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ABSTRACT. Evolutionary tracks for a $30 \mathrm{M}_{\odot}$ star with various mass loss rates (MLR) were evolved to core He exhaustion. The "overluminosity" of mass losing (ML) stars is explained in terms of the well known mass -1 uminosity ( $M-L$ ) law. A critical ZAMS MLR above which mass loss leads to evolution to fainter luminosities is derived. Two tracks showed reversals in their direction of evolution across the HR diagram. These have been shown to be a consequence of mass loss dominating over the effects of the shell source. An analytic criterion for this condition has been derived.

Evolutionary tracks (Fig. 1) for a 30 M star with $(X, Z)=$ ( $0.71,0.02$ ) and initial MLR of ( $0.0,1.0,2.5,5.8,10.0$ ) $\times 10^{-7} \mathrm{M} / \mathrm{yr}$ have been calculated to core He exhaustion or until the star reached the giant branch (track Tl06). The mass loss was governed by the McCrea (1962) algorithm, $\dot{\mathrm{M}}=-\mathrm{kLR} / \mathrm{M}$.

Nuclear burning during the main sequence (MS) phase always causes the stellar luminosity to increase (viz, the constant mass track T00 in Fig. 1). But, if mass is lost at a very high rate so that little nuclear fuel is burnt before the mass is significantly reduced, then the star will evolve down the ZAMS. Since the radius increases due to the growing composition discontinuity between the convective core and the envelope, the ML star will evolve redwards in the HR diagram within the wedge defined by the constant mass track and the ZAMS. At extremely high MLR, the radius may actually decrease but the star will never evolve bluer than the ZAMS unless the surface $H$ is reduced (de Loore et al. 1977, Chiosi et al. 1978, Dearborn et al.1978). However, for all the tracks in Fig. 1 it has been demonstrated (Falk and Mitalas 1981a) that the luminosity is given by the $M-\mathrm{L}$ law,

$$
\mathrm{L}=\mathrm{A}(\langle\beta\rangle)<\mu\rangle^{4}\langle\beta\rangle^{4} \mathrm{M}^{3}
$$

where $\langle\mu\rangle$ is the average mean molecular weight of the star, $\langle\beta\rangle$ is the mean ratio of gas to total pressure defined by the Eddington quartic,

$$
\begin{equation*}
(1-\langle\beta\rangle) /\langle\beta\rangle^{4}=0.00309\langle\mu\rangle^{4} \mathrm{M}^{2} \text {. } \tag{2}
\end{equation*}
$$

$\overline{\text { *present } a d d r e s s: ~ U n i v e r s i t y ~ o f ~ S u s s e x, ~ B r i g h t o n, ~ U . K . ~}$

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C. Chiosi and R. Stalio (eds.), Effects of Mass Loss on Stellar Evolution, 261-263. Copyright © 1981 by D. Reidel Publishing Company.
and $\left.A\left(\langle\beta\rangle_{0}\right)=0.00309(4 \pi c G / K)\left(M_{\odot} / L_{\odot}\right)(0.9989-0.5910<\beta\rangle_{0}^{2}\right)$ is a constant during the evolution. Even though the luminosity of a ML star is reduced by mass loss, it can never be less than that of an equivalent mass (at the instant of comparison) star which has evolved without mass loss. This 'overluminosity" can be derived from (1) and (2) to be

$$
\begin{equation*}
\frac{L(\dot{M}, M)}{L(\dot{M}=0, M)}=\frac{A\left(\left\langle\beta\left(M_{0}\right)\right\rangle_{0}\right)}{A^{\prime}\left(\langle\beta(M)\rangle_{0}\right)} \quad \frac{1-\langle\beta(\dot{M}, M)\rangle}{1-\langle\beta(\dot{M}=0, M)\rangle} \tag{3}
\end{equation*}
$$

Since $\langle\beta(\dot{M}, M)\rangle$ is always less than $\langle\beta(\dot{M}=0, M)\rangle$ due to the larger $\langle\mu>$ of the ML star and because $A(<\beta \gamma)$ increases with ZAMS mass, the ratio of luminosities in (3) is always greater than unity.

The maximum MLR which a ZAMS star can have if it is to evolve to brighter luminosities can be derived from the M-L law to be

$$
\begin{equation*}
\dot{M}(0)=-M(0) /\left(3.2 t_{n}\right), \tag{4}
\end{equation*}
$$

where $t_{n}$ is the nuclear timescale of the core.
The most striking post-MS features of the tracks shown in Fig. 1 are the "zig-zag" of track T257 and the reversal of track T507 back to the blue scale of the HR diagram before the star reached the giant branch. The latter track is quite interesting in that the star evolving along it spent $25 \%$ of its core He burning 1 ifetime at $\log \mathrm{T}_{\mathrm{e}}>4.6$, had a 1 uminosity characteristic of a $30 M_{\mathcal{Q}}$ star but ended this burning phase with a mass of about $10 \mathrm{M}_{\mathscr{O}}$. The low $\mathrm{H} / \mathrm{He}$ and high $\mathrm{N} / \mathrm{C}$ ratios at the surface suggest comparison to a WN star. Interestingly, the final age of this star is almost the same as that of the star which evolved without mass loss. Both the "zig-zag" of T257 and the reversal of T507 have been shown by Falk (1979) to be the result of mass loss dominating over the effects of the nuclear shell source (the second turn around of T257 is produced when gravitational contraction of the He depleted core is significant). The interplay of nuclear burning and mass loss during the core He burning stage was demonstrated by various experiments on the evolving models as well as by a theoretical analysis. If the density in the envelope decreases monotonically outwards from the value at the shell source, then one can derive from the conservation of mass that the stellar radius will depend on $M, M_{S}, R, R_{S}$ and $\rho_{S}$ where the subscript refers to the values at the shell source. One can also derive that $R \vee M \eta\left(\eta\right.$ is a constant) $\left(\right.$ Stein 1966) and that $\left(d \log \rho_{s} / d t\right)=-\zeta\left(d \log M_{S} / d t\right)$ (Falk 1979) where $\zeta$ is a constant. Since the luminosity is almost constant during the post-MS evolution the star will evolve back to the blue (dR/dt<0) if

$$
\begin{equation*}
\mathrm{t}_{\mathrm{ML}} \quad \frac{\mathrm{t}_{\mathrm{S}}}{\left[\zeta-(\zeta+1) \mathrm{q}_{\mathrm{S}}\right.} \tag{5}
\end{equation*}
$$

where $t_{M L}=-M /(d M / d t)$ and $\left.\left.t_{s}=-M_{S} / \mathrm{dM}_{S} / \mathrm{dt}\right)=-X_{S} / \mathrm{dX}_{\mathrm{s}} / \mathrm{dt}\right)$ are, respectively, the mass loss and the shell timescales and $\mathrm{q}_{\mathrm{S}}=\mathrm{M}_{\mathrm{S}} / \mathrm{M}$. ${ }^{\text {. }}$ This formula predicts that if there is no mass loss ( $\mathrm{t}_{\mathrm{ML}}+\infty$ ) then blueward evolution should occur whenever $q_{s}>.55$, for a $30 M_{0}$ star with $\zeta=1.2$. This value agrees very we11 with the $\mathrm{q}_{\mathrm{S}}=0.6$ found by Gianonne (1967) and Höppner and Wright (1973). A detailed discussion of the MS phase of evolution with mass loss is presented in Falk and Mitalas (1981a) and the post-MS phase is covered in Falk and Mitalas (1981b).


Fig.1. Theoretical HR diagram for a $30 \mathrm{M}_{\mathrm{O}}$ star evolving with initial mass loss rate of . (0.0, 1.0, 2.5, 5.0,10.0) $\times 10^{-7} \mathrm{M}_{0} / \mathrm{yr}$ for tracks T00, T107, T257, T507, T106 respectively.

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