7
QCD and its global invariance

7.1 $U(1)$ global invariance

$\mathcal{L}_{\rm QCD}(x)$ is invariant under the $U(1)_B$ global transformation :

$$\psi_i(x) \to \exp(-i\theta\mathbf{1})\psi_i(x) \,, \tag{7.1}$$

to which corresponds the conserved baryonic current:

$$J^\mu(x) = \sum_i \bar{\psi}_i \gamma^\mu \psi_i(x) \,, \tag{7.2}$$

and the baryonic charge generator of the $U(1)_B$ group:

$$B = \int d^3x \, J^0(\vec{x}, t) \,. \tag{7.3}$$

For massless quarks, $\mathcal{L}_{\rm QCD}(x)$ is also invariant under the axial $U(1)_A$ transformation:

$$\psi_i \to (-i\theta\mathbf{1}\gamma_5)\psi_i \,, \tag{7.4}$$

acting on quark-flavour components. The corresponding current:

$$J_5^\mu(x) = \sum_i \bar{\psi}_i \gamma^\mu \gamma_5 \psi_i(x) \,, \tag{7.5}$$

has an anomalous divergence:

$$\partial\mu J_5^\mu(x) = \frac{g^2}{4\pi^2}\frac{n}{8}\epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma} \,, \tag{7.6}$$

where the rate of the change of the associated axial charge:

$$\dot{Q}_5 = \int d^3x \; \partial_0 J_5^0(\vec{x}, t) \,, \tag{7.7}$$

is zero in the absence of instanton-type solutions [105].

https://doi.org/10.1017/9781009290296.013 Published online by Cambridge University Press

7.2 $SU(n)_L \times SU(n)_R$ global chiral symmetry

As we have already dicussed in Part 1, and we shall partly repeat here, $\mathcal{L}_{\rm QCD}(x)$ also possesses a $SU(n)_L \times SU(n)_R$ global chiral symmetry. In the massless quark limit ($m_j = 0$), it is invariant under the global chiral transformation:

$$\psi_i(x) \to \exp(-i\theta^A T_A)\psi_i(x) \,,$$
$$\psi_i(x) \to \exp(-i\theta^A T_A \gamma_5)\psi_i(x) \,, \tag{7.8}$$

where $T^A (A \equiv 1, \ldots, n^2 - 1)$ are the infinitesimal generators of the $SU(n)$ group acting on the quark-flavour components. The associated Noether currents are the vector and axial-vector currents:

$$V_\mu^A(x) = \bar{\psi}_i \gamma_\mu \, T_{ij}^A \psi_i(x) \,,$$
$$A_\mu^A(x) = \bar{\psi}_i \gamma_\mu \gamma_5 \, T_{ij}^A \psi_i(x) \,, \tag{7.9}$$

which are the ones of the algebra of currents of Gell-Mann [69,13]. The corresponding charges, which are the generators of $SU(n)_L \times SU(n)_R$ are:

$$Q_L^A = \int d^3x \left(V_0^A - A_0^A\right) \,,$$
$$Q_R^A = \int d^3x \left(V_0^A + A_0^A\right) . \tag{7.10}$$

The charges are conserved in the massless quark limit, and obeys the commutation relation:

$$\left[Q_L^\alpha, Q_L^\beta\right] = i f_{\alpha\beta\gamma} Q_L^\gamma \,,$$
$$\left[Q_R^\alpha, Q_R^\beta\right] = i f_{\alpha\beta\gamma} Q_R^\gamma \,,$$
$$\left[Q_L^\alpha, Q_R^\alpha\right] = 0 \,, \tag{7.11}$$

where $\alpha,\ \beta,\ \gamma = 1, \ldots n$. In the Nambu–Goldstone [17] realization of chiral symmetry, the axial charge does not annihilate the vacuum, which is the basis of the successes of current algebra and pion PCAC [13]. In this scheme, the chiral flavour group $G \equiv SU(n)_L \times SU(n)_R$ is broken spontaneously by the light quark (u, d, s) vacuum condensates down to a subgroup $H \equiv SU(n)_{L+R}$, where the vacua are symmetrical:

$$\langle \bar{\psi}_u \psi_u \rangle = \langle \bar{\psi}_d \psi_d \rangle = \langle \bar{\psi}_s \psi_s \rangle \,. \tag{7.12}$$

The Goldstone theorem states that this spontaneous breaking mechanism is accompanied by $n^2 - 1$ massless Goldstone P (pions) bosons, which are associated with each unbroken generator of the coset space G/H. For $n = 3$, these Goldstone bosons can be identified with the eight lightest mesons of the Gell-Mann eightfoldway ($\pi^+,\ \pi^-,\ \pi^0,\ \eta,\ K^+,\ K^-,\ K^0,\ \bar{K}^0$). On the other hand, the vector charge is assumed to annihilate the vacuum and the corresponding symmetry is achieved à la Wigner–Weyl [18]. In the vector case, the particles are classified in irreducible representations of $SU(n)_{L+R}$ and form parity doublets.

https://doi.org/10.1017/9781009290296.013 Published online by Cambridge University Press