This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background material in case the problem is unsolved. Send all communications concerning this department to I.G. Connell, Department of Mathematics, McGill University, Montreal, P.Q.

# ON THE SET OF ZERO DIVISORS OF A TOPOLOGICAL RING

Kwangil Koh

Let R be a topological (Hausdorff) ring such that for each $a \in R$, aR and Ra are closed subsets of R. We will prove that if the set of non-trivial right (left) zero divisors of R is a non-empty set and the set of all right (left) zero divisors of R is a compact subset of R, then R is a compact ring. This theorem has an interesting corollary. Namely, if R is a discrete ring with a finite number of non-trivial left or right zero divisors then R is a finite ring (Refer [1]).

THEOREM. Let R be a topological ring such that aR and Ra are closed subsets of R for any $a \in R$. If the set of non-trivial right (left) zero divisors of R is non-empty and the set of all right (left) zero divisors is compact then R is a compact ring.

Proof: Let A be a non-empty set of all non-trivial right zero divisors of R (i.e., $A = \{0 \neq x \in R \mid rx = 0$ for some $r \neq 0$ in $R\}$). Then for any $a \in A$, $aR \subseteq A \cup \{0\}$. aR is compact since it is a closed subset of the compact set $A \cup \{0\}$. Let $(a)^r = \{r \in R \mid ar = 0\}$. Then $(a)^r$ is a closed set since the left multiplication by a is a continuous function and the space R is a Hausdorff space. $(a)^r$ is a compact set since $(a)^r \subseteq A \cup \{0\}$. Since aR is homeomorphic to $R/(a)^r$, the quotient space of R by $(a)^r$ is compact. Thus R is compact by [2: (5.25) THEOREM, p.39].



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North Carolina State University
Raleigh, North Carolina




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