SANDRA MÜLLER (*formerly* UHLENBROCK), ***Pure and Hybrid Mice with Finitely Many Woodin Cardinals from Levels of Determinacy***, Westfälische Wilhelms-Universität Münster, Germany, 2016. Supervised by Ralf-Dieter Schindler. MSC: primary 03E45, secondary 03E60, 03E55. Keywords: inner models, Woodin cardinals, projective determinacy, hybrid mice.

Abstract

Mice are sufficiently iterable canonical models of set theory. Martin and Steel showed in the 1980s that for every natural number n the existence of n Woodin cardinals with a measurable cardinal above them all implies that boldface $\mathbf{\Pi}^1_{n+1}$ determinacy holds, where $\mathbf{\Pi}^1_{n+1}$ is a pointclass in the projective hierarchy. Woodin and Neeman (in the late 80s and early 90s) then proved an exact correspondence between mice and projective determinacy. Woodin showed that boldface $\mathbf{\Pi}^1_{n+1}$ determinacy implies that the mouse $M_n^\#(x)$ with n Woodin cardinals exists and is ω_1-iterable for all reals x, the converse is due to Neeman. In the first part of this thesis we prove this implication of the result, which is a so far unpublished result by W. Hugh Woodin. In fact, the following theorem is shown in the thesis.

THEOREM 1. *Let $n \geq 1$ and assume there is no $\mathbf{\Sigma}^1_{n+2}$-definable ω_1-sequence of pairwise distinct reals. Moreover, assume that $\mathbf{\Pi}^1_n$ determinacy and Π^1_{n+1} determinacy hold. Then $M_n^\#$ exists and is ω_1-iterable.*

As a consequence, we can obtain the following Determinacy Transfer Theorem for all levels n.

THEOREM 2 (Determinacy Transfer Theorem). *For $n \geq 1$, $\mathbf{\Pi}^1_{n+1}$ determinacy is equivalent to $\Game^{(n)}(< \omega^2 - \mathbf{\Pi}^1_1)$ determinacy.*

Following this, we consider pointclasses in the $L(\mathbb{R})$-hierarchy and show that determinacy for them implies the existence and ω_1-iterability of certain hybrid mice with finitely many Woodin cardinals, which we call $M_k^{\Sigma,\#}$. These hybrid mice are like ordinary mice, but equipped with an iteration strategy for a mouse they are containing, and they naturally appear in the core model induction technique. The results we proved are in fact more general because they hold for an arbitrary adequate pointclass Γ which is $\mathbb{R}$-parametrized and has the scale property and a premouse $\mathcal{N}$ capturing certain sets of reals which has an iteration strategy $\Sigma \in \forall^{\mathbb{N}}\Gamma$ which condenses well. We apply these results to the following setting in the $L(\mathbb{R})$-hierarchy.

THEOREM 3. *Let $\alpha < \beta$ be ordinals such that $[\alpha, \beta]$ is a weak Σ_1-gap, let $k \geq 0$, and let $A \in \Gamma = \mathbf{\Sigma}_n(J_\beta(\mathbb{R})) \cap \mathcal{P}(\mathbb{R})$, where $n < \omega$ is the least natural number such that $\rho_n(J_\beta(\mathbb{R})) = \mathbb{R}$. Moreover, assume that every $\mathbf{\Pi}^1_{2k+5}\Gamma$-definable set of reals is determined. Then there exists an ω_1-iterable hybrid Σ-premouse $\mathcal{N}$ which captures every set of reals in the pointclass $\Sigma^1_k(A)$ or $\Pi^1_k(A)$.*

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WILLIAM CHEN, ***Some Results on Tight Stationarity***, University of California, Los Angeles, USA, 2016. Supervised by Itay Neeman. MSC: 03E05. Keywords: mutual stationarity, pcf theory, Prikry forcing, singular cardinals, tight stationarity, tree-like scales.

Abstract

Fix an increasing sequence of regular cardinals $\langle \kappa_n : n < \omega \rangle$. Mutual and tight stationarity are properties akin to the usual notion of stationarity, but defined for sequences $\langle S_n : n < \omega \rangle$ with $S_n \subseteq \kappa_n$. This work focuses particularly on tight stationarity, providing a new characterization for it and comparing it to other concepts of stationarity.

Starting from a pcf-theoretic scale, we define a transfer function mapping a sequence of subsets to a single subset of a certain regular cardinal, the length of the scale. The transfer