NUMERICAL FILTERING OF REFRACTION COEFFICIENTS

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ABSTRACT

The fluctuations of refraction coefficients can be described by a stochastic process: a two-component process. One component of the process is caused by short-periodic (daily) climatic variations. The other component is influenced by long-periodic (yearly) climatic variations. The central moments of these two components are used to estimate the covariance matrix of time series of refraction coefficients. Different functional and stochastic models are tested in connexion with time averaging of refraction coefficients.

1. INTRODUCTION

For long distances the accuracy of vertical angle measurements depends mainly on the estimation of the parameters of the meteorological field along the ray path. On the basis of atmospheric physics equations can be found which describe the relations between the observations, the model errors and the systematic parameters. In general these equations are named "functional models". The functional models are one supposition for estimating systematic parameters by least-squares adjustment procedures.

Assumptions about the accuracy, the stochastic independence and the correlation of observations are called "stochastic models". The stochastic models are the second supposition for estimating parameters by least-squares adjustment procedures.

The loss of information in connexion with adjustment procedures will be kept small if the functional and stochastic models are estimated as exactly as possible. This paper deals with tests analysing the influence of different functional and stochastic models on the results of adjustment procedures. Though the following tests are based on time series of refraction coefficients the result will be typical for all adjustment procedures using observations influenced by the atmosphere.

E. Tengström and G. Teleki (eds.), Refractional Influences in Astrometry and Geodesy, 213–225.

2. FUNCTIONAL AND STOCHASTIC MODELS OF TIME SERIES OF REFRACTION COEFFICIENTS

Assumptions concerning the functional and stochastic models of observations can be found best, when a great number of measurements is available.Normally it is difficult to estimate the models directly from geodetic measurements, since for economic reasons only a small number of repeated measurements can be performed. In this paper only those fluctuations of vertical angle measurements will be considered, which are caused by variations of the meteorological field. It is shown (KAHMEN 1977) that these fluctuations can be estimated without direct geodetic measurements using only records of the meteorological field. Long-time records of meteorological parameters are available at many meteorological stations. With these records the physical causes of the fluctuations of the vertical angles can first be found. If the physical causes are well known the fluctuations of the vertical angles can easily be calculated by simple linear transformations. Causes of the fluctuations of vertical angles are the variations of the refraction coefficients.

The figures 2.1 and 2.2 show examples of such time series +). In figure 2.1 we see mean values for one hour of refraction coefficients, plotted daily from January 1962 to December 1963 using observations recorded between 12 a.m. and 1 p.m. . Figure 2.2 shows mean values for one hour of refraction coefficients, plotted every fifth day from January 1962 to April 1971 using observations recorded between 12 a.m. and 1 p.m. .

The figures 2.1 and 2.2 show that the fluctuations of the refraction coefficients can be described by a stochastic process.

+) Further time series have been calculated and will be published.

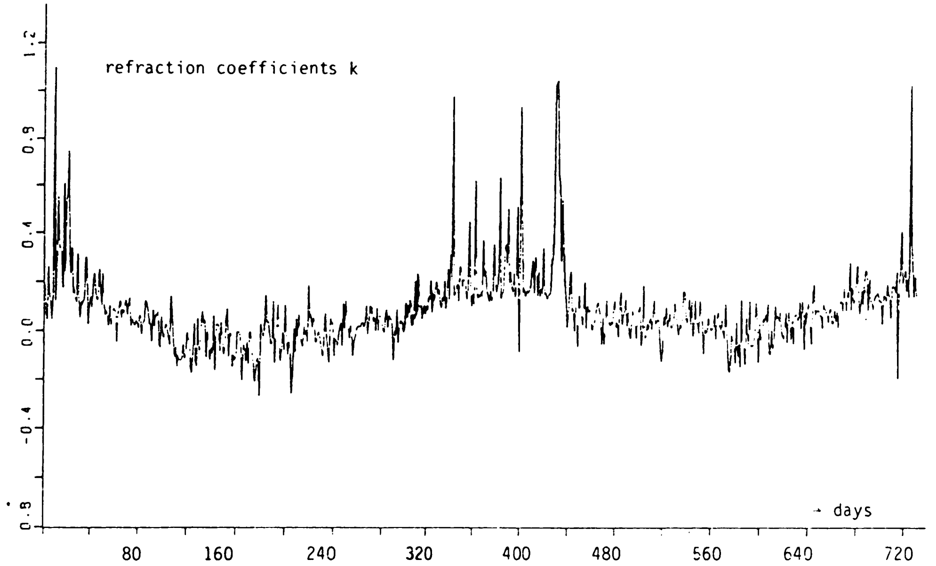

Figure 2.1 Refraction coefficients calculated from 1-1-1962 to 31-12-1963 (time-difference between the single values: 1 day)

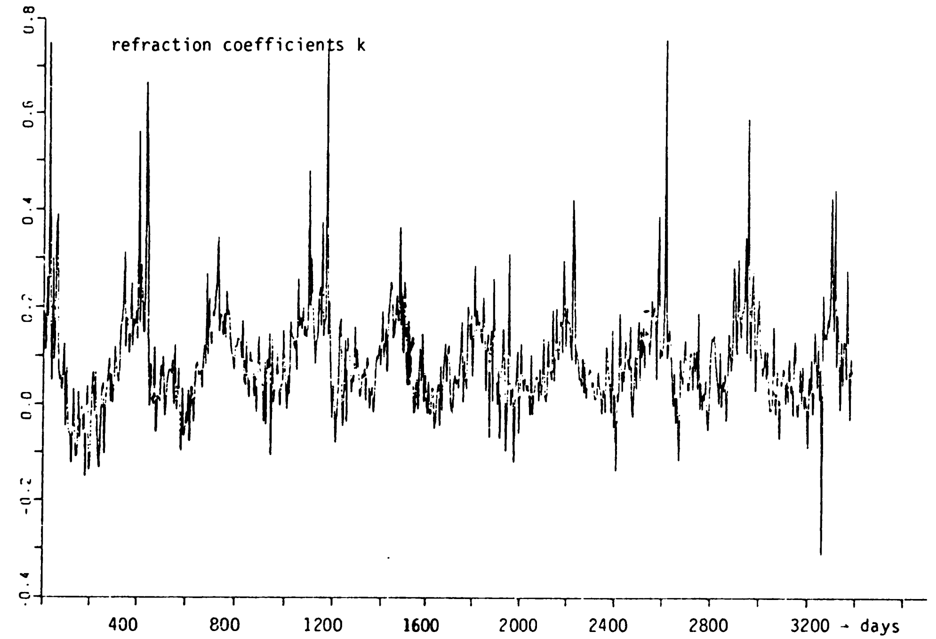

Figure 2.2 Refraction coefficients calculated from 1-1-1962 to 27-4-1971 (time-difference betwen the single values: 5 days)

Figure 2.3 indicates the structure of such a linear stochastic process.

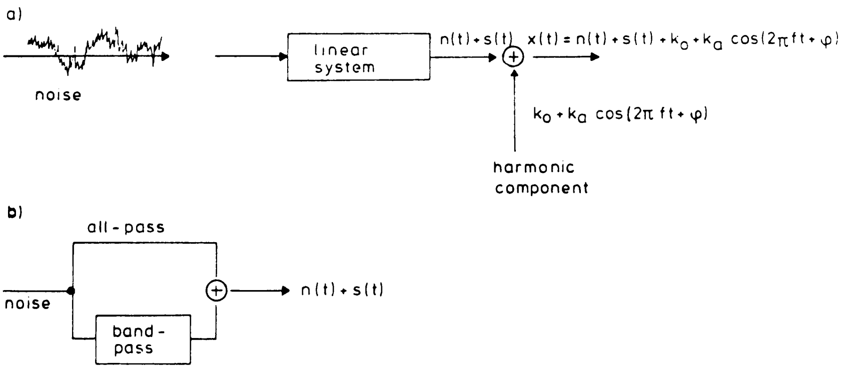

FIGURE 2.3 a) structure of a linear stochastic process
b) linear system

The linear system is composed of the parallel connection of an all-pass and a band-pass. Consequently the stochastic process, which is superimposed by a trend, can be described by the linear model

$$\mathbf{x}(\mathbf{t}) = \mathbf{n}(\mathbf{t}) + \mathbf{s}(\mathbf{t}) + \mathbf{k}_0 + k_a \cos\,(2\pi f\mathbf{t} + \boldsymbol{\varphi}) \qquad (2.1)$$

where

$\mathbf{x}(\mathbf{t}) = [x_1, x_2 \ldots]$ and $\mathbf{t} = [t_1, t_2 \ldots]$

$\mathbf{n}(\mathbf{t}) = [n_1, n_2 \ldots]$

$\mathbf{s}(\mathbf{t}) = [s_1, s_2 \ldots]$

$\mathbf{k}_0 \doteq [k_0, k_0 \ldots]$

$\boldsymbol{\varphi} = [\varphi, \varphi \ldots]$

$\mathbf{n}(\mathbf{t})$ and $\mathbf{s}(\mathbf{t})$ describe the non deterministic part of the process and $\mathbf{k}_0 + k_a \cos\,(2\pi f\mathbf{t} + \boldsymbol{\varphi})$ describes the deterministic part of the process.

$\mathbf{n}(\mathbf{t})$ is a short period random process and $\mathbf{s}(\mathbf{t})$ a long period random process. Figure 2.4 for example shows how one signal $x(\mathbf{t})$ is built by additive superposition of the functions $n(\mathbf{t})$, $s(\mathbf{t})$, $k_0 + k_a \cos\,(2\pi f\mathbf{t} + \boldsymbol{\varphi})$.

https://doi.org/10.1017/S007418090006602X Published online by Cambridge University Press

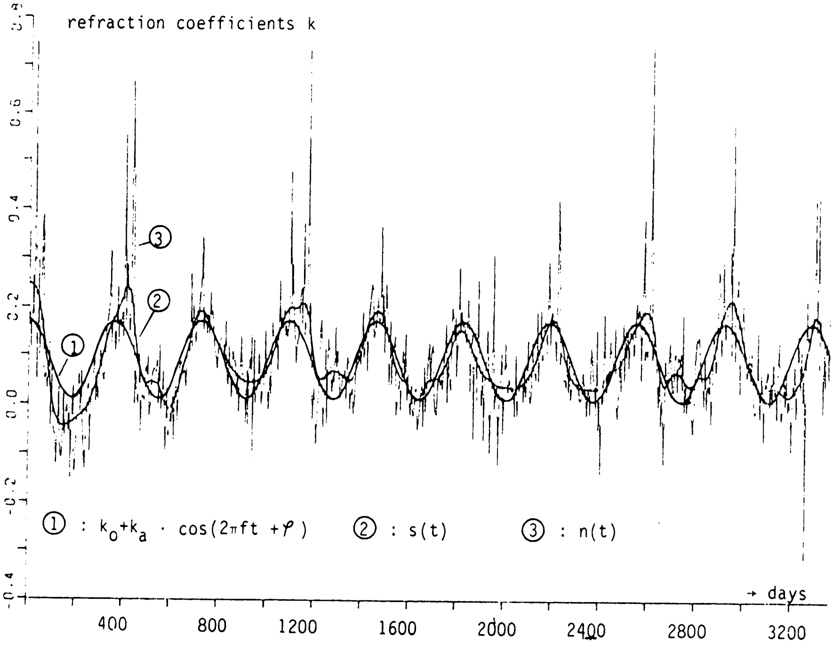

FIGURE 2.4 Superposition of the signals n(**t**), s(**t**), $k_o + k_a \cos(2\pi f\mathbf{t} + \varphi)$

For one signal $n(\mathbf{t}) = \mathbf{n}$ we can assume (KAHMEN 1977)

$$E\{\mathbf{n}\} = o \tag{2.2}$$

$$E\{\mathbf{n}\mathbf{n}^T\} = \sigma_n^2\,\delta(\tau) \tag{2.3}$$

(σ: standard deviation)

where

$$\delta(\tau) = \begin{cases} 1 & \text{for } \tau = 0 \\ 0 & \text{for } \tau \neq 0 \end{cases} . \tag{2.4}$$

τ describes the time-difference between single measurements.

$E\{\cdot\}$ is the statistical expectation.

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σ_n^2 has different values during the several periods of a year.

One signal $s(\mathbf{t}) \equiv \mathbf{s}$ can be approximated by a Gaussian narrow-band noise (KAHMEN 1977)

$$s(t) = s_o(t) \cos [\omega_o t + \Psi (t)] \tag{2.5}$$

where the probability density of $s_o(t)$ and $\Psi(t)$ is

$$p (s_o) = \frac{s_o}{\sigma^2} \exp (- \frac{s_o^2}{2\sigma^2}), \text{ (Raleigh-Distribution)} \tag{2.6}$$

$$p (\Psi) = \frac{1}{2\pi} [\delta_{\Gamma} (\Psi) - \delta_{\Gamma} (\Psi -2\pi)] \tag{2.7}$$

(δ_{Γ} = normed step-function).

Consequently we can assume:

$$E \{\mathbf{s}\} = o \tag{2.8}$$

$$E \{\mathbf{s}\mathbf{s}^T\} = \sigma_s^2 \exp (- a_o^2 \tau^2) \cos \omega_o \tau = r(\tau) \tag{2.9}$$

where τ describes the time-difference between single measurements and a_o is a constant.

3. ADJUSTMENT OF TIME SERIES OF REFRACTION COEFFICIENTS OR REFRACTION ANGLES

The basic equations for the coefficients k of refraction and the angles δ of refraction are (FEARNLEY 1884/85):

$$k = \frac{2}{S} \int_0^S \frac{S-S'}{S} \chi(S') \, dS' \tag{3.1}$$

$$\delta = \frac{\rho}{R} \int_0^S \frac{S-S'}{S} \chi(S') \, dS' \tag{3.2}$$

where S is the length of the curved path (the difference between the arc-length and the chord-length is neglected, S' describes the coordinates along the light path, R is the radius of the earth and

$\chi = \chi\ (S')$ is the local refraction coefficient. From (3.1) and (3.2) we get:

FIGURE 3.1 Angle of refraction

$$\delta = \frac{S\rho}{2R} \cdot k . \tag{3.3}$$

Approximate refraction free directions can be calculated from time series of vertical angles, if the systematic components of δ are eliminated by subtraction and if the stochastic components of δ are filtered by time averaging procedures. In the following the calculation of refraction free directions will be analysed with different functional and stochastic models. Equ. (3.3) shows that the tests can directly be made with the time series of refraction coefficients shown in figure 2.1 and 2.2 as there is a linear relation between δ and k.

With equ. (2.1) the general model for the vector of the observations $\mathbf{x} \equiv x\ (\mathbf{t})$ is (Wolf 1977)

$$\mathbf{x} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{s} + \mathbf{n} \tag{3.4}$$

where

$\hat{\mathbf{x}}$ ... vector of systematic parameters

$\mathbf{A}$ ... matrix relating $\mathbf{x}$ and $\hat{\mathbf{x}}$

$\mathbf{B}$ ... matrix relating $\mathbf{x}$ and $\mathbf{s}$.

The matrix $\mathbf{C}_{ss}$ = cov ($\mathbf{s}$) and the matrix $\mathbf{C}_{nn}$ = cov ($\mathbf{n}$) are given a priori as the vector $\mathbf{n}$ is chracterized by equ. (2.2), (2.3) and the vector $\mathbf{s}$ by equ. (2.8), (2.9). $\mathbf{C}_{ss}$ and $\mathbf{C}_{nn}$ have the form :

$$\mathbf{C}_{ss} = \begin{vmatrix} \sigma_s^2 & r_{12}\,\sigma_s^2 & \cdots & r_{1u}\,\sigma_s^2 \\ r_{12}\,\sigma_s^2 & \sigma_s^2 & \cdots & r_{2u}\,\sigma_s^2 \\ \vdots & \vdots & \ddots & \vdots \\ r_{u1}\,\sigma_s^2 & r_{u2}\,\sigma_s^2 & \cdots & \sigma_s^2 \end{vmatrix} \tag{3.5}$$

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$$\mathbf{C}_{nn} = \begin{vmatrix} \sigma^2_{n1} & 0 & \dots & 0 \\ 0 & \sigma_{n_2}^{} & \dots & 0 \\ \vdots & \vdots & & \\ 0 & 0 & \dots & \sigma^2_{n_u} \end{vmatrix} \tag{3.6}$$

For the following calculations we get $\mathbf{B} = \mathbf{I}$ ($\mathbf{I}$: unit matrix) so that we have instead of (3.4) the classical model of collocation

$$\mathbf{x} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{s} + \mathbf{n} \,. \tag{3.7}$$

With the side condition

$$\mathbf{s}^T \mathbf{C}_{cc}\, \mathbf{s} + \mathbf{n}^T \mathbf{C}_{nn}\, \mathbf{n} = \text{minimum} \tag{3.8}$$

and condition adjustment with unknowns we get

$$\hat{\mathbf{x}} = (\mathbf{A}^T \bar{\mathbf{C}}_0^{-1} \mathbf{A})^{-1} \mathbf{A}^T \bar{\mathbf{C}}_0^{-1} \mathbf{x} \tag{3.9}$$

where

$$\bar{\mathbf{C}}_0 = \mathbf{C}_{nn} + \mathbf{C}_{ss} \,. \tag{3.10}$$

4. NUMERICAL CALCULATIONS

The observations x are taken from the time series shown in figure 2.1. The estimations of the functional and stochastic models are also partly based on the time series shown in figure 2.1 and 2.2. In order to keep the discussion of the results clean only one parameter will be estimated: the time average of the refraction coefficients. Then the mean square error of the time average is (Wolf 1975):

$$m_{\hat{x}} = m_0 \sqrt{Q_{\hat{x}\hat{x}}} \tag{3.11}$$

$Q_{\hat{x}\hat{x}}$ is a diagonal element of the matrix $(\mathbf{A}^T \bar{\mathbf{C}}_0^{-1} \mathbf{A})^{-1}$ and

m_0 the mean square error of unit weight of the observations.

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The adjustment procedures are based on the following models:

model I (generally used for practical work)

a) $k = 0.13$

b) all observations have equal σ^2

c) the observations are uncorrelated

model II (assumptions based on the time series shown in figure 2.1 and 2.2)

a) $k = k_o + k_a \cos(2\pi ft)$
where $k_o = 0.09$, $k_a = 0.08$, $f = 1$ oscillation/year,
$t = 0$ at the beginning of January.

b) $\sigma_n = 0.16$ from November to February
$\sigma_n = 0.08$ from March to April
$\sigma_s = 0.07$ over the whole year

c) the observations are uncorrelated

model III

a) as in model II

b) as in model II

c) $r = 0.9$ for a time-difference τ up to 1 month
$r = 0$ " " " of 3 months
$r = 0.8$ " " " of 6 months
$r = 0$ " " " of 9 months
$r = 0.7$ " " " of 12 months

model IV

a) as in model II

b) as in model II

c) $r = 0.9$ for a time-difference τ up to 1 month
$r = 0$ " " " of 3 months
$r = 0.4$ " " " of 6 months
$r = 0$ " " " of 9 months
$r = 0.3$ " " " of 12 months

The results of the adjustment procedures are shown in Table 1 and Table 2. For all time-series the time averages $\hat{x}$ (the mean systematic errors of the refraction free directions) and the mean-square errors $m_{\hat{x}}$ become maximum, when model I is used. Comparing the $\hat{x}$ and $m_{\hat{x}}$ of the four different adjustments one can see that the values of $m_{\hat{x}}$ differ much more than those of $\hat{x}$. Analysing the $\hat{x}$ and $m_{\hat{x}}$ when the model II,

TABLE 1

time-series	date	k_i	date	k_i	date	k_i	date	k_i	date	k_i	number of the first... measurements	model I $\hat{x}$	model I $m_{\hat{x}}$	model II $\hat{x}$	model II $m_{\hat{x}}$	model III $\hat{x}$	model III $m_{\hat{x}}$	model IV $\hat{x}$	model IV $m_{\hat{x}}$
1	5-1-62	0.11	5-4-62	0.08	5-7-62	-0.02	5-10-62	-0.06	5-1-63	0.25	7	0.04	0.39	0.03	0.12	0.02	0.16	0.02	0.15
	6-1-62	0.05	6-4-62	-0.06	6-7-62	-0.06	6-10-62	0.11	6-1-63	0.16	9	-0.02	0.36	0.00	0.09	-0.01	0.12	-0.02	0.13
	7-1-62	0.15							7-1-63	0.14	11	-0.04	0.33	-0.01	0.07	-0.01	0.10	-0.01	0.11
	8-1-62	1.10							8-1-63	0.12	16	-0.03	0.27	-0.01	0.03	-0.01	0.08	-0.01	0.08
	9-1-62	0.12							9-1-63	0.17									
2	10-1-62	0.26	10-4-62	-0.03	10-7-62	-0.03	10-10-62	0.03	10-1-63	0.12	7	0.07	0.20	0.05	0.06	0.05	0.07	0.05	0.07
	11-1-62	0.55	11-4-62	0.02	11-7-62	0.03	11-10-62	-0.03	11-1-63	0.14	9	0.05	0.20	0.03	0.05	0.01	0.06	0.00	0.06
	12-1-62	0.34							12-1-63	0.35	11	0.01	0.20	0.00	0.04	0.00	0.05	0.00	0.05
	13-1-62	0.32							13-1-63	0.21	16	0.01	0.17	0.01	0.03	0.00	0.04	0.00	0.04
	14-1-62	0.31							14-1-63	0.16									
3	15-1-62	0.17	15-4-62	-0.06	15-7-62	-0.07	15-10-62	-0.04	15-1-63	0.18	7	0.13	0.25	0.11	0.08	0.10	0.10	0.10	0.10
	16-1-62	0.61	16-4-62	0.15	16-7-62	0.00	16-10-62	-0.02	16-1-63	0.25	9	0.06	0.27	0.06	0.06	0.03	0.09	0.03	0.09
	17-1-62	0.23							17-1-63	0.64	11	0.01	0.26	0.02	0.05	0.02	0.07	0.02	0.08
	18-1-62	0.54							18-1-63	0.13	16	0.03	0.24	0.03	0.05	0.02	0.07	0.02	0.08
	19-1-62	0.55							19-1-63	0.16									
4	20-1-62	0.75	20-4-62	-0.06	20-7-62	-0.10	20-10-62	0.02	20-1-63	0.15	7	0.04	0.29	0.01	0.09	0.00	0.11	0.00	0.11
	21-1-62	0.30	21-4-62	-0.15	21-7-62	-0.04	21-10-62	0.03	21-1-63	0.34	9	-0.02	0.29	-0.02	0.06	-0.05	0.09	-0.05	0.09
	22-1-62	0.23							22-1-63	0.35	11	-0.05	0.26	0.03	0.05	-0.04	0.07	-0.04	0.08
	23-1-62	0.34							23-1-63	0.27	16	0.01	0.24	0.01	0.04	-0.01	0.06	-0.02	0.07
	24-1-62	0.15							**24-1-63**	**0.50**									
5	25-1-62	0.17	25-4-62	-0.12	25-7-62	0.02	25-10-62	0.11	25-1-63	0.19	7	-0.08	0.14	-0.06	0.04	-0.07	0.04	-0.07	0.04
	26-1-62	0.13	26-4-62	-0.05	26-7-62	-0.09	26-10-62	-0.02	26-1-63	0.24	9	-0.11	0.14	-0.06	0.03	-0.07	0.04	-0.07	0.04
	27-1-62	0.11							27-1-63	0.17	11	-0.12	0.13	-0.06	0.03	-0.06	0.04	-0.06	0.04
	28-1-62	0.31							28-1-63	0.16	16	-0.08	0.12	-0.04	0.02	-0.05	0.03	-0.05	0.03
	29-1-62	0.12							29-1-63	0.23									

TABLE 2

time-series	date	k_i	date	k_i	date	k_i	date	k_i	date	k_i	number of the first ...measurements	model I $\hat{x}$	model I $m_{\hat{x}}$	model II $\hat{x}$	model II $m_{\hat{x}}$	model III $\hat{x}$	model III $m_{\hat{x}}$	model IV $\hat{x}$	model IV $m_{\hat{x}}$
6	1-2-63	0.15	1-5-63	-0.07	1-8-63	0.01	1-10-63	0.12	1-12-63	0.12	7	0.16	0.38	0.06	0.12	0.04	0.14	0.04	0.14
	2-2-63	0.29	2-5-63	-0.12	2-8-63	-0.08	2-10-63	0.08	2-12-63	0.02	9	0.09	0.36	0.02	0.09	-0.01	0.11	-0.02	0.12
	3-2-63	0.29							3-12-63	0.14	11	0.07	0.33	0.02	0.07	0.01	0.09	0.01	0.10
	4-2-63	0.46							4-12-63	0.14	16	0.04	0.27	0.01	0.05	0.00	0.07	-0.01	0.08
	5-2-63	1.02							5-12-63	0.12									
7	6-2-63	1.03	6-5-63	0.11	6-8-63	0.06	6-10-63	0.08	6-12-63	0.19	7	0.41	0.40	0.31	0.13	0.29	0.15	0.29	0.15
	7-2-63	1.05	7-5-63	0.11	7-8-63	0.03	7-10-63	0.19	7-12-63	0.14	9	0.30	0.41	0.21	0.10	0.18	0.13	0.16	0.15
	8-2-63	0.64							8-12-63	0.13	11	0.25	0.39	0.16	0.08	0.15	0.11	0.14	0.13
	9-2-63	0.59							9-12-63	0.05	16	0.17	0.34	0.13	0.06	0.12	0.10	0.12	0.11
	10-2-63	0.28							10-12-63	0.18									
8	11-2-63	0.54	11-5-63	0.02	11-8-63	-0.03	11-10-63	0.26	11-12-63	0.17	7	0.05	0.20	0.03	0.05	0.03	0.07	0.03	0.07
	12-2-63	0.32	12-5-63	0.03	12-8-63	0.03	12-10-63	0.14	12-12-63	0.17	9	0.01	0.19	0.01	0.04	0.01	0.06	0.00	0.06
	13-2-63	0.19							13-12-63	0.15	11	0.02	0.17	0.03	0.03	0.03	0.05	0.03	0.06
	14-2-63	0.12							14-12-63	0.24	16	0.03	0.14	0.03	0.02	0.03	0.04	0.03	0.04
	15-2-63	0.01							15-12-63	0.19									
9	16-2-63	0.08	16-5-63	-0.02	16-8-63	0.04	16-10-63	0.10	16-12-63	0.22	7	-0.03	0.09	-0.02	0.02	-0.02	0.03	-0.02	0.03
	17-2-63	0.20	17-5-63	0.02	17-8-63	-0.03	17-10-63	0.21	17-12-63	0.23	9	-0.05	0.09	-0.02	0.02	-0.02	0.02	-0.03	0.03
	18-2-63	0.24							18-12-63	0.40	11	-0.04	0.09	-0.01	0.02	-0.01	0.03	-0.01	0.03
	19-2-63	0.08							19-12-63	0.19	16	0.01	0.12	0.01	0.02	0.01	0.03	0.01	0.03
	20-2-63	0.09							20-12-63	0.21									
10	21-2-63	0.00	21-5-63	0.16	21-8-63	0.07	21-10-63	0.21	21-12-63	0.27	7	-0.06	0.08	-0.01	0.04	0.00	0.05	0.00	0.05
	22-2-63	0.10	22-5-63	0.05	22-8-63	0.04	22-10-63	0.12	22-12-63	0.24	9	-0.07	0.07	0.00	0.03	0.01	0.04	0.02	0.04
	23-2-63	0.06							23-12-63	0.53	11	-0.05	0.08	0.01	0.02	0.01	0.03	0.02	0.04
	24-2-63	-0.06							24-12-63	0.21	16	0.02	0.14	0.03	0.02	0.03	0.04	0.03	0.04
	25-2-63	0.15							25-12-63	0.19									

model III and model IV are used, we can notice that there is only a small difference between the $\hat{x}$ while there is a greater difference between the $m_{\hat{x}}$. Sometimes the $m_{\hat{x}}$ calculated with model IV are three times greater than the $m_{\hat{x}}$ calculated with model II. This difference increases with the number of the observations. There is no significant difference between the results of the adjustment based on model III and model IV. Consequently only a rough appraximation of the factor $\exp(-a_0{}^2\tau^2)$ of (2.9) is needed. The values of $m_{\hat{x}}$ calculated with model III and IV are nearly always larger than the values of $\hat{x}$. We will not find that as often when considering the values calculated with model II. Therefore the results we get with model III and IV appear more realistic.

5. OPTIMUM ARRANGEMENT FOR THE TIME OF MEASUREMENTS

The vector

$$\mathbf{z} = (\mathbf{A}^T \bar{\mathbf{C}}_0^{-1} \mathbf{A})^{-1} \mathbf{A}^T \bar{\mathbf{C}}_0^{-1}$$

of equ. (3.9) can be used to find an optimum arrangement of the time of the measurements, as its components characterize, how much the single measurements affect the time average. Table 3 for example shows the normed vector $\mathbf{z}_0$ for the first 7, 9, 11 and 16 measurements of the time series 1 ... 5 of table 1, when model III is used.

TABLE 3

number of the observations	$\mathbf{z}_{0_7}$	$\mathbf{z}_{0_9}$	$\mathbf{z}_{0_{11}}$	$\mathbf{z}_{0_{16}}$
1	0.090	0.050	0.041	0.033
2	0.090	0.050	0.041	0.033
3	0.090	0.050	0.041	0.033
4	0.090	0.050	0.041	0.033
5	0.090	0.050	0.041	0.033
6	0.275	0.207	0.129	0.117
7	0.275	0.207	0.129	0.117
8		0.167	0.138	0.102
9		0.167	0.138	0.102
10			0.129	0.117
11			0.129	0.117
12				0.033
13				0.033
14				0.033
15				0.033
16				0.033

For example the first column $\mathbf{z}_{07}$ shows that the first five Observations are less significant in the final result than the last two observations.

6. CONCLUSIONS

The foregoing considerations are model studies. As the results of the adjustment procedures are not derived form direct geodetic measurements they cannot generally be compared with those calculated with direct measurements. The model studies however can help us to estimate suitable functional and stochastic models for adjustment procedures, to interpret the results and finally to find an optimum arrangement for the time of the measurements.

LITERATURE:

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DISCUSSION

L. Hradilek: Can your method be used for a smaller number of observations, let us say 5 repetitions of vertical angle measurements within one day?

H. Kahmen: Yes, the arrangement of the measurements is included in my model. I have taken this measurement together to an alternative time equation. The recordings of the meteorological parameters were done between 12 a.m. and 1 p.m. and there were recordings every ten minutes. I time-averaged these values and the single refraction coefficients sought for were time-averages of one hour. So I think they include several measurements during one day.

B. Garfinkel: Can you give us a definition of the coefficient of refraction?