8

Electromagnetic interactions

In this section we discuss the interaction of nuclei, nucleons, or any finite quantum mechanical system with the electromagnetic field. Much of what we know about nuclei and nucleons comes from such interactions. We start with the general multipole analysis of the interaction of a nucleus with the quantized radiation field [Bl52, Sc54, de66, Wa95]. In the following $e_{\rm p} = |e|$ is the proton charge.

The starting point in this analysis is the total hamiltonian for the nuclear system, the free photon field, and the electromagnetic interaction

$$
\begin{aligned}
H_{\rm total} &= H_{\rm nuclear} + \sum_{\mathbf{k}} \sum_{\rho=1,2} \hbar\omega_k a^\dagger_{\mathbf{k}\rho} a_{\mathbf{k}\rho} \\
&\quad -\frac{e_{\rm p}}{c}\int \mathbf{J}_N(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})\, d^3x + \frac{e_{\rm p}^2}{8\pi}\int\int \frac{\rho_N(\mathbf{x})\rho_N(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d^3x\, d^3x'
\end{aligned} \tag{8.1}
$$

This is the hamiltonian of *quantum electrodynamics* (QED); it is written in the Coulomb gauge. $\mathbf{A}$ is the vector potential for the quantized radiation field, which in the Schrödinger picture takes the form

$$
\mathbf{A}(\mathbf{x}) = \sum_{\mathbf{k}} \sum_{\rho=1,2} \left(\frac{\hbar c^2}{2\omega_k \Omega}\right)^{1/2} [\mathbf{e}_{\mathbf{k}\rho} a_{\mathbf{k}\rho} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}] \tag{8.2}
$$

Here $\mathbf{e}_{\mathbf{k}\rho}$ with $\rho = (1, 2)$ represent two unit vectors transverse to $\mathbf{k}$ (see Fig. 8.1). The hermitian conjugate is denoted by h.c. We quantize with periodic boundary conditions (p.b.c.) in a large box of volume Ω, and in the end let $\Omega \to \infty$. With this choice

$$
\frac{1}{\Omega}\int_{\rm Box} d^3x\, e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{x}} = \delta_{\mathbf{k}_1,\mathbf{k}_2} \tag{8.3}
$$

where the expression on the right is a Kronecker delta.

https://doi.org/10.1017/9781009290616.011 Published online by Cambridge University Press

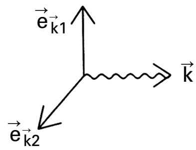

Fig. 8.1. Transverse unit vectors.

The only assumption made about the target is the existence of local current and charge density operators $\mathbf{J}_N(\mathbf{x})$ and $\rho_N(\mathbf{x})$. These quantities must exist for any true quantum mechanical system. H_{nuclear} could be given in terms of potentials, or it could be for a coupled baryon and meson system, or it could be for a system of quarks and gluons; it does not matter at this point.[1]

It is convenient to henceforth incorporate the explicit factor of $1/c$ in Eq. (8.1) into the definition of the current $\mathbf{J}_N(\mathbf{x})$ itself.

First go from plane polarization to circular polarization with the transformation (cf. Fig. 8.1).

$$\mathbf{e}_{\pm 1} \equiv \mp \frac{1}{\sqrt{2}}(\mathbf{e}_1 \pm i\mathbf{e}_2) \qquad\qquad \mathbf{e}_0 \equiv \mathbf{e}_z \equiv \frac{\mathbf{k}}{|\mathbf{k}|} \tag{8.4}$$

These circular polarization vectors satisfy the relations

$$\mathbf{e}^{\dagger}_{\mathbf{k}\lambda} = (-1)^{\lambda}\mathbf{e}_{\mathbf{k}-\lambda} \qquad\qquad \mathbf{e}^{\dagger}_{\lambda}\cdot\mathbf{e}_{\lambda'} = \delta_{\lambda\lambda'} \tag{8.5}$$

If, at the same time, one defines

$$a_{\mathbf{k}\pm 1} \equiv \mp \frac{1}{\sqrt{2}}(a_{\mathbf{k}1} \mp i a_{\mathbf{k}2}) \tag{8.6}$$

then the transformation is *canonical*

$$[a_{\mathbf{k}\lambda}, a^{\dagger}_{\mathbf{k}'\lambda'}] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'} \tag{8.7}$$

Since $\mathbf{e}_1 a_1 + \mathbf{e}_2 a_2 = \mathbf{e}_{+1}a_{+1} + \mathbf{e}_{-1}a_{-1}$ the vector potential takes the form

$$\mathbf{A}(\mathbf{x}) = \sum_{\mathbf{k}} \sum_{\lambda=\pm 1} \left(\frac{\hbar c^2}{2\omega_k \Omega}\right)^{1/2} [\mathbf{e}_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}] \tag{8.8}$$

The index $\lambda = \pm 1$ is the circular polarization, as we shall see, and only $\lambda = \pm 1$ appears in the expansion so $\nabla \cdot \mathbf{A}(\mathbf{x}) = 0$, characterizing the Coulomb gauge.

Now proceed to calculate the transition probability for the nucleus or

[1] Although Eq. (8.1) is correct in QCD, some models may have an additional term of $O(e^2\mathbf{A}^2)$ in the hamiltonian; the arguments in this section are unaffected by such a term.

nucleon to make a transition between two states and emit (or absorb) a photon. Work to lowest order in the electric charge e, use the *Golden Rule*, and compute the nuclear matrix element $\langle J_f M_f; \mathbf{k}\lambda | H' | J_i M_i \rangle$ where H' is here the term linear in the vector potential in Eq. (8.1); it is this interaction term that can create (or destroy) a photon. All that will be specified about the nuclear state at this point is that it is an eigenstate of angular momentum. It will be assumed that the target is massive and its position will be taken to define the origin; transition current densities occur over the nuclear volume and hence *all transition current densities will be localized in space*. Since the photon matrix element is $\langle \mathbf{k}\lambda | a^\dagger_{\mathbf{k}'\lambda'} | 0 \rangle = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}$, the required transition matrix element takes the form[2]

$$\langle J_f M_f; \mathbf{k}\lambda | \hat{H}' | J_i M_i \rangle = -e_{\rm p} \left(\frac{\hbar c^2}{2\omega_k \Omega} \right)^{1/2} \langle J_f M_f | \int e^{-i\mathbf{k}\cdot\mathbf{x}} \mathbf{e}^\dagger_{\mathbf{k}\lambda} \cdot \hat{\mathbf{J}}(\mathbf{x})\, d^3x | J_i M_i \rangle \tag{8.9}$$

This expression now contains all of the physics of the target. We proceed to make a multipole analysis of it. With the aid of the Wigner–Eckart theorem we will then be able to extract two invaluable types of information:

- The angular momentum selection rules
- The explicit dependence on the orientation of the target as expressed in (M_i, M_f)

The goal of the multipole analysis is to reduce the transition operator to a sum of *irreducible tensor operators* (ITO) to which the Wigner–Eckart theorem applies [Ed74].

We recall the definition of an ITO. It is a set of $2J+1$ operators $\hat{T}(J, M)$ with $-J \leq M \leq J$ that satisfy the following commutation relations with the three components $\hat{J}_i$ of the angular momentum operator

$$[\hat{J}_i, \hat{T}(J, M)] = \sum_{M'} \langle JM' | \hat{J}_i | JM \rangle\, \hat{T}(J, M') \tag{8.10}$$

The above is the infinitesimal form of the integral definition of an ITO (they are fully equivalent)

$$\hat{R}_{\alpha\beta\gamma}\, \hat{T}(J, M) \hat{R}^{-1}_{\alpha\beta\gamma} = \sum_{M'} \mathscr{D}^J_{M'M}(\alpha\beta\gamma)\, \hat{T}(J, M') \tag{8.11}$$

Here $\hat{R}_{\alpha\beta\gamma}$ is the rotation operator, and $\mathscr{D}^J_{M'M}(\alpha\beta\gamma)$ are the rotation matrices, defined by [Ed74]

$$\begin{aligned} \hat{R}_{\alpha\beta\gamma} &\equiv e^{i\alpha\hat{J}_3} e^{i\beta\hat{J}_2} e^{i\gamma\hat{J}_3} \\ \mathscr{D}^J_{M'M}(\alpha\beta\gamma) &= \langle JM' | e^{i\alpha\hat{J}_3} e^{i\beta\hat{J}_2} e^{i\gamma\hat{J}_3} | JM \rangle \end{aligned} \tag{8.12}$$

We proceed to the multipole analysis.

[2] For clarity we now use a notation where a caret over a symbol denotes an operator in the target Hilbert space.

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