

# A nonextensive approach for the angular momentum loss rate in low-mass stars

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**Abstract.** The present study demonstrates that behavior of rotational velocity as a function of stellar age is consistent using Tsallis' nonextensive formalism, resulting in a new approach to understanding the stellar rotational scenario.

## 1. Introduction

The pioneering study by Skumanich (1972) showed that  $v \sin i$  of G-type Main-Sequence (MS) stars for Hyades and Pleiades measured by Kraft (1967) were consistent with the  $v \propto t^{-1/2}$ . This relationship is consistent with simple theories of angular momentum loss from rotating stars, where an ionized wind is coupled to the star by a magnetic field Schatzman (1962). Indeed, angular momentum loss due to stellar winds is generally believed to be responsible for the Skumanich relationship, but the exact dependence of rotation on age is not yet completely described Kawaler (1988) and Krishnamurthi *et al.* (1997). Barry *et al.* (1987) and Soderblom *et al.* (1991) reported similar qualitative results for solar-type stars, but with power-law presenting exponents ranging from  $-1/2$  (corresponding to the Skumanich relation) to  $-4/3$ . More recently, Pace and Pasquini (2004) claimed that these power-laws do not fit the age-activity-rotation of G dwarf stars in open clusters. According to these authors, a  $t^{-5/2}$  law is more consistent with the observations. Despite the differences in rotation-activity-age relationships obtained to date, most of the data strongly suggest that this relationship is indeed deterministic and not merely a statistical artefact.

In this study, we present a new nonextensive approach to the study of stellar rotational braking in F and G type stars, connecting angular momentum loss rate by magnetic stellar wind Kawaler (1988), Chaboyer *et al.* (1995) with Tsallis' nonextensive statistical mechanics Tsallis (1988). For this, we revisit parametric models for angular momentum loss by magnetic stellar wind, with an emphasis on a modified Kawaler model. The accepted version for publication is available in de Freitas and De Medeiros (2013).

## 2. Nonextensive approach for stellar rotation-age relation

This section details a nonextensive approach for the modified Kawaler model divided into two classes: saturated and  $\beta$ -saturated models. Both have the same unsaturated model, but with different saturated timescale  $t_{sat}$ .

### 2.1. ( $\beta$ )-Saturated and unsaturated regimes

In saturated domain, we have that

$$v(t) = v_0 \exp[-\lambda_1(t - t_0)] \quad (t_0 \leq t < t_{sat}) \quad (2.1)$$

and

$$\lambda_1 = f_{sat} \omega_{sat}^{4aN/3}, \quad (2.2)$$

where

$$f_{sat} = \frac{K_w}{I_{\odot,k}} \left( \frac{R}{R_{\odot}} \right)^{-N} \left( \frac{M}{M_{\odot}} \right)^{-1-N/3} \left( \frac{\dot{M}}{10^{-14}} \right)^{1-2N/3}, \quad (2.3)$$

On the other hand, for unsaturated regime, we have

$$v(t) = v_{sat} [1 + (q - 1)\lambda_q(t - t_{sat})]^{1/q} \quad (t \geq t_{sat}), \quad (2.4)$$

with the entropic index given by

$$q = 1 + \frac{4aN}{3}, \quad (2.5)$$

and

$$\lambda_q = \frac{f_{sat}}{R^{q-1}}, \quad (2.6)$$

in our context  $\lambda_1$  and  $\lambda_q$  denotes the braking strength.

As reported by Ivanova and Taam (2003), high rotation rates can differ significantly from the exponential decay, without which the angular momentum loss rate necessarily saturates the magnetic field. In this case the time dependence of  $v$  is given by Chaboyer *et al.* (1995).

$$v(t) = v_0 [1 + (\beta - 1)\lambda_{\beta}(t - t_0)]^{1/\beta} \quad (t_0 \leq t < t_{sat}), \quad (2.7)$$

where the function  $\lambda_{\beta}$  is denoted by

$$\lambda_{\beta} = \frac{f_{sat}\omega_{sat}^{q-\beta}}{R^{\beta-1}}. \quad (2.8)$$

### 3. Results and Conclusions

In summary, our study presents a new statistical approach for stellar rotational braking. The present analysis shows that the rotation-age relationship can be well reproduced using a nonextensive approach from the statistical mechanics, namely the Tsallis nonextensive models. The nonextensive analysis proposed here considers several parameters, such as magnetic field geometry and mass loss, and offers the possibility to study the stellar rotational braking behaviour for different classes of stars, on the basis of the same approach. The present work has been awarded as the Best Poster Presentation at referred Symposium.

### References

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