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Symmetry and complexity in dynamical systems

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Historically, static symmetric bodies and ornaments are geometric idealizations in the Platonic tradition. Actually, symmetries are locally and globally broken by phase transitions of instability in dynamical systems generating a variety of new order and partial symmetries with increasing complexity. The states of complex dynamical systems can refer to, for example, atomic clusters, crystals, biomolecules, organisms and brains, social and economic systems. The paper discusses dynamical balance as dynamical symmetry in dynamical systems, which can be simulated by computational systems. Its emergence is an interdisciplinary challenge of nonlinear systems science. The philosophy of science analyses the common methodological framework of symmetry and complexity.

Symmetry and complexity in early culture and philosophy

In all ancient cultures, we find symmetric symbols with cosmic and mythological meaning.[1] A famous mystic diagram of Indian cosmology is the Shrî Yantra (yantra of eminence) with concentric rings of rotational symmetries, symbolizing the birth of the universe in several phases from a central point (bindu) (Figure 1). The two kinds of triangles in the centre with upward and downward pointing vertices indicate the anti-symmetric energies of the goddess Shakti and the god Shiva. In meditations, the phase transitions of the material world from the shapeless origin can be followed in a backward direction, starting with our experiences of the material world until our mind arrives at the bindu of shapeless origin and unifies with the final state of harmony and happiness (sarva ânandamaya). In Chinese natural philosophy, rotational symmetries of trigrams from the I Ching symbolize diametrically opposed natural forces and entities like fire (li) and water (khan), sea (tui) and mountain (kên), sky (chhien) and earth

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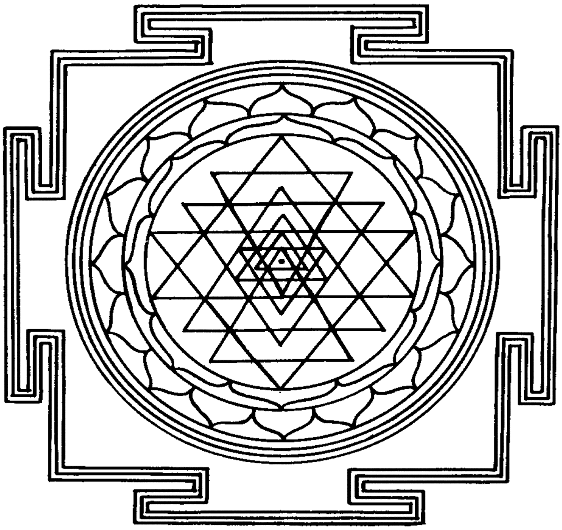

Figure 1. Symmetries of Indian cosmology (Shrî-Yantra)

(khun), wind (sun) and thunder (chen) which are also represented on old Chinese coins.

In Greek cosmology, Plato's cosmos is a centrally-symmetrically-ordered system with the Earth in the centre. Sun, moon and planets turn on spheres. The most external shell carries the sphere of the fixed stars. According to the Platonic-Pythagorean conception, the motion of each one produces a sound so that the tones of the movements of the spheres jointly form a harmony of the spheres in the sense of a well-ordered musical scale, like the strings of a lyre. In Platonic tradition, the observed retrograde movements (e.g. Mars along the ecliptic) are considered symmetry breaking of cosmic harmony. Thus, they must be explained by uniform, circular movements in order to 'save' the symmetry of heaven.[2]

According to Apollonius of Perga (ca. 210 BC), the planets rotate uniformly on spheres (epicycles) whose imagined centres move uniformly on great circles (deferents) around the centre (Earth). By appropriately proportioning the speed and diameter of the circular-motions, the observed retrograde movements can be produced, as it is seen by an observer on the central Earth. The variation in planetary brilliance can be explained by the varying distance between the Earth and the looped path of a planet. By the epicycle–deferent technique, a multiplicity of elliptical orbits, mirror symmetry curves, periodic curves, and also non-periodic and asymmetrical curves can be reduced to uniform and circular movements ('symmetry') and approximated to observational data. Thus, in the Middle Ages, a complicated 'computus' of celestial movements was generated, but without physical explanation.

Symmetry was not only assumed in the macrocosmos of the universe, but also in the microcosmos of natural elements such as fire, air, water, earth, and the quintessence of the cosmos. They are characterized by the five regular bodies ('Platonic bodies') of the three-dimensional Euclidian space. In the Renaissance, Kepler even used the Platonic bodies to determine the distances of celestial spheres in the heliocentric system of planets (*Mysterium cosmographicum*). Symmetry is still a topic in the Leonardo-world of modern times.

Mathematical concept of symmetry

In modern times, symmetries are defined mathematically by group theory.[1,3] The symmetry of a set (e.g. points, numbers, functions) is defined by the group of self-mappings ('automorphisms') that leaves unchanged the structure of the set (e.g. proportional relations in Euclidean space, arithmetical rules of numbers). In general, the composition of automorphisms satisfies the axioms of a mathematical group: (1) the identity I that maps every element of a set onto itself, is an automorphism; (2) for every automorphism T an inverse automorphism T' can be given with $T \cdot T' = T' \cdot T = I$; (3) if S and T are automorphisms, then so is the successive application $S \cdot T$. An example is mirror symmetry with the identical inverse automorphism $T \cdot T = I$. The group of similarities leaves the shape of a figure unchanged (invariant). The size of a figure is invariant with respect to the group of (proper and improper) congruencies. Improper congruencies have the additional property of mirror symmetry.

In one dimension, ornaments of stripes are classified by seven frieze groups, which are systematically produced by periodic translations in one direction and reflections transverse to the longitudinal axis of translations (Figure 2). In two dimensions, there are 17 wallpaper groups, produced by translations in two directions, reflections, inversions and rotations. In order to classify the discrete space groups, one starts with the regular Platonic bodies of Euclidean geometry. In general, one may ask which of the finite point groups of movements leave spatial grids invariant. There are 32 groups that are of great importance in crystallography, but if one considers translations too, there are 230 space groups with three independent translations first classified by Federov and Schönflies.

In mathematics, there is a major difference between discrete and continuous groups. Examples of discrete groups are the finite rotation groups of polygons, ornaments, and crystals. An example of a continuous group with infinitely many infinitesimal transformations is the rotation of a circle or Lie groups, and these have importance in physics. In geometry, symmetric properties of figures and bodies indicate invariance with respect to automorphisms like rotations, translations and reflections. In general, geometric theories can be classified under the viewpoint of geometric invariants, which remain unchanged by metric, affine,

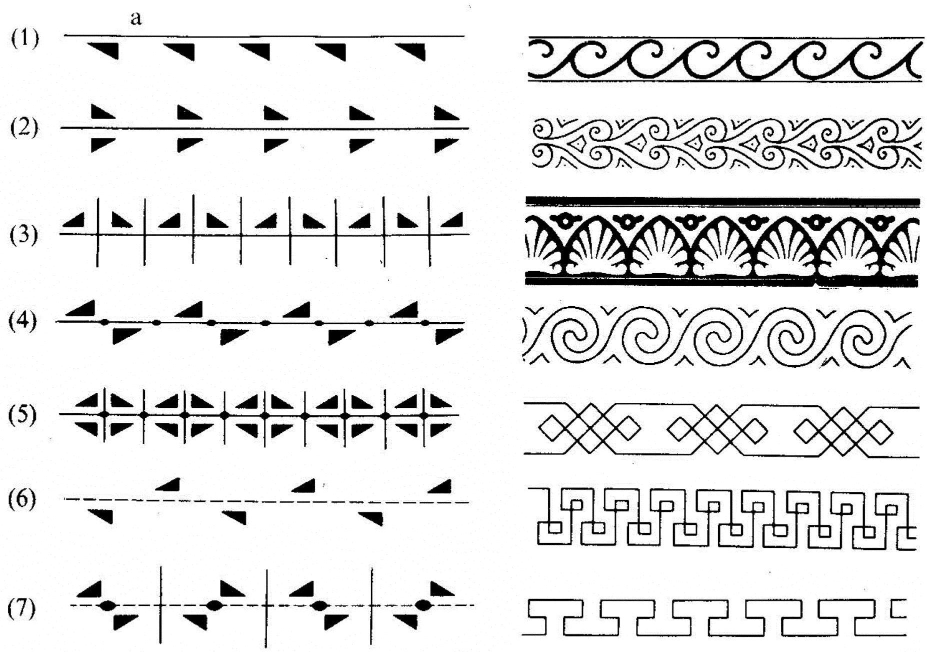

Figure 2. Symmetries and ornaments (frieze groups)

projective, topological and other transformations. As an example, the concept of a regular triangle is an invariant of Euclidean, but not of projective, geometry: it remains invariant with respect to metric transformations, while a projective transformation distorts the triangle's sides. In algebra and number theory, solutions of equations can also be characterized by symmetric properties. Thus, symmetry becomes a universal property of mathematical structures.

Global and local symmetry in dynamical systems

Applications of symmetries in the natural sciences refer to dynamical systems. Dynamical systems (e.g. planetary systems, fluids, organisms) consist of elements i ($1 \leq i \leq n$) (e.g. planets, atoms, molecules, cells) in states s_i (e.g. a planet's state of movement with location and momentum at time t). The state dynamics $s_i(t)$ at time t is determined by time-depending differential equations (e.g. equations of motion for planets). In a first example, symmetries of space and time can be introduced as form-invariance of the equations of dynamical systems with respect to transformations between reference systems of space and time. In classical mechanics, inertial systems are reference systems for force-free bodies moving along straight lines with uniform velocity (Newton's *lex inertiae*). Mechanical

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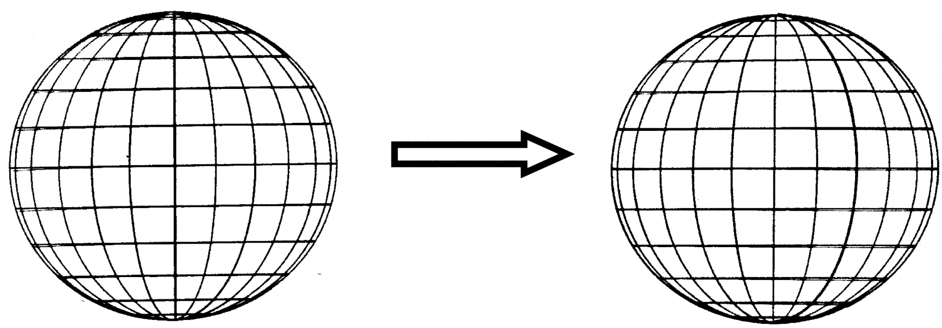
laws are preserved (invariant) with respect to Galilean transformations between all inertial systems moving uniformly relative to one another (Galilean invariance).

Intuitively, Galilean invariance means that a natural law of mechanics is true independently of the particular inertial system of an observer. Einstein's special relativistic space–time unifies Maxwell's electrodynamics and classical mechanics by the common Poincaré transformation group, which leaves the Minkowskian geometry of both theories invariant. Geometrical symmetries of space and time have the remarkable property of implying conservation laws of physical quantities. Examples are the conservation of linear momentum and energy, which are implied by the homogeneity of space and time, i.e. spatial and time translation.

Classical mechanics and special relativity are examples of global symmetry. In this case, the form of a natural law is preserved with respect to a common transformation of all coordinates. Analogously, the shape of an elastic balloon remains unchanged (invariant) after global symmetry transformation, since all points were rotated by the same angle (Figure 3). For a local symmetry, the elastic balloon must also retain its shape when the points on the surface are rotated independently of one another by different angles. Thereby, local forces of tension occur on the surface of the elastic balloon between the points caused by the local changes (Figure 4). They compensate the local changes and preserve the shape of the sphere ('saving its symmetry'). Analogously, in general relativity, the principle of equivalence demands that local gravitational fields can be compensated by the choice of an appropriate accelerated reference system. This fact is well known in the local loss of gravity in falling aeroplanes. The relativistic field equations are form-invariant ('covariant') with respect to these local symmetry transformations. We can also say that gravitation is introduced by



original sphere with untransformed coordinates

global symmetry transformation

Figure 3. Global symmetry

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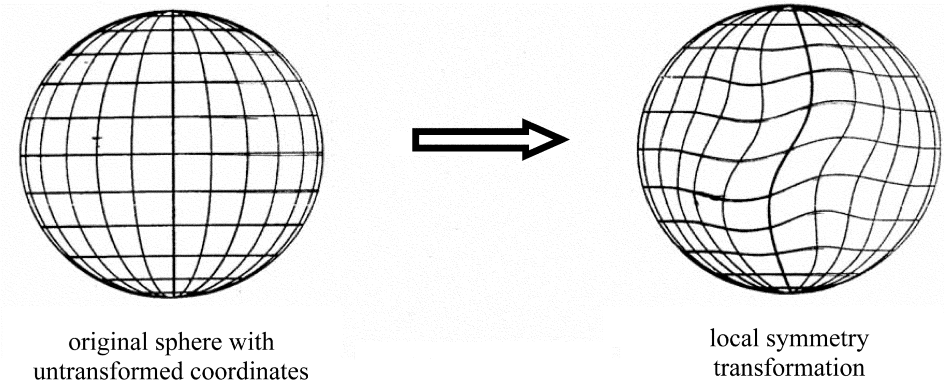

Figure 4. Local symmetry

demanding form-invariance (covariance) of field equations with respect to local symmetry.

The application of global and local symmetry to gravitation can be generalized for all physical forces. If field laws are form-invariant with respect to transformations of global symmetry, form-invariance with respect to local symmetry can only be realized by the introduction of new fields leading to new forces of interaction. According to Weyl, these are called gauge fields, because, historically, he referred to transformations of scales and their invariance ('gauge invariance'). In this sense, gravitation is the gauge field of local Poincaré-invariance and the force of gravitation is a consequence of local Poincaré-symmetry. In general, the gauge principle demands form-invariance of field equations with respect to local symmetry ('gauge') transformations. In quantum field theories, gauge groups are used to define electromagnetic, weak, and strong interaction.[1,4]

In general, the gauge principle only determines the form of the coupling term of physical interaction. But the existence of a physical force is an empirical question which, of course, cannot be derived from an a priori demand of local symmetry. A gauge group characterizes a physical interaction mathematically in terms of local symmetry. It is epistemically remarkable that only gauge-invariant quantities have observable effects. For example, field force tensors are gauge-invariant, but not gauge fields (potentials). Local phase transformations do not change any measurable observable. Therefore, the gauge principle or demand for local symmetry can epistemically be considered as a filter of observables in a theory of physical interactions.

Actually, the fundamental physical forces of interaction can be characterized by local gauge symmetries. In the standard model, gravitation, elctromagnetic, weak and strong interaction are represented by local Poincaré-, $U(1)$-, $SU(2)$- and $SU(3)$-gauge groups. Research on unified theories tries to unify the fundamental

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forces step by step in states of higher energy characterized by unified local symmetries. In 1954, the Yang–Mills theory tried at first to unify the proton and neutron by a gauge theory of isospin-symmetry. But the Yang–Mills theory only forecasted massless gauge particles of interaction, in contradiction to empirical observations. Later on, Goldstone and Higgs introduced the mechanism of spontaneous symmetry breaking in order to give appropriate gauge particles the desired mass. The intuitive idea is that a symmetric theory can have asymmetric consequences. For example, the equations of a ball and the wheel of a roulette are symmetric with respect to the rotation axis, but the ball always keeps lying in an asymmetric position. In a first step, electromagnetic and weak forces could already be unified at very high energies in an accelerator ring. This means that at a very high energy the particles of the weak interaction (electrons, neutrinos, etc) and electromagnetic interaction are indistinguishable and transformed into one another. Their transformations are described by the same symmetry group $U(1) \times SU(2)$. At a critical value of lower energy the symmetry spontaneously breaks apart into two partial symmetries $U(1)$ of electromagnetic force and $SU(2)$ of weak interaction. The gauge particles of weak interaction get their mass by the Higgs mechanism, but the photon of electromagnetic interaction remains massless.

After the successful unification of electromagnetic and weak interaction, physicists tried to realize the 'big' unification of electromagnetic, weak and strong forces, and in a last step the 'superunification' of all four forces. There are several research strategies for super-unification, such as supergravity and superstring theories. Mathematically they are described by extensions of richer structures of local symmetries and their corresponding gauge groups. On the other hand, the variety of elementary particles is actualized by spontaneous symmetry breaking. The concepts of local symmetry and symmetry breaking play a central role in cosmology. During cosmic expansion and the cooling temperature, the initial unified supersymmetry of all forces broke apart into the subsymmetries of physical interactions, and the corresponding elementary particles were crystallized in phase stages leading to more variety and complexity.

The phases of cosmic expansion are determined by properties of symmetry breaking. For example, in the case of the weak interaction, neutrinos occur only in a left-handed helix, but not a right-handed one, which means parity violation. This kind of antisymmetry or dissymmetry seems also to be typical for molecular structures of life. Protein analysis shows that amino acids have an antisymmetrical carbon atom and occur only in the left-handed configuration. Weak interaction takes part in chemical bonds. Thus, cosmic parity violation of weak interaction is assumed to cause the selection of chiral molecules. The reason is that the left-handed (L) and right-handed (D) examples of chiral molecules can be distinguished by a tiny parity violating energy difference ΔE_{pv}. The energetically

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stable examples (e.g. L-form of amino acids) are preserved. However, this assumption is only based on theoretical calculations (e.g. Hartree–Fock procedures in physical chemistry). We still miss exact measurements of experiments because of the tiny small parity violation energy difference ΔE_{pv} (e.g. 4×10^{-14}(hc)cm^{-1} (H_2O_2), 1×10^{-12}(hc)cm^{-1} (H_2S_2)), although there are proposed spectroscopic experiments.[5]

In classical physics, dynamical systems are invariant with respect to the discrete symmetry transformations of parity P, charge C and time T. But quantum systems are, in general, only invariant with respect to the combination PCT. For example, parity violation with only left-handed neutrinos, but right-handed antineutrinos with PC-symmetry satisfies PCT-symmetry. The PCT-theorem is a very general result of quantum mechanics. But PCT-violation is not excluded forever. T-violation during the decay of Kaons are first hints.

From symmetry breaking to complexity in dynamical systems

In classical physics, every orbit in a configuration space possesses one and only one time-reversed counterpart: Let $s(t)$ be the time-depending state (e.g. in Hamiltonian mechanics $s(t) = (q_i(t), p_i(t))_{i=1 \dots 3N}$ a point in the 6N-dimensional phase space), then its time-reversed orbit $s_T(-t) = (q_i(-t), -p_i(-t))$ is a solution of the equations of motion too. In quantum mechanics, $s(t) = \psi(t)$ is the Schrödinger function, and $s_T(-t) = \psi^*(-t)$ is Wigner's time-reversal transformation. But in thermodynamics, we observe asymmetry of time. A glass of water falls down from a table, splits up in many parts, energy dissipates; but the time-reversal process was never observed. According to Clausius, irreversible processes are distinguished by increasing entropy: the change of the entropy S of a physical system during time dt consists of the change d_eS of the entropy in the environment and the change d_iS of the intrinsic entropy in the system, i.e. $dS = d_eS + d_iS$. For isolated systems with $d_eS = 0$, the second law of thermodynamics requires $d_iS \geq 0$ with increasing entropy ($d_iS > 0$) for irreversible thermal processes and $d_iS = 0$ for reversible processes in the case of thermal equilibrium.

According to Boltzmann, entropy S is a measure of the probable distribution of microstates of elements (e.g. molecules of a gas), generating a macrostate (e.g. a gas), i.e.

$$S = k_B \ln W$$

with k_B the Boltzmann-constant and W the number of probable distributions of microstates, generating a macrostate. According to the second Law, entropy is a measure of increasing disorder in isolated systems. The reversible process is extremely improbable. In a statistical description, irreversible processes are of the form

improbable state $\rightarrow$ probable state

However, for statistical reasons, there are just as many processes of the type

probable state $\rightarrow$ improbable state

If the entropy S of a state s is given by $S = F(s)$ with $F(s) = F(s_T)$ and T-symmetric dynamics, then Loschmidt's objection to Boltzmann's second law means that for every solution with $dS/dt > 0$ one has precisely another one with $dS/dt < 0$, and vice versa. In order to understand the thermodynamical arrow of time, one therefore has to explain the initial improbable state.

The explanation is delivered by quantum cosmology. According to unified theory, the expansion of the universe starts with an initial quantum state of supersymmetry, followed by symmetry breaking and generating elementary particles, leading to increasing diversity and complexity of galactic structures. High symmetry and order means a distinguished state of less entropy and, according to Boltzmann, less probability. Thus, the cosmic arrow of time can be explained by transitions from the improbability of high order ('symmetry') to the probability of increasing disorder and entropy ('symmetry breaking') without contradiction to the T-symmetric laws of physics.[6,7]

In the expanding universe of globally increasing entropy, local islands of new order and less entropy emerge like, e.g. stars, planets, and life with increasing diversity and complexity. The local emergence of order is made possible by phase transitions (symmetry breaking) of equilibrium states in open systems interacting with their environment. The emergence of order by symmetry breaking can be studied in many examples: a ferromagnet is a complex system of upwards and downwards pointing dipoles. During annealing the temperature to a critical value (Curie point), the old state becomes unstable and changes into a new state of equilibrium with one of two possible regular patterns of upwards or downwards pointing dipoles ('symmetry breaking'). In the same way, the emergence of ice crystals can be explained. During annealing to the freezing point, water molecules arrange themselves in regular patterns of rotational and mirror symmetries, breaking the full symmetry of a homogeneous distribution.

Order and structure do not only emerge by decreasing, but also by increasing energy.[9,17] In a Bénard-experiment, during heating a fluid from below, regular patterns of convection-rolls emerge spontaneously at a critical value of instability with two possible directions. The selection of order depends sensitively on tiny initial fluctuations ('spontaneous symmetry breaking'). In fluid dynamics and aerodynamics, one can study the emergence of new order and structure by symmetry breaking when a dynamical system is driven further and further away

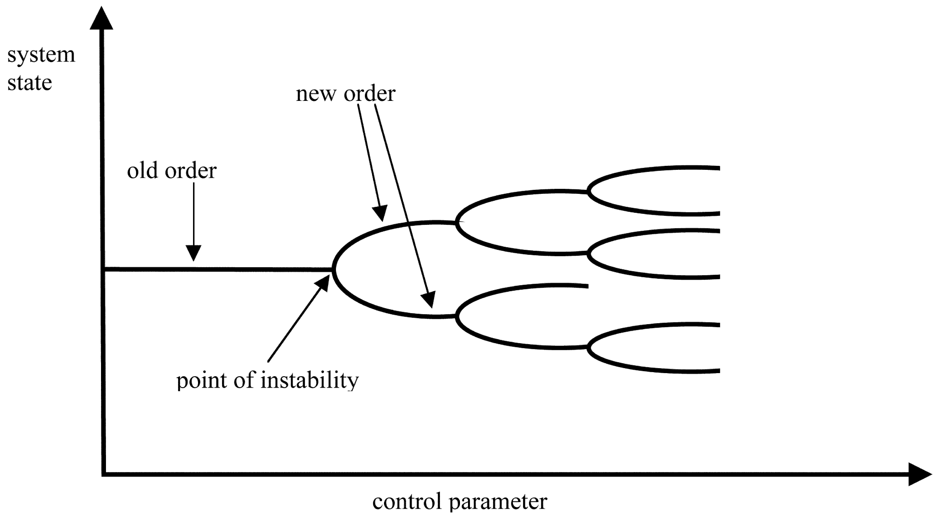
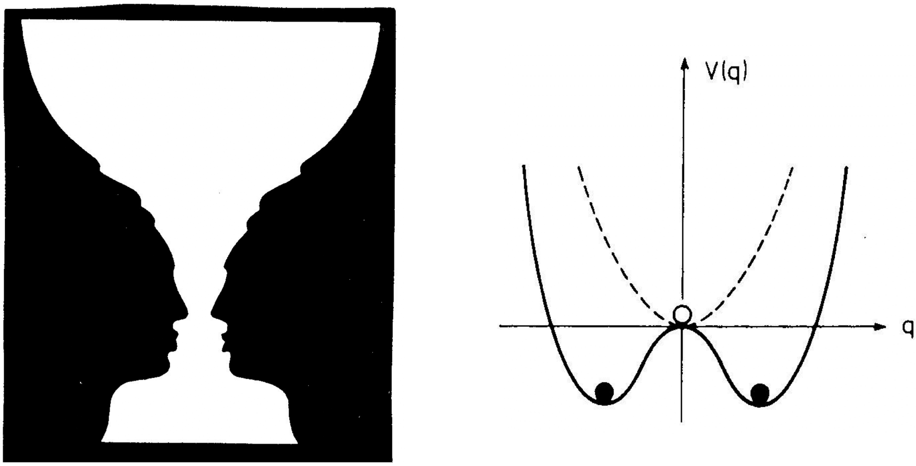


Figure 5. Bifurcation tree of symmetry breaking (non-equilibrium dynamics)

from thermal equilibrium ('non-equilibrium dynamics'). During increasing fluid velocity, old states of equilibria break down, and new fluid patterns in new states of local equilibria emerge with increasing complexity, from a homogeneous surface of a stream behind an obstacle (fixed point attractor) to oscillating vortices (limit cycles) and structures of turbulence (chaos). Symmetry breaking in non-equilibrium dynamics is a fundamental procedure in understanding the emergence of new order and structure with increasing complexity. In general, the emergence of order by symmetry breaking is explained by phase transitions of complex dynamical systems, which can be illustrated by a bifurcating tree of local equilibria (Figure 5). By increasing control parameters (e.g. temperature, fluid velocity), old equilibria become unstable at critical points of instability, break down and new branches of local equilibria with new order emerge, which, again, can become unstable etc.

The mathematical formalism of symmetry breaking in nonlinear dynamics does not depend on physical applications, but can be generalized for biological and even social models.[8–10] For example, Darwin's evolution of species can be represented as a bifurcation tree of local equilibria. The states of species become unstable by random fluctuations such as mutations, generating bifurcating branches. Selections are the driving forces in the branches for further local equilibria with new species of increasing complexity.

Complex organisms are generated with new functional symmetries, which are more or less optimally adapted to their ecological niches. An example is the mirror symmetry of flying, swimming, and walking animals.



(a) (b)

Figure 6. Symmetry breaking of cognition (a) and its mathematical model (b)

There is a remarkable relationship between the formation of order in nature and the recognition of order in brains by spontaneous symmetry breaking. Do we see a symmetric vase or two diametrically opposed faces with mirror symmetry (Figure 6(a))? The ambiguity of perceptions and the spontaneous decision of the cognitive system of the human brain for one interpretation is a psychological example of symmetry breaking, depending on tiny fluctuations of awareness for details, e.g. in the foreground or background of the picture.

Mathematically, spontaneous symmetry breaking means that, at the maximum of a potential, a state (e.g. position of a ball at the top of a symmetric curve) is symmetric but unstable, and tiny initial fluctuations decide which of the two possible stable states of minima the state will finally reach (Figure 6 (b)).

Even in social dynamics, the emergence of order can be modelled by symmetry breaking. For example, in economy, two competing firms provide a bifurcation of an initial equilibrium state into a winner and a loser. If, at a critical point of competition, a product has some tiny competitive advantages on the market, then the market leader will dominate in the long run and even enlarge his/her market shares without necessarily offering the better product: the winner takes all. Symmetry and complexity are general features of culture and society. In ancient cultures, social symmetry was identified with the cosmic symmetry of the universe. In modern societies, symmetry breaking and socio-economic transitions are related to critical instabilities and shifts of historical, economic, and political developments with increasing complexity.

Symmetry and complexity in computational systems

In the age of computers with increasing computational power, a fundamental relationship between dynamical systems and computational systems becomes obvious. Dynamical systems are modelled by time-dependent differential equations, which can be digitalized as the computational processes of computers (Ref. 8, chapter 7). Thus, in the tradition of G.W. Leibniz, the principle of computational equivalence demands that all processes in nature and society can be viewed as computations on universal computational systems (e.g. Turing machines). According to Church's thesis, there are several computational systems that are equivalent to Turing machines. Cellular automata have turned out to be appropriate discrete and quantized models of complex systems with nonlinear differential equations describing their evolution dynamics.

Imagine a chessboard-like plane with cells. A state of a one-dimensional cellular automaton consists of a finite string of cells, each of which can take several states (e.g. 'black' (0) or 'white' (1)) and is connected to a neighbourhood of cells (e.g. only to its two nearest neighbours), with which it exchanges information about its state. The following (later) states of a one-dimensional automaton are the following strings on the space–time plane, each of which consists of cells taking one of the allowed states, depending on their preceding (earlier) states and the states of their distinguished neighbourhood. In the case of binary states and two direct neighbours inclusively of the cell itself (i.e. a neighbourhood of three cells), there are $2^3 = 8$ possible rules of transition for each automaton. Each rule is characterized by the eight-digit number of the states that each cell of the following string can take. These binary numbers can be ordered by their corresponding decimal numbers. There are $2^8 = 256$ one-dimensional automata of this simple type. The time evolution of these simple rules already produces all kinds of complex patterns we observe in nature, regular and symmetric ones, random ones and complex patterns with localized structures.

Cellular automata can generate time-symmetric (reversible) and time-asymmetric (irreversible) patterns of behaviour like dynamical systems (Figure 7).[12] In the first class, cellular automata always evolve after finite steps to a uniform pattern of rest, which is repeated for all further steps in the sense of a fixed point attractor. The fixed-point dynamics do not depend on the initial states of the automata: All initial states lead to the same attractor. Therefore, we have no chance to go backwards from the attractor and reconstruct the initial states from which the automata actually started. In the second class, the development of repeated patterns is obviously reversible for all future developments. In random patterns of the third class, all correlations have decayed, and therefore, the evolution is irreversible. For localized complex structures of a fourth class, we have the chance

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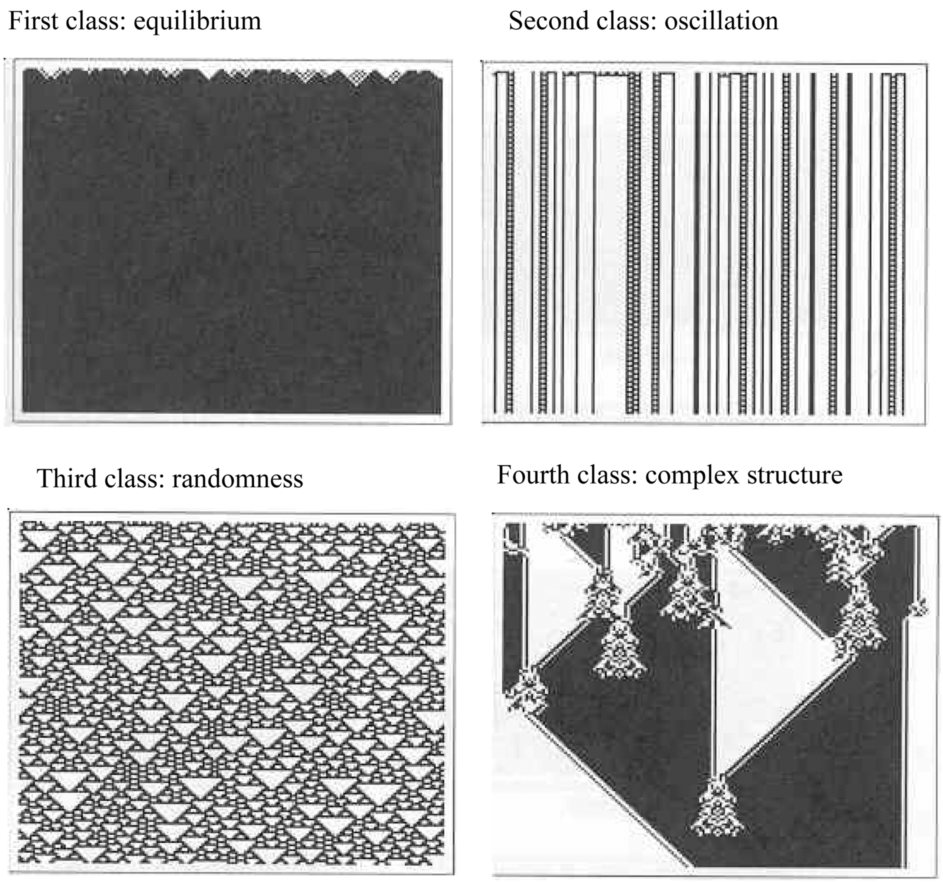

Figure 7. Cellular automata with reversible and irreversible dynamics

to recognize strange or chaotic attractors, which are locally complex and correlated patterns, like islands of complex order in a sea of randomness with complete loss of structure. In the third and fourth classes, the evolution of patterns sensitively depends on the initial states (like the butterfly effect in nonlinear dynamics). Obviously, these classes of cellular automata correspond to the increasing degrees of complexity in dynamical systems we discussed in the last section, from fixed point attractors to periodic oscillations (limit cycles) and finally chaos and randomness.

We can simulate the thermodynamical arrow of time and Loschmidt's reversibility objection by cellular automata with reversible rules. Reversible rules remain the same when turned upside-down. In this case, the rules are affected by the dependence on states two steps back. For example, let us take the eight rules of the cellular automaton with the decimal number 122 and add the eight additional reversible cases:

1	1	1	1	1	1	1	1
111	110	101	100	011	010	001	000
1	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0
111	110	101	100	011	010	001	000
0	1	1	1	1	0	1	0

According to the second law of thermodynamics, increasing disorder and randomness ('entropy') is generated from simple and ordered conditions of closed dynamical systems. On the macrolevel, irreversibility is highly probable in spite of the time-symmetric laws of molecular interactions on the microlevel ('microreversibility'). Some cellular automata with reversible rules generate patterns of increasing randomness, starting from simple and ordered initial conditions. For example, the reversible cellular automaton of rule 122R can start from an initial condition in which all black cells or particles lie in a completely ordered pattern at the centre of a box (Figure 8).[12] Running downwards, the distribution seems to become increasingly random and irreversible, in accordance with the second law.

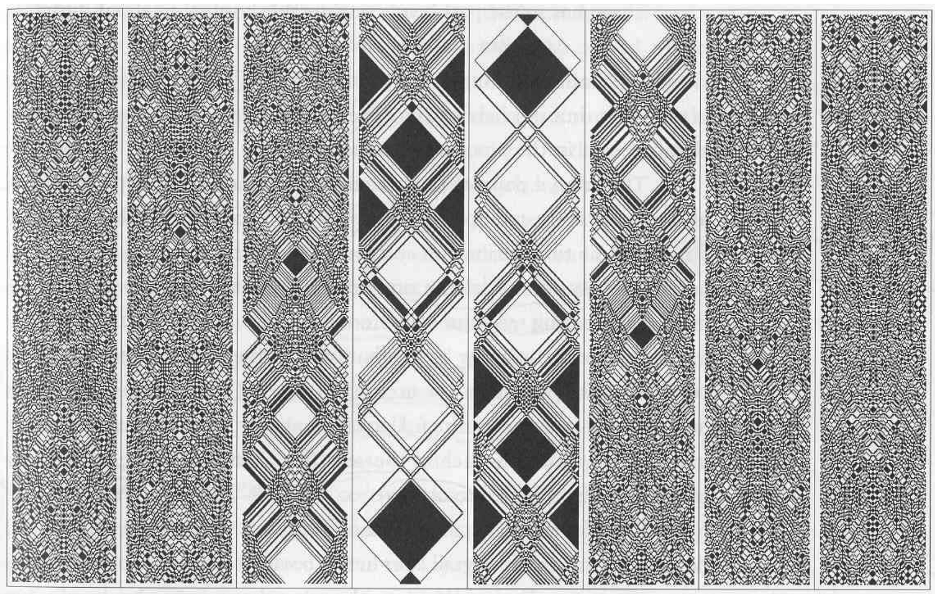

Figure 8. Second law of thermodynamics and computational irreversibility

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In principle, with respect to Loschmidt's reversibility objection, a symmetric solution is also possible. Tracing backwards through the actual evolution of a reversible system one can find initial conditions leading to decreasing randomness. But it can take an irreversible amount of computational work. In this case, there is no shortcut to predict future behaviour: we must wait for the actual development step by step. Further on, it is a remarkable fact of computational systems that, on the macrolevel, complex patterns with randomness, turbulence, and chaos emerge, although there is microreversibility at the microlevel of interacting cells, as in dynamical systems. Consequently, computational systems are not necessarily computable and decidable like dynamical systems in nature and society. In the case of randomness, there is no shortcut to the actual dynamics. Modern sciences of complexity are basically characterized by computational irreducibility. Even if we know all the laws of behaviour on the microlevel, we cannot predict the development of a random system on the macrolevel in a shorter way than the actual process. And that means: even if, one day, we will know all laws of symmetry in a unified theory, the world will be too complex for total computability.

Symmetry and complexity in the philosophy of science

Figure 9 shows a classification of symmetries we have discussed. There are the main distinctions of discrete symmetries (e.g. *P*, *C*, *T*-symmetry) and continuous

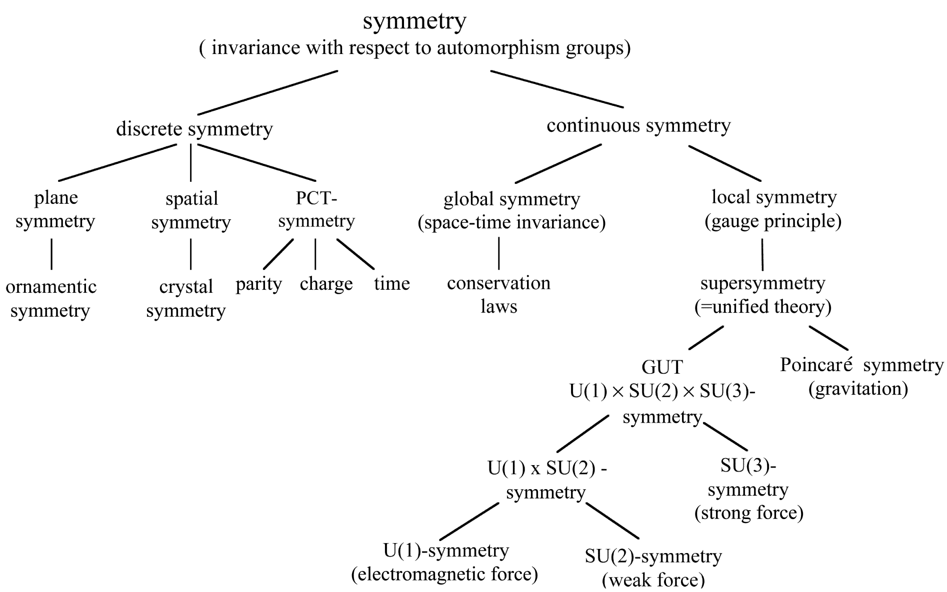

Figure 9. Classification of symmetry

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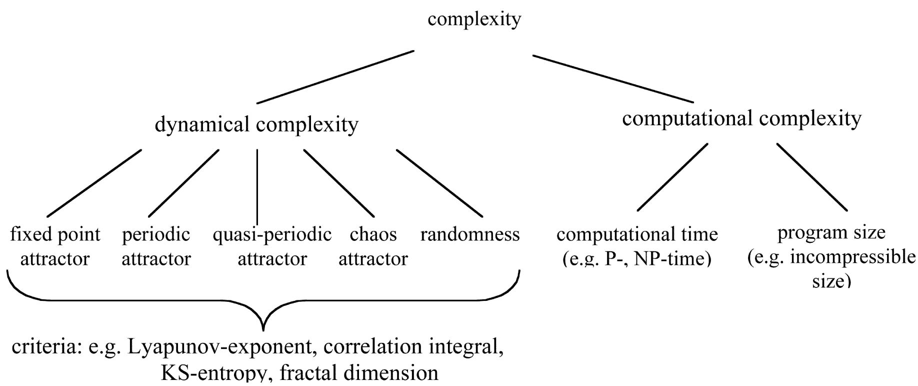

Figure 10. Further classification of complexity

symmetries, global symmetries (e.g. space–time symmetries and conservation laws) and local symmetries (gauge principle). Figure 10 is a classification of complexity with the main distinction of dynamical and computational complexity. Dynamical complexity refers to increasing digression from fixed point attractors to periodic and quasi-periodic behaviour (limit cycles), chaos attractors and finally randomness. Randomness means the complete decay of all correlations and complete irregularity. Thus, there is no chance of any prediction. In the case of deterministic chaos, predictions depend sensitively on initial data, which can be measured by Lyapunov-exponents. Another measuring method of complexity degrees is the fractal dimension of an attractor. In our context, it is remarkable that the fractal dimension measures the degree of symmetry in the sense of self-similarity. On different time scales, the fractal dimension determines the self-similarity of a time series (e.g. stocks, weather, climate), which is a necessary (but not sufficient) condition of a strange attractor. Computational complexity relates to computational time and program size in order to compute the equations and functions of dynamical systems.[9]

From a logical point of view, symmetry and complexity are syntactical and semantical properties of theories and their models (Figure 11). For example, consider a mathematized theory of a four-dimensional Riemannian manifold with an automorphism group of infinitesimal isometries and form-invariance of certain tensors. By an appropriate physical interpretation of the terms and quantities, we can get a model of a homogeneous and isotropic vector field of expanding galaxies which is, more or less approximately, confirmed by observational data of the real world. The expanding galaxies are represented by the model of a dynamical system satisfying the laws and axioms of a mathematized theory. Symmetry refers to group-theoretical form-invariance of mathematical terms and equations. In this sense, symmetries are meta-laws of mathematized theories.

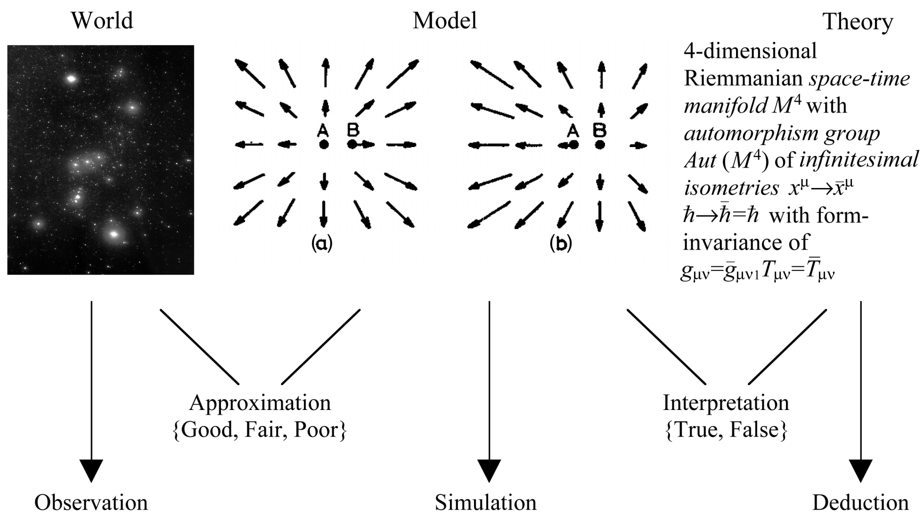

Figure 11. Symmetry and complexity as structural properties of theories, models and the world

According to the principle of computational equivalence, the models of dynamical systems can also be considered computational systems with different degrees of computability. A Turing machine can be interpreted in the framework of classical and relativistic deterministic physics. But stochastic computing machines and quantum computing machines, representing stochastic and quantum systems, do not compute functions in that sense. The output state of a stochastic machine is random, with only a probability function for the possible outputs depending on the input state. The output state of a quantum machine, although fully determined by the input state (e.g. by a Schrödinger equation), is not an observable and so the observer cannot in general discover its label. In general, computational systems must not be computable and decidable. Uncertainty, uncomputability, undecidability, and incompressibility of information are fundamental insights of modern sciences of complexity.

The epistemic question arises of whether symmetry and complexity are only syntactical and semantical properties of scientific theories and their models, or are they real structures of the world. Empirical structuralism defends a strict empirical view:[13] symmetry and complexity only refer to syntactical and semantic properties of mathematical structures, which are inventions of the human mind. But if they are only syntatical and semantical constructions, why do observations, measurements and predictions display these regularities? It seems to be a wonder or a miracle. Putnam put it in the 'no miracle-argument' of scientific realism: 'The positive argument for realism is that it is the only philosophy that doesn't make

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the success of science a miracle'.[14] But realism of what? Is it the entities, the abstract structural relations, the fundamental laws or what?

Structural realism assumes that mathematical structures refer to real structures of the world, independent of syntactical and semantical representations in the human mind. The question is which mathematical terms and models refer to ontological structures.[15] I have argued that gauge groups of local symmetries can be considered epistemic filters of observables. Only gauge-invariant quantities such as field forces have measurable effects. But what about field potentials that are not gauge-invariant? Are they only theoretical terms and inventions of the human mind, like the epicycles of Greek astronomy to 'save' the symmetry of the universe? In this case, the precise confirmation of field theories by modern accelerators seems to be a fantastic miracle. On the other hand, measurable field forces are derivations of field potentials, which also determine the freedom of gauging in a field theory.

Complex degrees of dynamical systems have been considered as attractors that are invariant structures of the corresponding phase spaces. Obviously, phase spaces are not observable, like height, depth and breadth of a room. They are mathematical constructions, referring to the state dynamics of a dynamical system. But attractors have measurable consequences on predicting future developments of dynamical systems. They seem to be miracles if we neglect structurally invariant features of their system dynamics. The principle of computational equivalence demands that dynamical systems can be considered computational systems with different degrees of complexity. Otherwise, the application of mathematics and computational procedures in science is an obscure miracle. Algorithmic procedures refer to structural features of dynamical processes with different degrees of complexity. Obviously, deterministic Turing machines are only approximations like deterministic dynamical systems in classical physics. But quantum computers refer to the dynamics of basic building blocks of the universe. Statistical procedures represent stochastic processes in nature and society. Thus, in the tradition of Leibniz, it seems to be not only a metaphoric *facon de parler* that even 'every organic body of a living being is a kind of divine machine or natural automaton surpassing all artificial automata infinitely' (*Monadology*).

On the other hand, one of the principal arguments against realism is the hypothetical status of theories and models: ontological commitments can be refuted by empirical tests. A mathematized theory may contain more or less structural relations and constraints than are satisfied by measurements and observations of the real world. Therefore, from an epistemic point of view, structures of the world remain more or less uncertain and indeterminate without denying their existence. My view of symmetry and complexity defends an epistemic structuralism between structural realism and empiristic structuralism.

Symmetries of theories and their models open insights into invariant structures of the world. Symmetry-breaking opens insights into the variety and complexity of the world. There seems to be a complementary relationship of symmetry and symmetry breaking that was already recognized by the pre-Socratic philosopher Heraclitus who emphasized: 'What is opposite strives toward union, out of the diverse there arises the most beautiful harmony.'[16]

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