symmetry and higher central concentration of galaxies than do the late type clusters. The galactic content of clusters also correlates with position in the cluster sequence (cf. Morgan 1962, Oemler 1974, Melnick and Sargent 1977, Bahcall 1977b). Late type clusters are generally rich in spirals (see Table 1), with a galactic distribution similar to that of field galaxies, while the intermediate type clusters are poor in spirals but rich in SO galaxies. The SO galaxies are believed to be spirals stripped of their gas by galaxy collisions in the dense cluster cores (Spitzer and Baade 1951) and by ram-pressure of the intracluster gas (Gunn and Gott 1972; Gisler 1979). Early-type clusters, which are dominated by central supergiant galaxies, have no spirals in their cores and contain a much higher proportion of ellipticals in the central regions than do the other cluster types. The higher fraction of ellipticals in the early-type clusters is not clearly understood; explanations range from galaxy collisions to initial conditions or early interstellar gas sweeping (e.g., Gott 1977, Rood and Norman 1979, Ostriker and Thuan 1975, and references therein). Typical percentages of E, SO and Sp galaxies in the different type clusters are summarized in Table 1. The ratio of spirals to elliptical galaxies in a cluster may also depend on magnitude (Shapely 1950) and on distance from the cluster center (Melnick and Sargent 1977). The brightest few galaxies of any cluster, however, are almost always of the E or SO type.

The cD galaxies frequently found at the centers of early-type clusters are supergiant galaxies with large diffuse envelopes; they occasionally contain multiple nuclei. These systems may be explained by the merging of bright galaxies in the dense centers of the compact clusters (Ostriker and Hausman 1977). The merging process may also explain the apparently nonstatistical features seen at the bright end of the cluster luminosity function, including the rather constant observed luminosity of the brightest galaxy in rich compact clusters.

II. DYNAMICAL EVOLUTION

The early to late-type cluster sequence (§ I, Table 1) can be explained qualitatively, and to some extent quantitatively, as a sequence of increasing dynamical evolution, with the denser systems progressing at a faster rate. The early-type, compact, dense, spiral-poor clusters may thus represent a higher degree of dynamical evolution than do the late-type, irregular, spiralrich clusters.

Some of the main dynamical processes involved in the evolution of clusters of galaxies are briefly summarized below (see Bahcall 1977c for a more detailed discussion). The rates of most of these processes are proportional to the galaxy density in the clusters and would therefore proceed faster in the denser clusters.

The collapse time scale of a typical compact cluster ($M = 10^{15} M_\odot$, $R \sim 3$ Mpc, $H_o = 50$) is $\sim 5 \times 10^9$ years. Since this is shorter than a Hubble time, it is believed that rich, compact-type clusters are collapsed systems that have undergone violent quasi-relaxation (Lynden-Bell 1967). Numerical models of the collapse (Aarseth 1963, 1969; Peebles 1970; White 1976b and references therein) indicate that violent dynamical evolution may occur for a short time immediately after collapse; after this period ($\gtrsim 1.5\ T_{collapse}$) the cluster may be essentially quiescent.

The two-body relaxation time for galaxies in clusters is typically too long ($\gtrsim 2 \times 10^{10}$ year) to play a significant role in dynamical evolution, except for the most massive galaxies in the cores of the compact clusters (where it is $\sim 10^9 - 10^{10}$ years). Therefore, no strong evidence for spatial and/or velocity segregation of galaxies by mass is either expected or observed, with the possible exception of massive galaxies in the cores of compact clusters (See Bahcall 1977c).

Galaxy interactions with gas (ram-pressure of the intracluster medium; Gunn and Gott 1972, Gisler 1979) are one of the main processes by which spirals can be stripped of gas, leaving a higher percentage of SO galaxies (and possibly E galaxies). A higher fraction of SO (and E) galaxies is indeed observed in high density, high velocity regions; SO and E galaxies are more frequent in clusters than in the field, relatively more common in compact clusters than in irregular ones, and more common in the cores than in the outer regions of rich clusters. Other processes such as collisions between spiral galaxies (Spitzer and Baade 1951) and thermal evaporation of galactic gas heated by a very hot intracluster gas (Cowie 1977) are also capable of stripping spirals and producing SO systems. If early and efficient gas removal by any of the above mechanisms prevents formation of galaxy discs (cf. Ostriker and Thuan 1975), leaving only the spheroidal bulge components of the would-be spirals, galaxies indistuinguishable from ellipticals may form. Thus, the same process that increases the fraction of SO galaxies in clusters may increase the proportion of ellipticals.

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Tidal interactions between galaxies will also proceed at a faster rate the higher the density and the higher the galaxy velocity. In the dense central regions of a compact cluster, the collision time scale is typically 10^8 to 10^9 years for a galaxy with a radius of $\sim$ 10 kpc. In the lower density regions of irregular clusters and outer regions of compact clusters the time between collisions is much longer ($> 10^{10}$ years). Richstone (1976) finds that if the galaxies in a dense cluster start out with extended halos 500 kpc in extent that encompass all the mass in the cluster, about 90% of the mass and 25% of the luminosity is liberated in a Hubble time. Tidal interactions would tend to reduce the tidal radius of the galaxies in the high-density, high-velocity regions; such an effect may be observationally hard to detect, but was recently reported in several rich clusters (Strom and Strom 1978a, 1978b).

As a natural consequence of dynamical friction, massive galaxies will tend to accumulate in the center of dense clusters (Ostriker and Tremaine 1975; White 1976a, 1977); if the density is high and velocity is low, the massive galaxies can merge to yield more luminous, larger galaxies of low surface brightness. This process of merger (or cannibalism; Gunn and Tinsley 1976, Ostriker and Hausman 1977) can explain the supergiant cD galaxies observed at the centers of early type clusters. These galaxies frequently contain multiple nuclei, which are explained, according to this model, as "undigested" victims of the cannibalism. The merging process can also explain the high and nearly constant observed luminosity of the cD galaxies, their low surface brightness, and the larger ΔM_{12} and ΔM_{13} observed in early-type versus late-type clusters. The larger gap of ΔM_{12} is due to the fact that the brightest galaxies merge first.

All the dynamical processes summarized above proceed at a faster rate for higher density systems and therefore tend to increase the dynamical evolution of early-type clusters as compared with late-type systems. Most of the observed properties of the cluster sequence (Table 1) can be understood in this way.

A larger than expected fraction of blue galaxies was observed by Butcher and Oemler (1978) in two compact, distant ($z = 0.4$) clusters; the ratio of blue to red galaxies increases with increasing distance from the cluster center. This suggests that relatively more young (spiral?) galaxies are observed in these clusters than in comparable compact-type nearby clusters (see also § III. Gisler 1979; Sarazin 1979). A similar trend of faint galaxies appearing bluer than expected was recently reported by Kron (1978) and by Katgert, de Ruiter and Van der Laan (1979). The latter study found from optical identification of a deep Westerbork sample of radio sources that at an estimated $z \gtrsim 0.5$ (or $m_R \gtrsim 19^m$) the radio galaxies become strikingly blue and more numerous compared to $z \lesssim 0.3$. A detailed study of faint galaxies ($z \gtrsim 0.3$) both in rich clusters and in the "field" is needed in order to better understand the nature of the suggested evolution.

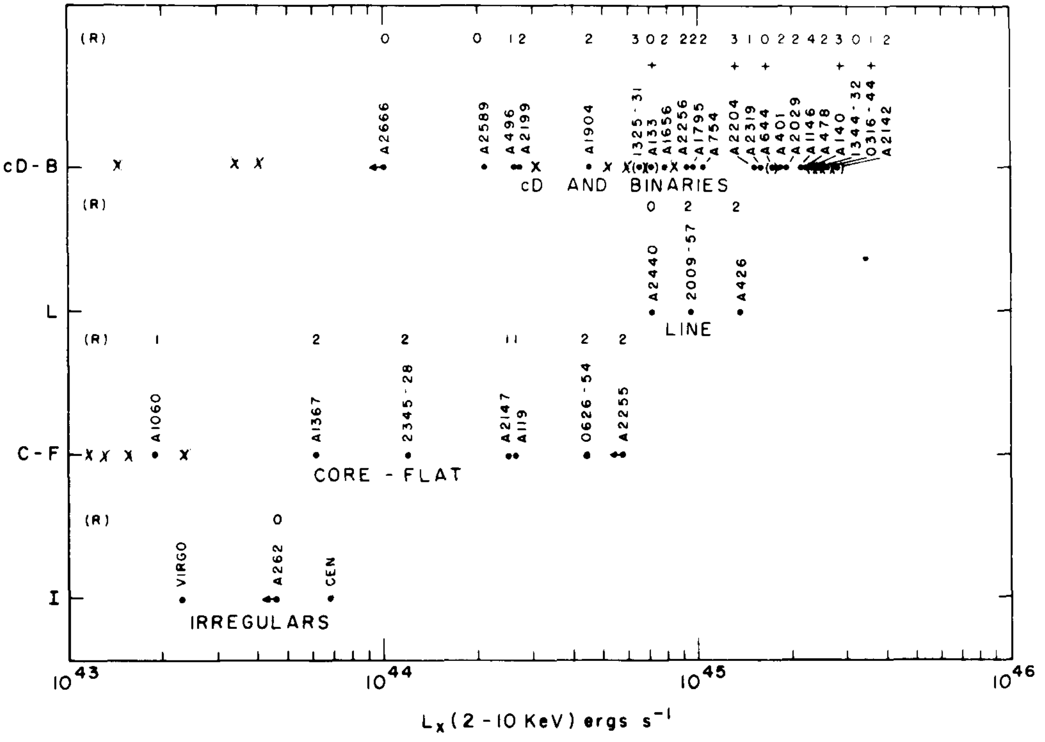

Figure 1. (From Bahcall 1977a). X-ray luminosity, L_x, of all 35 clusters in the Ariel-5 catalogue are plotted as a function of their R-S classification groups. Each cluster is identified by its name. Points in parenthesis are clusters with estimated redshift. The richness group (R) of each cluster is also identified. Recent detections by the Einstein satellite of nearby Abell clusters are marked by the x's.

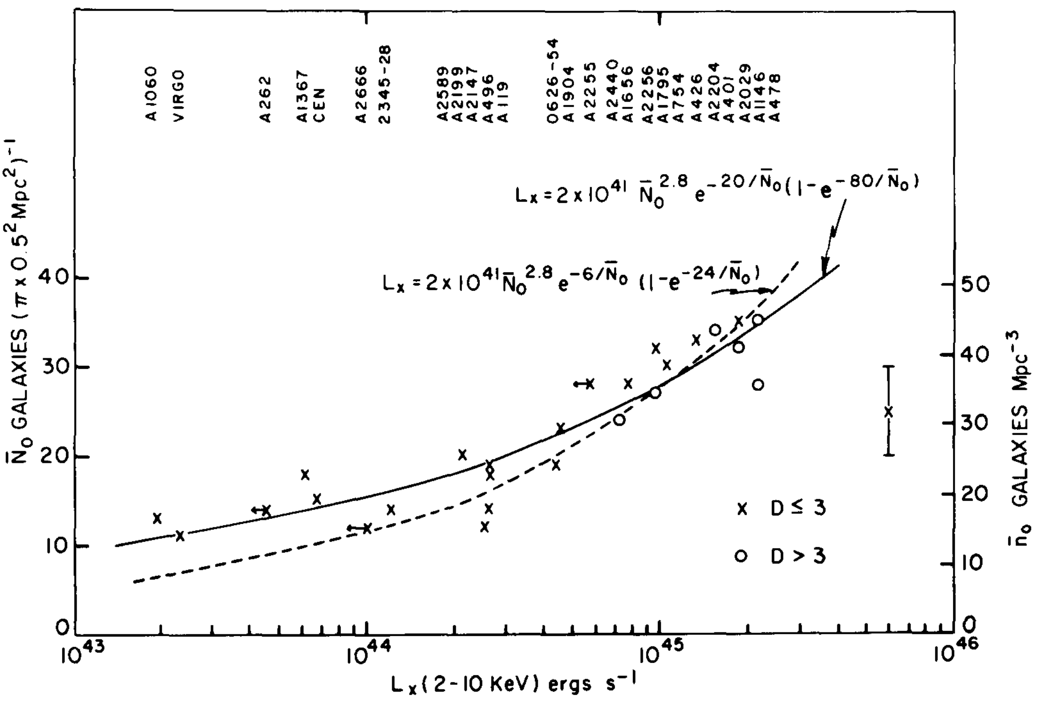

Figure 2. (From Bahcall 1977a). Central galaxy density within 0.5 Mpc radius of the cluster center, N_o, is plotted against L_x. A typical error-bar for N_o is shown. The curves are the expected dependence from a thermal bremsstrahlung model with $T \propto \upsilon^2 \propto \bar{N}_o$. The dashed curve represents an a priori fixed exponent, $B_E = 6$; the solid curve corresponds to B_E being a free parameter.

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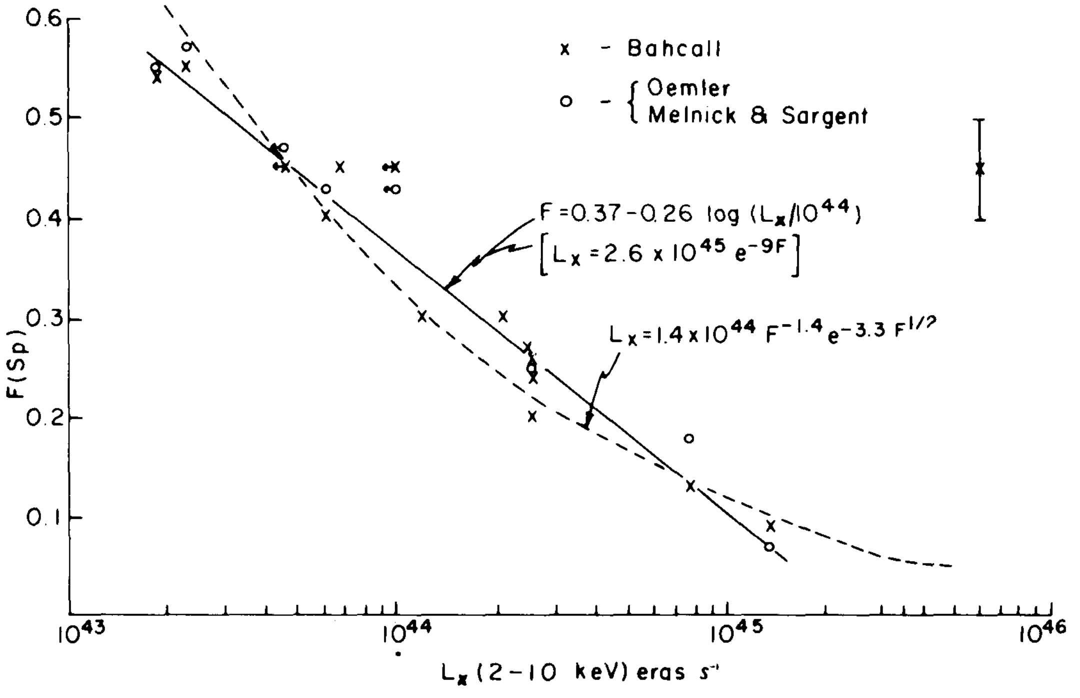

Figure 3. (From Bahcall 1977b). The fraction of spiral galaxies, $F(S_p) \equiv S_p/All$, is plotted against X-ray luminosity. A typical error-bar is shown. Solid and dashed lines, respectively, are the expected functional forms as given by equations (3) and (5) of Bahcall (1977b); these assume thermal bremsstrahlung emission from an intracluster gas whose ram-pressure strips the spiral galaxies.

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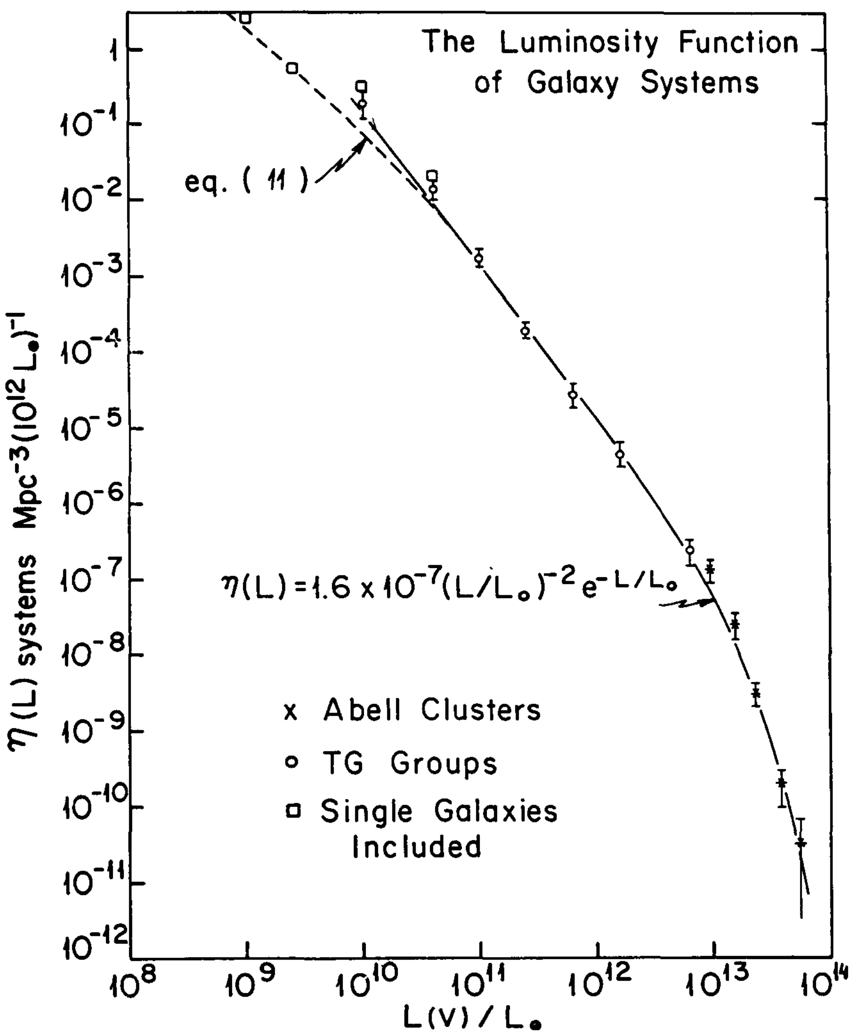

Figure 4. (From Bahcall 1979a). The luminosity function of all galaxy systems, from single galaxies and small groups to rich clusters. The Abell clusters are marked by the X's, the TG groups - by the circles, and squares represent the LF when "single" galaxies are added (where they have non-vanishing contribution). The solid line represents the best fitting curve to the group-cluster LF. The dashed line is the all-galaxy-system (AGS) LF, $\eta(L) = 1.6 \times 10^{-7} (L/L_o)^{-2} e^{-L/L_o} (1 + 1.6 \times 10^{10}/L)^{-0.75}$ Mpc^{-3} $(10^{12} L_{\odot})^{-1}$, with $L_o \cong 1 \times 10^{13} L_{\odot}$, (eq. 11 of Bahcall 1979a). It includes single galaxies and extrapolates, at faint luminosities, to the properly normalized Schechter's field-galaxy LF; at high luminosities (L > L*) it is identical to the groupings LF (solid line).

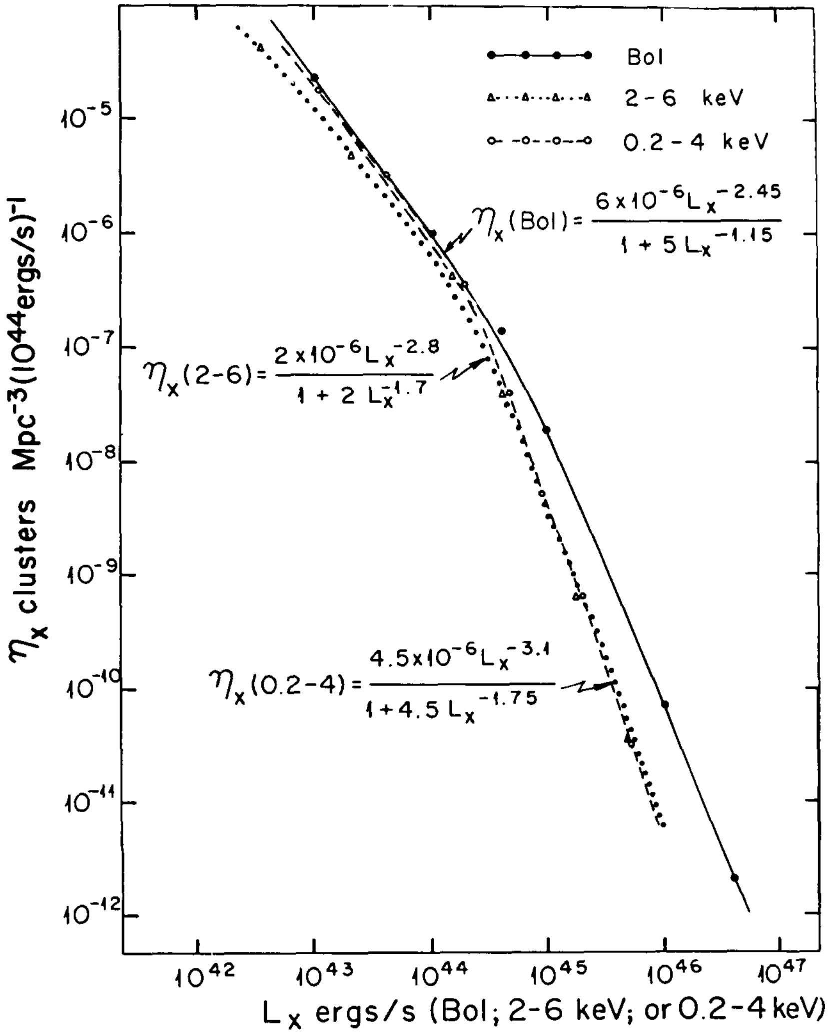

Figure 5. (Bahcall 1979b). Calculated X-ray luminosity-function of clusters of galaxies for three X-ray energy bands: bolometric, i.e., very wide; 2-6 kev; and 0.5-3 kev. The actual numerical calculations are represented by dots (Bol.), triangles (2-6 kev), and open circles (0.5-3 kev). The curves represent analytic expressions (Bahcall 1979b) that describe each appropriate LF (as indicated in the figure). The X-ray luminosity L_x represents L_x (Bol) for η_x(Bol), L_x(2-6) for η_x(2-6), and L_x(0.5-3) for η_x(0.5-3).

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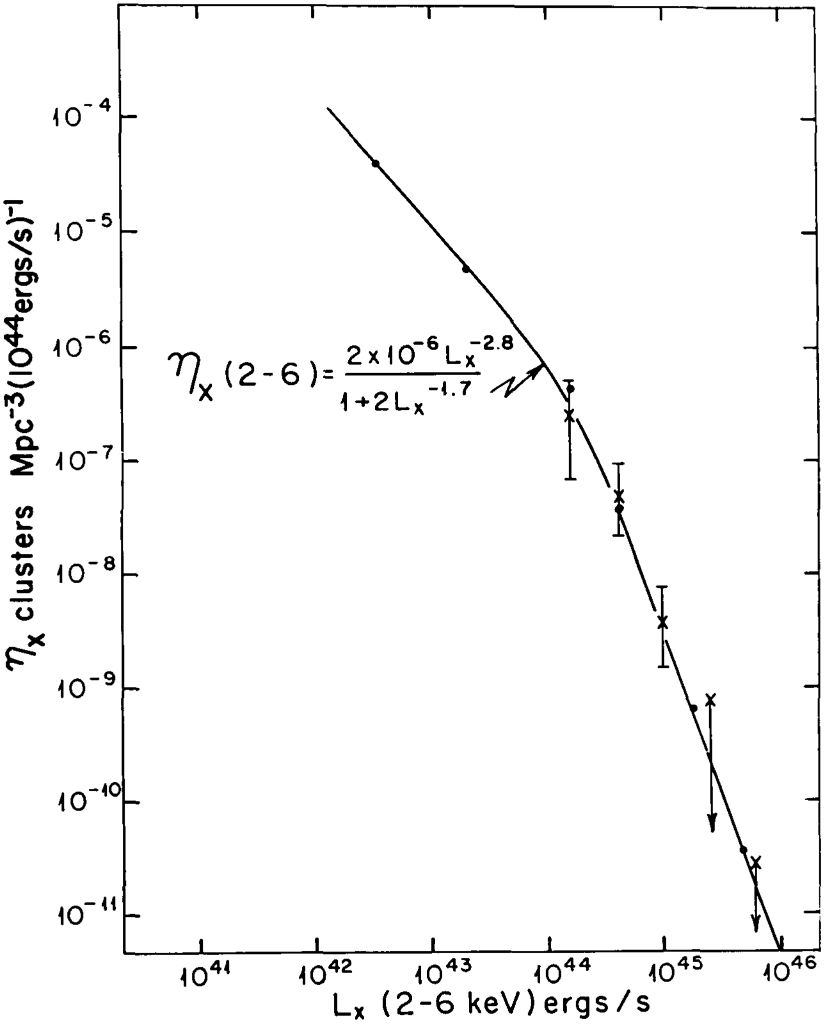

Figure 6. (Bahcall 1979b). Calculated (curve and dots) and observed (X's) LF in the range 2-6 kev. The dots represent the numerical calculations (Figure 5), the curve is the descriptive analytic expression, and the X's are the available observations (as given by Schwartz 1978).

III. CORRELATIONS OF OPTICAL AND X-RAY PROPERTIES

The idea that the diffuse X-ray emission from clusters of galaxies (Uhuru: Giacconi et al. 1974, Forman et al. 1978; Ariel-V: Cooke et al. 1978) correlates with the cluster sequence described above was suggested and studied by Bahcall (1974, 1977a, 1977b). These investigations yielded three main correlations between the X-ray luminosity of a cluster and the optical properties related to the cluster sequence. The correlations are illustrated in Figures 1 through 3.

The first correlation is between X-ray luminosity and optical cluster morphology (Figure 1). Bahcall (1974, 1977a) found that early-type clusters have, *on the average*, higher X-ray luminosity than do the late-type clusters; intermediate-type clusters have intermediate X-ray intensities. She suggested that this indicates a high intracluster gas density and temperature in the early-type clusters.

It was suggested by Bahcall (1977a) that since most of the hot intracluster gas appears to have originated from the galaxies (the iron in the X-ray emitting gas has approximately a solar abundance: Serlemitsos et al. 1977; Bahcall and Sarazin 1977; Mushotzky et al. 1978), the hot intracluster gas density should be roughly proportional to the galaxy density. Bahcall determined the galaxy density at the centers of all 26 X-ray clusters with measured redshift and found, as expected, a correlation between X-ray luminosity and average galaxy (or gas) central density (Figure 2). The correlation found agrees well with the theoretical expectation (curves in Figure 2) of a thermal bremsstrahlung model in which the gas temperature T is proportional to the square of the galaxy velocity, υ^2 (or equivalently, to the central galaxy density). This correlation is fully consistent with, and is more quantitative than, the luminosity-morphology relation discussed above.

Since a high galaxy and gas density in the central region of a compact cluster implies a higher rate of collisions and ram-pressure (Spitzer and Baade 1951, Gunn and Gott 1972), many of the spiral galaxies in these high density regions should be stripped of their interstellar matter; therefore, fewer spirals than average should be found in the high density, high X-ray luminosity clusters. Bahcall (1977b) determined the fraction of spiral galaxies, F(Sp), in all (then known) X-ray clusters with $z \leq 0.05$. She found a strong correlation, as expected, between X-ray luminosity and F(Sp); F(Sp) decreases sharply with increasing X-ray luminosity (i.e. density) (Figure 3). The curves in Figure 3 represent theoretically expected L_X - F(Sp) relations assuming a combined model of thermal bremsstrahlung emission for the X-rays and stripping of the spiral galaxies by the same intracluster gas (see Bahcall 1977b) for more details). The agreement between the observed and expected dependence is good; this correlation therefore provides support for both the thermal bremsstrahlung model and the ram-pressure stripping of spiral galaxies. (See also McHardy 1978 whose results are consistent with the above correlations.)

Spectra and temperatures of a large sample of X-ray clusters determined by Mushotzky et al. (1978), Smith et al. (1979), and Forman and Jones (1978), yield consistent agreement with the above correlations; the observed temperatures are typically higher for high density clusters, as expected. The velocity dispersion in the clusters appears to be also correlated with the temperature, but the spread is still rather large (e.g. Smith et al. 1979).

Recent measurements by Bahcall (1979c) of galaxy densities and F(Sp) of low-luminosity rich clusters recently detected by Einstein (Murray et al. 1979) appear to be consistent with, and strengthen, the correlations discussed above (Fig. 1-3).

The core radii of the hot cluster gas responsible for the X-ray emission was determined by Murray et al. (1979) for a sample of low luminosity clusters detected by the Einstein satellite. These sizes appear to be similar to the radii determined optically by Bahcall for the distribution of the brightest ~3 mag cluster galaxies (same reference). These core-radii range from ~0.2 Mpc to ~0.4 Mpc (for H_o = 50 km/s Mpc^{-1}), with a typical average value of ~0.3 Mpc.

IV. STRUCTURE AND CORE-RADII

The density profile of the distribution of galaxies in a rich cluster can be described to a good approximation by a truncated isothermal profile (Zwicky 1957, Bahcall 1972, 1975), or by a King-type curve (see Bahcall 1977c for a review). These functions define two main structural parameters of a cluster: a central galaxy density and a core radius. The core radius of a cluster is usually defined as the radius at half maximum surface density. A combined surface distribution of 15 rich clusters is shown in Figure 2 of Bahcall (1977c). In some clusters the galaxy distribution is less regular and the fit to the above standard curves is less good.

The core radii of rich compact clusters are found by Bahcall (1975, 1977c) to be rather constant, with an average value of $R_c = 0.25 \pm 0.05$ Mpc (H_o = 50 km/s Mpc). This value is the size measured for the distribution of the brightest ~3 magnitude galaxies in the cluster. Dressler (1976) measured a sample of rich clusters to much fainter magnitudes (~6^m) and found a similar constancy in the core size, but with a larger value of typically ~0.4 Mpc. Quintana (1979) and Sarazin (1979b) have carefully measured the core radii of clusters as a function of galaxy magnitude (Sarazin with an improved maximum-likelihood method which involves only the direct galaxy positions) and find that the core radius increases with magnitude. They find that the core radius is indeed ~0.2 Mpc for the brightest galaxies, as obtained by Bahcall, but increases to ~0.3 Mpc, or possibly ~0.4 Mpc, for the much fainter galaxies. The dependence of core radius on magnitude interval may be due to real physical phenomena (such as mass segregation; tidal inter-

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actions; sub-clumps of faint galaxies around massive ones) or selection effects (e.g., obscuration of the very faint galaxies in the dense cluster cores). The possible dependence of core-size on magnitude should be taken into account when using core-sizes of clusters in various connections.

V. OPTICAL LUMINOSITY FUNCTION OF CLUSTERS OF GALAXIES

The optical luminosity function of galaxy systems - from single galaxies and small groups to rich clusters of galaxies - was recently determined by Bahcall (1979a). The catalogues of Abell (1958) and Turner and Gott (1976) were used for the rich clusters and small groups respectively. The total luminosity of an Abell cluster was assumed to be proportional to the number of member galaxies within a specific luminosity range, and was normalized to the total observed luminosity of the Coma cluster (see Bahcall 1979a for details). The luminosity function of rich (Abell) clusters was found to be a steep function of luminosity. It can be well represented by a Schechter (1976) function with a power-law slope of $\alpha = -2$, and an exponential cutoff $L_o \simeq 10^{13} L_\odot$. Alternatively, if represented as a power-law dependence alone, it roughly satisfies $\eta(L) \simeq 2 \times 10^{-7} (L/10^{13})^{-5}$ clusters $Mpc^{-3} (10^{12} L_\odot)^{-1}$. The rich cluster LF is represented by the x's in Figure 4.

The LF of rich clusters was combined with that of small groups to yield a general galaxy groupings LF (Bahcall 1979a). A smooth transition between the groups and rich clusters occurs near $\simeq 10^{13} L_\odot$. The overall LF can be represented well by a Schechter function:

$$\eta(L) = 1.6 \times 10^{-7} (L/L_o)^{-2} \exp(-L/L_o) \ \text{cls Mpc}^{-3} (10^{12} L_\odot)^{-1},$$

with $L_o \simeq 10^{13} L_o$ (Figure 4). The "break" at L_o occurs at the typical luminosity of rich Abell-type clusters. The best-fit slope α is found to depend on the density enhancement factor with which the groups are selected, but the dependence is relatively weak. The functional form is consistent with the Press-Schechter (1974) model of galaxy formation and clustering, and with the recent Silk-White (1978) model.

When single galaxies are added to the groupings LF, the LF of all galaxy systems (AGS) flattens to $\eta(L) \propto L^{-1.25}$ below L^*, as expected from the field galaxy LF of Schechter (1976). The AGS at higher luminosities ($> L^*$) is well represented by the above-described groupings LF (Figure 4).

The luminosity density in the universe as calculated by integrating the above AGS LF is found to be (Bahcall 1979a) $\mathcal{L}(B(0)) \simeq 10^8 L_\odot(B)$ Mpc^{-3}, in agreement with the recent value reported by Felten (1976). About 5% of the luminosity is contributed by rich (Abell) clusters, $\simeq$ 75% by small groups, and $\simeq$20% by "single" galaxies.

VI. X-RAY LUMINOSITY FUNCTION OF CLUSTERS OF GALAXIES

The X-ray luminosity function of clusters of galaxies was calculated by Bahcall (1979b) using the optical LF of clusters and groups discussed above (§ V), and a relation between X-ray and optical luminosity based on a thermal bremsstrahlung model. The derived X-ray LF applies to clusters whose X-ray emission is mostly due to thermal bremsstrahlung from a smooth, hot intracluster gas. The predicted LF for three typical X-ray energy bands of observation (Bolometric; 2-6 kev; and 0.5 - 3 kev (Einstein)) are presented in Figure 5; included in the figure are simple analytic representations $\eta_x(L_x)$ to describe these functions. The main features predicted by the LF are (see also Bahcall 1979b) a change of slope in the log η_x - log L_x plane at $L_x \sim 10^{44}$ ergs/s, with a moderately steep slope of ~ -2.5 at high luminosities ($\sim 10^{44} - 10^{46}$ ergs/s), and a flatter slope of ~ -1.3 below 10^{44} ergs/s. Below $\sim 10^{42-43}$ ergs/s, the potential well of the associated cluster is weaker and the smoothly distributed thermal emission may not dominate the total X-ray emission since gas may clump around the massive cluster galaxies (Bahcall and Sarazin 1977). This low-luminosity region is not predicted by the model.

The calculated LF is in good agreement with available X-ray data (Figure 6). The observed upper limits at $\sim 10^{45-46}$ ergs/s may be close to the actual density of high luminosity clusters, as predicted by the calculated LF. Note that the calculated LD does *not* decay exponentially at the bright end ($<10^{46}$ ergs/s) *if* the optical LF of § V is indeed a good representation of this high luminosity end.

With the above assumptions, the author estimates that most Abell clusters have diffuse thermal bremsstrahlung luminosities in the range $\sim 10^{43} - 10^{46}$ ergs/s (2-6 kev). The Einstein satellite observations may be able to check this prediction.

REFERENCES

Aarseth, S.J.: 1963, M.N.R.A.S. 126, 223.
Aarseth, S.J.: 1969, M.N.R.A.S. 144, 537.
Abell, G.O.: 1958, Ap.J.Suppl. 3, 211.
Bahcall, J.N. and Sarazin, C.L.: 1977, Ap.J.Lett. 213, L99.
Bahcall, N.A.: 1972, A.J. 77, 550.
Bahcall, N.A.: 1974, Ap.J. 193, 529.
Bahcall, N.A.: 1975, Ap.J. 198, 249.
Bahcall, N.A.: 1977a, Ap.J.Lett. 217, L77.
Bahcall, N.A.: 1977b, Ap.J.Lett. 218, L93.
Bahcall, N.A.: 1977c, Ann.Rev.Astron.Astrophys. 15, 505.
Bahcall, N.A.: 1979a, Ap.J. 232, 689.
Bahcall, N.A.: 1979b, Ap.J.Lett. 232, L83.
Bahcall, N.A.: 1979c, in preparation.
Bautz, L.P. and Morgan, W.W.: 1970, Ap.J.Lett. 162, L149.
Butcher, H. and Oemler, A.: 1978, Ap.J. 219, 18.

https://doi.org/10.1017/S1539299600004627 Published online by Cambridge University Press

Cooke, B.A. et al.: 1978, M.N.R.A.S. 182, 489.
Cowie, L.: 1977, Nature 266, 501.
Dressler, A.: 1976, Ph.D. Thesis, U.C. Santa Cruz.
Felten, J.E.: 1976, Ap.J. 207, 700.
Forman, W. et al.: 1978, Ap.J.Suppl. 38, 357.
Gisler, G.R.: 1979, preprint.
Giacconi, R. et al.: 1974, Ap.J.Suppl. 27, 37.
Gott, J.R.: 1977, Ann.Rev.Astron.Astrophys. 15, 235.
Gunn, J.E. and Gott, J.R.: 1972, Ap.J. 176, 1.
Gunn, J.E. and Tinsley, B.: 1976, Ap.J. 210, 1.
Jones, C. and Forman, W.: 1978, Ap.J. 224, 1.
Katgert, P., de Ruiter, H.R. and Van der Laan, H.: 1979, Nature 230, 20.
Kron, R.: 1978, Ph.D. Thesis, U.C. Berkeley.
Lynden-Bell, D.: 1967, M.N.R.A.S. 136, 101.
McHardy, I.: 1978, M.N.R.A.S. 184, 783.
Melnick, J. and Sargent, W.L.W.: 1977, Ap.J. 215, 401.
Morgan, W.W.: 1962, Ap.J. 135, 1.
Murray, S.S. et al.: (in preparation)
Mushotzky, R.F., Serlemitsos, P.J., Smith, B.W., Boldt, E.A. and Holt, S.S.: 1978, Ap.J. 225, 21.
Oemler, A.: 1974, Ap.J. 194, 1.
Ostriker, J.P. and Hausman, M.A.: 1977, Ap.J.Lett. 217, L125.
Ostriker, J.P. and Thuan, T.X.: 1975, Ap.J. 202, 353.
Ostriker, J.P. and Tremaine, S.D.: 1975, Ap.J.Lett. 202, L113.
Peebles, P.J.E.: 1970, A.J. 75, 13.
Press, W.H. and Schechter, P.: 1974, Ap.J. 187, 425.
Quintana, H.: 1979, A.J. 84, 15.
Richstone, D.O.: 1976, Ap.J. 204, 642.
Rood, H.J. and Sastry, G.N.: 1971, P.A.S.P. 83, 313.
Roos, N. and Norman, C.A.: 1979, Astron.Astrophys. 76, 75.
Sarazin, C.L.: 1979a, preprint.
Sarazin, C.L.: 1979b, preprint.
Schechter, P.: 1976, Ap.J. 203, 297.
Schwartz, D.A.: 1978, Ap.J. 220, 8.
Serlemitsos, P.J., Smith, B.W., Boldt, E.A., Holt, S.S. and Swank, J.H.: 1977, Ap.J.Lett. 211, L63.
Shapely, H.: 1950, Publ. Uni. Michigan Obs. 10, 79.
Silk, J.C. and White, S.D.M.: 1978, Ap.J.Lett. 223, L59.
Smith, B.W., Mushotzky, R.F. and Serlemitsos, P.J.: 1979, Ap.J. 227, 37.
Spitzer, L. and Baade, W.: 1951, Ap.J. 113, 413.
Strom, S.E. and Strom, K.M.: 1978a, A.J. 83, 732.
Strom, S.E. and Strom, K.M.: 1978b, Ap.J.Lett. 225, L93.
Turner, E.L. and Gott, J.R.: 1976, Ap.J.Suppl. 32, 409.
White, S.D.M.: 1976a, M.N.R.A.S. 174, 19.
White, S.D.M.: 1976b, M.N.R.A.S. 177, 717.
White, S.D.M.: 1977, M.N.R.A.S. 179, 33.
Zwicky, F.: 1957, "Morphological Astronomy", Berlin: Springer-Verlag.
Zwicky, F. et al.: 1961, "Catalogue of Galaxies and Clusters of Galaxies", 6 volumes (1961-1968). Pasadena: California Institute of Technology.