1

Introduction

Turbulence is a state of fluid motion that is spatially and temporally complex. One might say that its study began about five centuries ago, when Leonardo da Vinci observed that water falling into a pond creates coherent eddies or whirls, which he compared to the curls of a woman's braided hair; see Fig. 1.1. The insight that eddies are the building blocks of turbulence was confirmed in 1839 by Hagen, who visualized the vortical motion of water with sawdust (Eckert, 2019), and by Reynolds (1883) in his pioneering studies of turbulent pipe flow. Reynolds visualized the flow of water by injecting a narrow jet of dye into the center of the flow along a larger glass pipe. He found that the dyed jet remains narrow and distinct when the water velocity is small; the dye breaks up and diffuses rapidly when the velocity is increased, filling the cross-section of the glass pipe at some point that marks the appearance of turbulence. A short-exposure photograph taken with an electric spark reveals that, instantaneously, the turbulent water consists of vortices; see Fig. 1.2. Turbulence as a subject of study has grown substantially since then because of its importance to a variety of fields such as astrophysics, geophysics, and various engineering applications including such large-scale projects as controlled fusion; what we have in front of us is the result of a rich combination of efforts of theorists, experimentalists, and simulation experts alike.

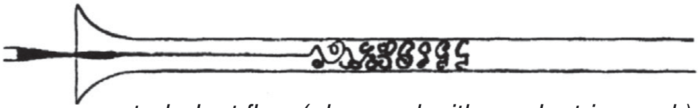
While Leonardo understood that the flow of water shapes the natural landscape, today we appreciate that turbulence also shapes processes that surround us (from jet engines to interstellar medium) or are contained inside us (from the blood flow in the aorta to airflow in lungs). Its various facets have continually been understood better over the last few decades, but it remains an unfinished problem. The subject is moving forward constantly, yet some of its essential problems have remained as current as ever. For example, the existence or breakdown of the smoothness of the Navier–Stokes equation has remained an unclaimed prize among the Millennium Problems of the Clay Mathematics Institute. And, as an illustration on the practical side, one cannot yet predict the pressure drop in a pipe using the Navier–Stokes

Figure 1.1 Leonardo da Vinci's studies of water (circa 1507), pen and ink on paper. (Top) Turbulent water in a pond. (Bottom) Leonardo's analogy between turbulent water flow and braided hair. Leonardo da Vinci, Public domain, via Wikimedia Commons.

equations alone. Because of its intellectual challenges and practical importance, it continues to challenge the attention of physicists, mathematicians, and engineers. One reason why so many scientists from various disciplines have to deal with one aspect or another of fluid turbulence is that the phenomenon spans an enormous range of scales – more than thirty orders of magnitude – from astronomical, when turbulent description is useful for explaining the shapes of spiral galaxies, the

laminar flow

turbulent flow



turbulent flow (observed with an electric spark)

Figure 1.2 The experiment of Osborne Reynolds (1883). A dye, injected into the center of the glass pipe, visualizes the flow of water from left to right. (Top) At small velocity, the flow is laminar and the jet of dye remains narrow. (Middle) At large velocities, the flow becomes turbulent and the dye spreads into the entire pipe cross-section. (Bottom) The short exposure photograph obtained using an electric spark reveals that turbulence consists of coherent eddies.

behavior of interstellar wind, the stream of interstellar gas and dust that is moving past the solar system, on the one hand, to the tiny core of a quantized vortex in superfluid helium on the other. Moreover, phenomena occurring at scales differing by many orders of magnitude can be linked together by the same underlying physics, such as the theoretically predicted Kibble–Zurek mechanism for the generation and decay of cosmic strings and quantized vortices in helium superfluids.

This book is devoted to the study of turbulence in quantum fluids displaying superfluidity, mainly in superfluid ^{4}He and ^{3}He-B. In general, superfluid helium possesses many extraordinary physical properties. The fundamental laws of physics become manifest when the temperature of the quantum liquid decreases, allowing us to use a helium droplet as a condensed matter model system for the early universe (Volovik, 2007). Turbulence in quantum fluids, which we shall refer to as *quantum turbulence*, is a relatively new area of research. A number of reviews are available on this subject, starting from the pioneering article by Vinen and Niemela (2002), to the works of Nemirovskii (2012); Skrbek and Sreenivasan (2012); Barenghi *et al.* (2014a); Tsubota *et al.* (2017), and to the more recent perspective of Skrbek *et al.* (2021). The aim of this book is to provide a coherent account of recent developments and attempt to draw qualitative and quantitative connections between quantum turbulence and turbulence in ordinary fluids, or the *classical turbulence*.

As stated in the Preface, the term *quantum turbulence* was first introduced in 1982 in the Ph.D. thesis of Barenghi (1982) and popularized by Donnelly and Swanson (1986). We shall base this book on quantum turbulence on the current work by distinct scientific communities: low-temperature physicists, condensed-matter physicists, fluid dynamicists, and atomic physicists. One of our aims is indeed to bring these communities together. We shall take the evidence and draw inferences from experiments, theory, and numerical simulations.

Quantum fluids are so called because their physical properties depend on quantum physics, classical physics being insufficient to describe them. The most studied quantum fluids are the low temperature phases of liquid helium $\left({}^4\mathrm{He}\text{ and }{}^3\mathrm{He}\right)$ and, more recently, ultracold atomic gases. We shall concentrate particularly on liquid helium, for which more experimental information is available, using results from the study of atomic gases only when relevant to the general problem of turbulence in quantum fluids.

From the standpoint of fluid dynamics, quantum fluids differ from classical fluids (such as air or water) in three respects:

(i) at nonzero temperature (or in the presence of impurities) they exhibit a two–fluid behavior;
(ii) under suitable conditions they can flow freely, without the dissipative effect of viscous forces, hence the term *superfluidity*;
(iii) their local rotation is constrained to thin *vortex lines* (also called *quantized vortices* or *superfluid vortices*);[1] unlike eddies in ordinary fluids, which are continuous and can have arbitrary size and strength, vortex lines are discrete, individual structures whose circulation and core thickness are determined by quantum mechanical constraints.

From the point of view of turbulence the fundamental property is quantized vorticity, which is an extraordinary manifestation of quantum mechanics at macroscopic length scales. This property arises because of the existence of a macroscopic complex wavefunction (sometimes called an order parameter), which governs the dynamics of the system and endows quantum turbulence with some attractive properties.

How these three properties of quantum fluids endow flows with certain distinguishing characteristics, how these characteristics are similar to, and different from, those of classical fluids are the topics that we take up in succeeding chapters. We expect our account to serve as both introductory material for graduate students and

---

[1] Exceptions exist, such as the superfluid ^{3}He-A phase where continuous vortices or vortex sheets can be formed.

a reference material for expert researchers. After reading the first three chapters, the remaining chapters can be read reasonably independently, but a chronological reading is recommended for a systematic understanding of the subject.

One further point is useful. Superfluid ^{4}He, ^{3}He, and atomic condensates exist only at very low temperatures (of the order of a kelvin, a millikelvin and a microkelvin, respectively). However, very small absolute temperatures are not necessary for the existence of quantum fluids: for example, exciton–polaritons condensates are superfluid at room temperature, and the interior of neutron stars is extraordinarily hot by normal standards. Quantum mechanics is required when the temperature is significantly lower than some characteristic temperature of the system, such as the *Fermi temperature* T_{F} in a gas of fermions.