STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES. I. THE RELATIONSHIP BETWEEN "CORRECTED" DEATH RATES AND LIFE TABLE DEATH RATES.

BY JOHN BROWNLEE, M.D., D.Sc.

(With 2 Diagrams.)

CIRCUMSTANCES in connection with the recent census have again directed my attention to the laws which govern human life. I have long been of the opinion that the old ideas that birth rates and death rates had no biological relationship beyond the obvious ones, that many infants mean more deaths, etc., usually found stated in public health text books, were based on a very imperfect induction. On one aspect of this I published a paper a number of years ago in the Transactions of the Royal Philosophical Society of Glasgow (1), and last summer Sir Shirley Murphy (2) read a paper on another aspect of the same subject before the Sanitary Congress held at York. But I have hitherto refrained from publishing theories because I believe that until quantitative measures are applied no scientific results worthy of discussion can be obtained. Now that such seem to be possible, I propose to discuss in a series of papers the different relationships which I have investigated. No mathematics will be introduced in the earlier papers, but the results of all will be summarised and dealt with in their mathematical and physico-chemical relationships in a concluding communication. The first paper relates to the connection between the "corrected" death rates and the "true" death rates as found by constructing a life table.

Many conclusions in public health work depend on the use of death rates as a means of comparing the state of health in different districts. The death rate commonly used is termed the "crude" death rate, and is obtained by dividing the total number of deaths in a district in one year

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

by the total number of inhabitants. The result is given in parts per thousand. This criterion has long been known to be of doubtful worth. The mortality varies with age and sex, and even in adjacent districts the distribution of persons according to these categories is so different as to preclude the comparison. Thus towns contain a very large number of young adults attracted by the opportunity of obtaining work, and as these constitute the healthiest part of the community the "crude" death rate is correspondingly reduced. To meet this, the refinement of the "corrected" death rate has been introduced. The mortalities for each sex at each age period having been ascertained, these are applied severally to the different age and sex groups of the population in the whole country (termed in this connection a "standard population"). This gives the figure which would be found if the mortalities in the district could be assumed to hold for the whole country. Such figures obviously admit of more certain comparison among themselves than those obtained by the older method. The drawback to this method is evidently the fact that the standard population is not a stationary but an increasing population, in which there is, more or less consistently, a smaller number of persons living as age is approached than would be present in a population in which the death rate equals the birth rate. More infants exist, it is true, than in a stationary population, but the period of the high mortality in childhood is short, not more than five years, and the next few quinquennia have a very low mortality, while at the later ages where the mortality is high there are relatively fewer persons living. Thus a "corrected" death rate is not a real death rate. It may be fictitiously low. This point is very important since many conclusions are daily being drawn from such figures. We hear of garden cities with mortalities of 7 per 1000, etc., though even 12 per 1000 is a death rate to be interpreted only with knowledge and discrimination. Neither, I fancy, can under the best conceivable conditions have any real meaning. This is obvious when we consider that in a stationary population the average age of the population in years at death or the expectation at birth is obtained by dividing the number of persons living by the number of deaths per annum. Thus if the population be 1000 and the annual number of deaths 20, the average age at death is 50 years, a possible result. Twelve per 1000 means the average age of 83 years, 7 per 1000 an average age of 143 years, both sufficiently ridiculous. The usual way to attain the truth is to construct a life table, but that is a process requiring both labour and mathematical skill. Were this the only solution it would be well-nigh impracticable

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

to press its use, but a considerable number of life tables have now been calculated, and by the use of these the true death rate may easily be estimated if the "corrected" death rate be known.

The life tables utilised in the calculations made for this paper are given in the following table. For convenience each is hereafter referred to by the letter placed opposite its description.

TABLE I.

Life Table of England (Farr) 1838–1851	E_1
,, ,, ,, (Ogle) 1871–1881	E_2
,, ,, ,, (Tatham) 1881–1891	E_3
,, ,, ,, (Tatham) 1891–1901	E_4
Healthy District Life Table of England (Farr) 1849–1851 ...	F
,, ,, ,, ,, ,, (Tatham) 1881–1890	H_1
,, ,, ,, ,, ,, (Tatham) 1891–1901	H_2
Brighton Life Table (Newsholme) 1881–1890	B
Manchester Life Table (Tatham) 1881–1890	M
London Life Table (Murphy) 1891–1900	L
Scottish Life Table (Adam) 1891–1900	S
Glasgow Life Table (Chalmers) 1891–1900	G

The number of tables is twelve. The first eleven give in all respects absolutely concordant results. The last, that for Glasgow, shows some differences, due I think to the fact that all the deaths at high ages belonging to Glasgow are not included. At the period for which it was constructed there was no mechanism by which deaths occurring in institutions outside Glasgow could be returned to the city, and the number of institutions outside Glasgow was very considerable. This criticism is borne out by the results of the recent census, and by a comparison with the death rates in Glasgow at the present time.

A life table as usually understood may be defined as the numerical construction of a stable population which possesses the same mortalities at each age as those in the population to be examined. By this means irregularities in the proportions of persons of different ages and sexes, due to varying birth rates and to immigration and emigration, are eliminated. The death rate of any population either "crude" or "corrected" will not be that of the life table. Generally it will be below the latter. Exceptionally, if the death rate is sufficiently high as to annul the natural increase and bring about a stationary or a declining population the "crude" or "corrected" death rate is found to be equal or greater than that obtained when a life table has been constructed.

Two examples will illustrate this. The corrected death rate of the males in the healthy district life table H_2 is found to be 13·49. The actual expectation of life is 52·87 years, giving a death rate of 18·91, both quite conceivable figures. A death rate of 13·49 gives, however, on a stationary population a mean life of $\frac{1000}{30 \cdot 49}$ or 74 years, a figure quite impossible. Manchester on the other hand has a corrected death rate of 28 per 1000. The expectation of life is 34·71 years, giving a death rate of 28·81 on a stationary population. In other words, we do not really have variations of the death rate from 13·5 to 28, but from 18·9 to 28·8, a difference much more easily understood.

For certain reasons, which will be discussed in a later paper, I think that the very highest mean age possible is about sixty years, and this represents a real death rate of 16·3.

How then is this true or life table death rate to be obtained? The method, which partly depends on the biological response of mankind to unhealthy conditions and is partly a pure arithmetical necessity, is expressed in the statement that the relationship between the true death rate and the corrected death rate is linear; that is, given the latter, the former is obtained by multiplying by a constant fraction and adding a definite constant. Thus if D_2 be the real death rate for the whole population of a district and D_1 the "corrected" death rate,

$$D_2 = \cdot 6842 D_1 + 9 \cdot 65.$$

D_2 is thus equal to D_1 when both are equal to 30·5. For diagrammatic purposes the difference between the true and the corrected death rates is the better figure to choose, in which case the above formula may be written

$$D_2 - D_1 = - \cdot 3158 D_1 + 9 \cdot 65.$$

But the theorem is yet more general. It is not necessary to begin at birth, any age is equally appropriate. To obtain the expectations of life at each period, all that it is necessary to know is the "corrected" death rate for all persons above that age.

These are severally calculated in exactly the same manner as the "corrected" death rate itself is calculated. The multiplications are the same, the only difference being that the sum is made by stages. Thus the two products at 75— and 65—75 are added together, then the product at 55—65 to the latter sum, and so on, so that we have a series of sums each to be divided by corresponding numbers obtained by summing the standard population in the same way. Corresponding to this series of death rates we have from the life tables a similar series of real death rate figures obtained by dividing 1000 by the expectation of life at the corresponding ages. At each age the relationship between the two series of figures is linear, the general equations, which are equivalent, being

$$D_2 - D_1 = -m\ D_1 + C$$

and

$$D_2 = (1 - m)D_1 + C.$$

The series of values of m and C from 0 to 55 years is given for both sexes in the accompanying table (Table II).

TABLE II.

	Males			Females		
Age	m	$1-m$	C	m	$1-m$	C
0	·3188	·6811	9·54	·3151	·6849	9·32
5	·5441	·4559	12·68	·5246	·4754	12·05
10	·5414	·4586	13·49	·5670	·4330	13·36
15	·4868	·5132	13·53	·5666	·4334	14·35
20	·5140	·4860	14·95	·5400	·4600	15·06
25	·4883	·5117	15·74	·5246	·4754	15·98
35	·4953	·5047	19·16	·4922	·5078	18·05
45	·4953	·5047	23·63	·4397	·5603	20·54
55	·3052	·6948	20·18	·3575	·6425	22·66

The data on which these figures are based are given in Table III. In parallel columns the corrected death rate D_1 the true death rate D_2 obtained from the life table, the actual difference being between the true death rate and the life table death rate, $D_2 - D_1$, and the theoretical value of the latter obtained by fitting the best straight lines, either by the method of least squares or by inspection, are given. The differences found between the real and the theoretical values are tabulated next, and the value of the square root of the mean of the squares of these differences, denoted by Δ, is added. This last figure gives the measure of the difference between the real and theoretical values. It rarely exceeds one per cent. The relationship however is best seen in diagrams. For this purpose the male death rates at 15 years (Diagram I) and at 55 years (Diagram II) have been chosen. Both illustrate well the concordance of fact and theory, and though the divergence is greater in the latter case than in the former the relationship is obviously truly linear.

When the great range of the corrected death rates is considered, *e.g.*, at birth in the case of males, from 13 per 1000 to 28 per 1000, this must be considered a very small error in a prediction of the true death rate. In fact the values obtained by the method here given may with some confidence be taken as probably more accurate than those obtained directly by constructing a life table, inasmuch as they represent the average of many life tables. The figures of one life table are at best

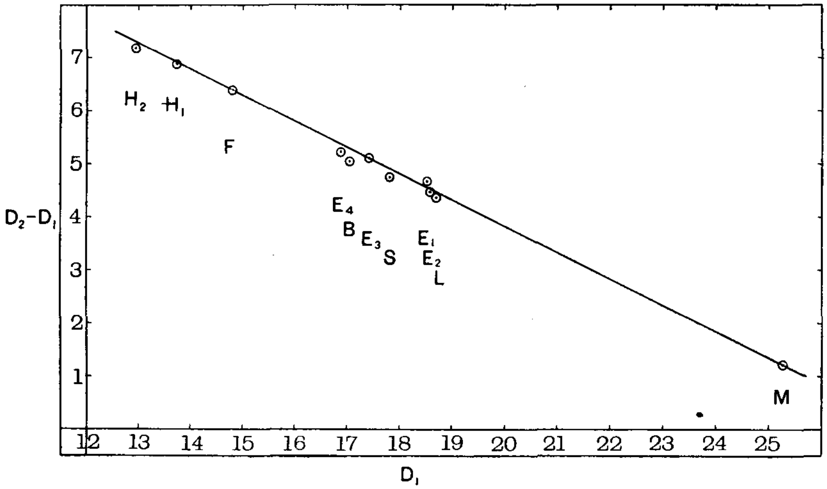

Diagram I.

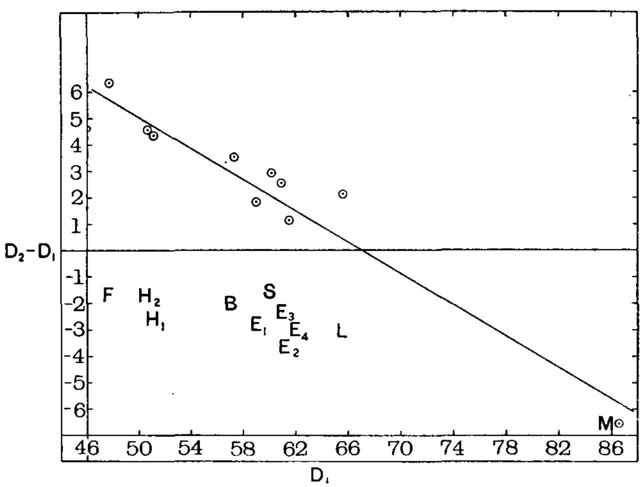

Diagram II.

but a close approximation. So much immigration at certain ages takes place from country to town, *i.e.*, from healthy to unhealthy conditions, that the average death rates at the ages of migration represent not one

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

phenomenon but a mixture of phenomena. The same is more or less true at all ages, since methods of living vary from class to class and all classes are grouped together in the one table. The error due to these factors, however, cannot be great on the whole, but may, as the above tables seem to show, be taken as probably not more than one per cent. That special populations such as Brighton should show some differences at high ages is not more than would be naturally expected, since the

TABLE III.

Showing the values of D_2 and D_1 obtained from the Life Tables and from the "standard" population and their relationships.

	Age 0. Males.					Age 0. Females.				
	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	13·49	18·91	5·42	5·29	−·13	12·49	17·95	5·46	5·38	−·08
H_1	14·26	19·42	5·16	5·05	−·11	13·40	18·50	5·10	5·10	·00
F	16·03	20·59	4·56	4·49	−·13	15·95	20·22	4·27	4·29	+·02
E_4	19·32	22·66	3·34	3·45	+·11	17·14	20·93	3·79	3·92	+·13
E_3	19·79	22·90	3·11	3·30	+·19	17·74	21·19	3·45	3·73	+·28
E_2	21·64	24·18	2·54	2·72	+·18	19·40	22·41	3·01	3·36	+·35
E_1	22·30	25·06	2·76	2·51	+·05	21·00	23·90	2·90	2·90	·00
B	19·75	22·94	3·19	3·31	+·12	16·05	20·41	4·36	4·26	−·10
S	19·21	22·36	3·15	3·58	+·43	17·31	21·06	3·75	3·88	−·13
L	21·82	24·40	2·58	2·66	+·08	18·49	22·06	3·57	3·49	−·08
M	28·00	28·81	0·81	0·91	−·10	24·46	26·02	1·56	1·64	+·08
G	26·43	28·43	2·00	1·20	−·80	24·15	26·52	2·37	1·71	−·66
Δ (excluding G)					·17					·15
	$D_2=·6811D_1+9·54$.					$D_2=·6849D_1+9·32$.				

	Age 5. Males.					Age 5. Females.				
	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	10·29	17·16	6·87	7·08	+·21	10·11	16·80	6·69	6·74	+·05
H_1	10·99	17·53	6·54	6·70	+·16	10·92	17·24	6·32	6·32	·00
F	12·21	18·39	6·18	6·04	−·14	13·18	18·54	5·36	5·13	−·23
E_4	13·41	18·69	5·28	5·38	+·10	12·50	17·92	5·42	5·49	+·07
E_3	13·98	18·96	4·98	5·07	+·09	13·29	18·21	4·92	5·07	+·15
E_2	15·14	19·66	4·52	4·44	−·08	14·32	18·84	4·52	4·53	+·01
E_1	15·58	20·12	4·54	4·20	−·34	15·94	19·87	3·93	3·68	−·25
B	13·57	18·54	4·97	5·30	+·33	11·29	17·57	6·28	6·12	−·16
S	14·25	19·10	4·85	4·93	+·08	13·38	18·51	5·13	5·12	−·01
L	14·85	19·38	4·53	4·60	+·07	12·85	18·18	5·33	5·31	−·02
M	20·22	21·93	1·71	1·68	−·03	18·43	20·81	2·38	2·38	·00
Δ					·18					·13
	$D_2=·4559D_1+12·68$.					$D_2=·4754D_1+12·05$.				

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

Age 10. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	11·37	18·46	7·09	7·33	+·24
H_1	12·07	18·84	6·77	6·96	+·19
F	13·04	19·50	6·46	6·43	+·03
E_4	14·79	20·15	5·36	5·48	+·12
E_3	15·29	20·49	5·20	5·21	+·01
E_2	16·42	21·09	4·67	4·60	−·07
E_1	16·56	21·26	4·70	4·53	−·17
B	14·90	20·09	5·19	5·42	+·23
S	15·69	20·58	4·89	5·00	+·11
L	16·38	20·90	4·52	4·62	+·10
M	22·13	23·39	1·26	1·51	+·25
Δ					·16

$D_2 = \cdot 4586 D_1 + 13\cdot 49$.

Age 10. Females.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
11·08	18·03	6·95	7·16	+·21
11·92	18·52	6·60	6·60	·00
14·04	19·65	5·61	5·40	−·21
13·64	19·24	5·60	5·62	+·02
14·42	19·57	5·15	5·18	+·03
15·46	20·10	4·64	4·59	−·05
16·93	20·97	4·04	3·76	−·28
12·25	18·81	6·56	6·41	−·15
14·53	19·85	5·32	5·12	−·20
13·92	19·42	5·50	5·46	−·04
19·98	22·01	2·03	2·03	·00
				·17

$D_2 = \cdot 4330 D_1 + 13\cdot 36$.

Age 15. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	12·97	20·13	7·16	7·22	+·06
H_1	13·73	20·57	6·84	6·85	+·01
F	14·80	21·19	6·39	6·33	−·06
E_4	16·88	22·12	5·24	5·31	+·07
E_3	17·38	22·49	5·11	5·06	−·05
E_2	18·57	23·05	4·48	4·49	+·01
E_1	18·50	23·16	4·66	4·52	−·14
B	17·03	22·08	5·05	5·24	+·19
S	17·79	22·55	4·76	4·87	+·11
L	18·68	23·04	4·36	4·44	+·08
M	25·26	26·47	1·21	1·23	+·02
Δ					·09

$D_2 = \cdot 5132 D_1 + 13\cdot 53$.

Age 15. Females.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
12·44	19·58	7·14	7·30	+·16
13·34	20·13	6·79	6·79	·00
15·47	21·25	5·78	5·58	−·20
15·35	21·00	5·65	5·65	·00
16·17	21·48	5·31	5·18	−·13
17·27	21·92	4·65	4·56	−·09
18·71	22·78	4·07	3·75	−·32
13·75	20·18	6·43	6·55	+·12
16·21	21·62	5·41	5·15	−·26
15·68	21·23	5·55	5·46	−·09
22·50	24·10	1·60	1·60	·00
				·16

$D_2 = \cdot 4334 D_1 + 14\cdot 35$.

Age 20. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	14·85	22·04	7·19	7·31	+·12
H_1	15·68	22·52	6·84	6·89	+·05
F	...	...	...	...	...
E_4	19·34	24·38	5·04	5·00	−·04
E_3	19·84	24·83	4·99	4·75	−·24
E_2	21·08	25·38	4·30	4·11	−·19
E_1	20·65	25·33	4·68	4·34	−·34
B	19·46	24·34	4·88	4·95	+·07
S	19·60	24·73	5·13	4·88	−·25
L	21·50	25·56	4·06	3·90	−·16
M	28·99	28·90	−·09	−·05	+·04
Δ					·17

$D_2 = \cdot 4860 D_1 + 14\cdot 95$.

Age 20. Females.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
13·97	21·31	7·34	7·51	+·17
14·91	21·92	7·01	7·01	·00
...	...	...	...	...
17·36	23·02	5·68	5·69	+·01
18·18	23·57	5·39	5·24	−·15
19·30	24·00	4·70	4·63	−·07
20·57	24·82	4·25	3·95	−·30
15·61	22·34	6·73	6·63	−·10
18·11	23·58	5·47	5·28	−·19
17·85	23·38	5·53	5·41	−·12
25·52	26·79	1·27	1·27	·00
				·13

$D_2 = \cdot 4600 D_1 + 15\cdot 06$.

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

Age 25. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	16·94	24·20	7·26	7·47	+ ·21
H_1	17·77	24·76	6·99	7·06	+ ·07
F	17·88	24·76	6·88	7·01	+ ·13
E_4	22·19	27·02	4·83	4·90	+ ·07
E_3	22·66	27·56	4·90	4·68	– ·22
E_2	23·82	28·02	4·20	4·11	– ·09
E_1	22·87	27·72	4·85	4·57	– ·28
B	22·33	26·94	4·61	4·84	+ ·23
S	22·14	27·21	5·07	4·93	– ·14
L	24·82	28·60	3·78	3·62	– ·16
M	33·38	32·58	– ·80	– ·56	+ ·24
Δ					·18

$D_2 = \cdot 5117 D_1 + 15\cdot 74.$

Age 25. Females.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
15·94	23·33	7·39	7·61	+ ·22
16·83	23·98	7·15	7·15	·00
18·65	24·89	6·24	6·29	+ ·05
19·94	25·40	5·46	5·51	+ ·05
20·72	25·97	5·25	5·10	– ·15
21·81	26·33	4·52	4·53	+ ·01
22·87	26·99	4·12	3·98	– ·14
18·06	24·70	6·64	6·50	– ·14
20·53	25·89	5·36	5·20	– ·16
20·72	26·00	5·28	5·11	– ·17
29·41	29·96	·55	·55	·00
				·12

$D_2 = \cdot 4754 D_1 + 15\cdot 98.$

Age 35. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	22·78	30·00	7·22	7·88	+ ·66
H_1	23·63	30·58	6·95	7·46	+ 51
F	22·76	30·58	7·82	7·89	+ ·07
E_4	29·96	34·20	4·24	4·32	+ ·08
E_3	30·15	34·59	4·44	4·23	– ·21
E_2	31·11	34·92	3·81	3·75	– ·06
E_1	29·39	34·01	4·62	4·60	– ·02
B	29·69	33·96	4·27	4·45	+ ·18
S	29·41	34·13	4·72	4·59	– ·13
L	33·51	36·70	3·19	2·56	– ·63
M	44·64	42·09	– 2·55	– 2·95	– ·40
Δ					·35

$D_2 = \cdot 5047 D_1 + 19\cdot 16.$

Age 35. Females.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
21·40	28·74	7·34	7·51	+ ·17
22·12	29·28	7·16	7·16	·00
23·49	29·88	6·39	6·49	+ ·10
26·84	31·73	4·89	4·83	– ·06
27·37	32·09	4·72	4·57	– ·15
28·39	32·36	3·97	4·07	+ ·10
28·53	32·69	4·16	4·00	– ·16
24·35	30·79	6·44	6·06	– ·38
26·88	31·87	4·99	4·81	– ·18
28·22	32·87	4·65	4·16	– ·49
39·33	38·02	– 1·31	– 1·31	·00
				·22

$D_2 = \cdot 5078 D_1 + 18\cdot 05.$

Age 45. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	32·59	39·23	6·64	7·49	+ ·85
H_1	32·28	39·70	7·42	7·64	+ ·22
F	31·38	39·14	7·76	8·09	+ ·33
E_4	41·77	45·05	3·28	2·94	– ·34
E_3	41·50	45·34	3·84	3·08	– ·76
E_2	42·17	45·31	3·14	2·74	– ·40
E_1	39·96	42·09	2·13	3·84	+ 1·71
B	40·40	44·37	3·97	3·62	– ·35
S	40·90	44·97	4·07	3·37	– ·70
L	45·83	48·43	2·60	·96	– 1·64
M	60·67	53·19	– 7·48	– 6·42	+ 1·06
Δ					·91

$D_2 = \cdot 5047 D_1 + 23\cdot 63.$

Age 45. Males.

D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
30·16	37·37	7·21	7·28	+ ·07
30·53	37·65	7·12	7·12	·00
31·52	37·79	6·27	6·68	+ ·41
37·10	41·32	4·22	4·23	+ ·01
37·33	41·58	4·25	4·13	– ·12
38·34	41·57	3·23	3·68	+ ·45
37·81	41·57	3·76	3·92	+ ·16
33·46	39·89	6·43	5·83	– ·60
36·66	41·20	4·54	4·42	– ·12
38·66	42·93	3·27	3·54	+ ·27
53·52	50·53	– 2·99	– 2·99	·00
				·24

$D_2 = \cdot 5603 D_1 + 20\cdot 54.$

Age 55. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	50·57	55·19	4·62	5·25	+ ·63
H_1	51·25	55·56	4·31	5·04	+ ·73
F	47·74	54·08	6·34	6·11	− ·23
E_4	61·79	63·33	1·54	1·82	+ ·28
E_3	60·93	63 53	2·60	2·08	− ·52
E_2	61·57	62·70	1·13	1·89	+ ·76
E_1	59·04	60·89	1·85	2·66	+ ·81
B	57·28	60·83	3·55	3·20	− ·35
S	60·10	63·09	2·99	2·34	− ·65
L	65·59	67·75	2·16	·66	− 1·50
M	86·59	80·06	− 6·53	− 5·75	+ ·78
Δ					·72

$D_2 = \cdot6948 D_1 + 20\cdot18.$

Age 55. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	46·14	52·30	6·16	6·16	·00
H_1	46·26	52·47	6·21	6·12	− ·09
F	47·71	51·97	4·26	5·60	+ 1·34
E_4	54·82	58·00	3·18	3·05	− ·13
E_3	54·95	58·04	3·09	3·01	− ·08
E_2	56·35	57·70	1·35	2·50	+ 1·15
E_1	56·65	57·37	0·72	2·40	+ 1·68
B	48·85	54·11	5·26	5·19	− ·07
S	53·66	57·40	3·74	3·47	− ·27
L	55·91	59·81	3·90	2·67	− 1·23
M	76·63	71·89	− 4·74	− 4·74	·00
Δ					·83

$D_2 = \cdot6425 D_1 + 22\cdot66.$

numbers on which the life table is based at these ages are not large enough to permit of certain conclusions. The result of the above investigation is in accordance with the view that the figures on which the Glasgow life table was based were probably not quite trustworthy. Had the life table for Scotland as a whole agreed with that of Glasgow we might have surmised that different conditions held in the two countries of England and Scotland, but the latter falls into line with the other English life tables. The other possibility, that the large numbers of Irish extraction present in Glasgow, approximately one-sixth to one-fourth of the whole population, have had a disturbing effect on the local death rates is of course open to consideration, but the absence of any life table for Ireland itself leaves us without the appropriate data to determine whether the latter hypothesis will bear examination.

In conclusion a few notes are necessary regarding the exact method in which the calculations discussed in the previous portion of the paper should be made. If only the "true" male and "true" female death rate is desired, all that is required is to calculate the corrected death rate for males and females in the usual way, using the proportionate population of England in the years 1891 to 1900 on account of the fact that all the constants of the "corrected" death rates in the above columns have been calculated on these figures. This gives at once the "true" death rates, with a probable error of not more than one per cent.

In order to facilitate the working of the complete method, all the figures necessary for its application are given in Table IV, namely, the proportionate age distribution in the population of England from 1891 to 1900 for males and females, and in parallel columns the

TABLE IV.

Showing the proportions of each sex in the standard population, namely England 1891–1900 *and in parallel columns the sums from each age upwards.*

Numbers in standard population			Sums from each age upwards		
Age period	Males	Females	Age	Males	Females
0– 5	59052	59468	0–	484057	515943
5–10	56000	56289	5–	425005	456475
10–15	53521	53550	10–	369005	400186
15–20	49986	50814	15–	315484	346636
20–25	44106	49419	20–	265498	295822
25–35	74159	81938	25–	221392	246403
35–45	57412	61276	35–	147233	164465
45–55	41980	45629	45–	89821	103189
55–65	27212	31184	55–	47841	57560
65–75	15026	18596	65–	20629	26376
75–	5603	7780	75–	5603	7780

TABLE V.

Table showing the calculation of Life Table for Liverpool Registration District 1891–1900.

Males.

Age period	Death rates	Age	Sum products above each age	Corrected death rates above each age	True death rates	Expectation of life in years
0– 5	121·49	0–	18332245	37·872	35·34	28·30
5–10	9·42	5–	11158018	26·253	24·65	40·56
10–15	4·64	10–	10630498	28·808	26·70	37·45
15–20	7·43	15–	10382160	32·908	30·42	32·87
20–25	9·78	20–	10010764	37·705	33·28	30·05
25–35	16·63	25–	9579407	43·268	37·88	25·72
35–45	28·89	35–	8346145	56·686	47·77	20·93
45–55	44·95	45–	6687512	74·453	61·20	16·34
55–65	71·00	55–	4800511	100·34	89·90	11·12

Females.

Age period	Death rates	Age	Sum products above each age	Corrected death rates above each age	True death rates	Expectation of life in years
0– 5	107·67	0–	17049658	33·045	31·95	31·30
5–10	8·40	5–	10646738	23·323	23·13	43·25
10–15	4·16	10–	10173910	25·422	24·37	41·04
15–20	5·04	15–	9951142	28·703	26·79	37·30
20–25	6·73	20–	9695040	32·773	30·13	33·19
25–35	13·05	25–	9362450	37·996	34·04	29·39
35–45	24·73	35–	8293159	50·425	43·66	22·90
45–55	38·70	45–	6777804	65·683	57·34	17·44
55–65	60·20	55–	5011961	87·073	78·60	12·72

https://doi.org/10.1017/S0022172400005362 Published online by Cambridge University Press

sum of these figures from birth and any age thereafter up to old age. To illustrate the method the expectations of life at ages 0–55 are calculated for the registration district of Liverpool for the same period. The process is shown in Table V. The death rates at each age for both sexes are given in the first column, next follow the sum of these products from each definite age upwards. Parallel to these are the corrected death rates at each age from 0 to 55, obtained by

TABLE VI.

Expectation of life at different ages.

	H_2	H_1	F	E_4	E_3	E_2	E_1	B	S	L	M
						Males.					
0	52·87	51·48	48·56	44·13	43·66	41·35	39·91	43·59	44·71	40·98	34·71
5	58·26	57·05	54·39	53·50	52·75	50·87	49·71	52·87	52·36	51·60	45·59
10	54·16	53·07	51·28	49·63	49·00	47·60	47·05	49·12	48·60	47·84	42·75
15	49·67	48·62	47·20	45·21	44·47	43·41	43·18	44·67	44·34	43·40	38·78
20	45·37	44·41	43·40	41·02	40·27	39·40	39·48	40·55	40·43	39·13	34·62
25	41·32	40·39	39·93	37·01	36·28	35·68	36·12	36·51	36·75	34·96	30·69
35	33·32	32 70	32·90	29·24	28·91	28·64	29·40	29·02	29·30	27 25	23·76
45	25·49	25·19	25·65	22·20	22·06	22·07	22·76	22·36	22·24	20·65	17·80
55	18·12	18·00	18·49	15·79	15·74	15·95	16·45	16·48	15·85	14·76	12·49
						Females.					
	H [H_2]	H [H_1]	·F	E [E_4]	E [E_3]	E [E_2]	E [E_1]	B	S	L	M
0	55·71	54·04	49·45	47·77	47·18	44·62	41·85	49·00	47·47	45·33	38·44
5	59·53	58·01	53·93	55·79	54·92	53·08	50·33	56·92	50·02	55·12	48·06
10	55·46	54·01	50·88	51·97	51·10	49·76	47·67	53·15	50·39	51·49	45·43
15	51·06	49·68	47·04	47·61	46·55	45·63	43·90	49·07	46·26	47·10	41·50
20	46·93	45·62	43·50	43·44	42·42	41·66	40·29	47·76	42·41	42·77	37·33
25	42·86	41·71	40·17	39·37	38·50	37·98	37·04	40·48	38·63	38·46	33·38
35	34·79	34·16	33·46	31·52	31·16	30·90	30·59	32·48	31·37	30·42	26·30
45	26·84	26·56	26·46	24·20	24 05	24·06	24·06	25·07	24·27	23·29	19·79
55	19·12	19·06	19·24	17·24	17·23	17·33	17·43	18·48	17·42	16·72	13·91

dividing the former by the corresponding sums from Table IV. The true death rate figures are then calculated by the formulae given in the earlier part of the paper (Table III), and the expectation of life at each age obtained by dividing 1000 by each of the latter. It will be noticed that in Liverpool the true death rate is less than the corrected death rate, in other words that during the years referred to, Liverpool was using up life to a greater extent than she was creating it.

This includes all that need be said in the present communication, but a concluding table (Table VI) showing the expectation of life in all the life tables used above may not be without interest as few persons possess access to all the data. These expectations are limited by the year 55. The ages above that come under a somewhat different law, and will be discussed in a subsequent paper.

The calculations in this article were made with the help of an Arithmometer supplied by the Carnegie Trustees.

REFERENCES.

(1) BROWNLEE, J. (1908). Germinal Vitality. *Trans. Roy. Phil. Soc. Glasgow.*

(2) MURPHY, SHIRLEY (1912). Some points in the Decline of the Birth and Death-Rates. *Journ. Roy. San. Inst.* XXXIII. 337.