THE COMPUTATION OF MEAN POSITIONS AND PROPER MOTIONS

Carl S. Cole
U. S. Naval Observatory
Washington, DC

ABSTRACT: This paper investigates the estimation of mean positions and proper motions given independent solar-system barycentric positions observed at different epochs, reduced to the same equator and equinox and freed of systematic errors. Past practices are reviewed and the relative quality of the data is studied to determine the appropriate form of the model.

1. INTRODUCTION

A program is in progress at the U.S. Naval Observatory which will result in a new fundamental catalog of approximately 40,000 stars. This catalog will establish a dynamical reference frame of stellar positions and proper motions based on conventional values of astronomical constants and a new analysis of observations of solar system objects incorporating corrections to current planetary theories. The catalog will also provide a more direct connection between the optical and radio reference frames by combining the current fundamental stars (FK5) and fainter reference stars into a single fundamental system. As part of this work, an investigation into the method of estimating mean positions and proper motions was conducted.

2. PAST PRACTICES

Traditional practice has been summarized by Newcomb (1906). One uses initial estimates of the position and proper motions (α_0, δ_0, μ and μ') and computes corrections to these quantities ($\Delta\alpha_0$, $\Delta\delta_0$, $\Delta\mu$ and $\Delta\mu'$). For each independent observation, observed minus computed residuals ($\Delta\alpha_\iota$ and $\Delta\delta_\iota$) are calculated. Condition equations are formed:

$$\Delta\alpha_\iota = \frac{d\alpha}{d\alpha_0}\Delta\alpha_0 + \frac{d\alpha}{d\delta_0}\Delta\delta_0 + \frac{d\alpha}{d\mu}\Delta\mu + \frac{d\alpha}{d\mu'}\Delta\mu',$$

$$\Delta\delta_\iota = \frac{d\delta}{d\alpha_0}\Delta\alpha_0 + \frac{d\delta}{d\delta_0}\Delta\delta_0 + \frac{d\delta}{d\mu}\Delta\mu + \frac{d\delta}{d\mu'}\Delta\mu' \tag{1}$$

and solved for by weighted least squares. The partial derivatives being evaluated at the epoch of observation.

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Newcomb used this method for 4 northern polar stars in his "Catalogue of Fundamental Stars" (Newcomb 1898). Unfortunately, this is the only catalog compilation solving for both coordinates simultaneously, subsequent compilers either being unaware of or ignoring Newcomb's work.

Several simplifying approximations in the application of this method are still in general use:

$$\frac{d\delta}{d\alpha_0} = \frac{d\alpha}{d\delta_0} = \frac{d\delta}{d\mu} = \frac{d\alpha}{d\mu'} = 0,$$

$$\frac{d\alpha}{d\alpha_0} = \frac{d\delta}{d\delta_0} = 1 \text{ and} \tag{2}$$

$$\frac{d\alpha}{d\mu} = \frac{d\delta}{d\mu'} = \tau_i$$

where τ_i is the epoch difference between the observation and initial estimate. Thus, the two coordinates are decoupled and solved for separately. Although the use of normal points to combine data is no longer common, many authors still use catalog weights, rather than residuals, to determine error estimates (cf. Cole in press).

With recent advances in computational ability, the assumptions of equations (2) are no longer necessary. Once the relationships between the observed coordinates and the star parameters are stated, they can be linearized similar to equations (1) and the star parameters, with the associated covariance matrix, can be estimated using least squares. However, the use of subjective judgment is still required to form these relationships.

3. THE FORM OF THE MODEL

The question arises how best to model the relationship between the coordinates (α and δ) at any arbitrary epoch and the relevant star parameters (α_0, δ_0, μ and μ'). (It is assumed that the measured epoch of observation is exact.) Several options are uniform motion in each coordinate, uniform motion on a great circle, uniform rectilinear motion and circular motion about the center of the galaxy.

The rotation of the galaxy and the finite speed of light (cf. Stumpff 1985) were ignored and uniform rectilinear motion was assumed. The ignored effects were deemed insignificant given the time span and accuracy of the observations. The problem with the model of uniform rectilinear motion is that six parameters are needed to describe the motion, and parallax and radial velocity data are scarce. One could estimate six parameters from the positional data given at least six angular measurements, but this would reduce the error degrees of freedom by two, and would give very poor estimates of the radial parameters.

Schlesinger (1917) suggested solving for radial velocities given accurate parallaxes and Eichhorn (1981, 1982) suggested solving for parallaxes given accurate radial velocities. If radial data (parallaxes and radial velocities) were available, they could be combined with the angular measurements into a single adjustment. This approach was rejected for use in the current program. The angular measurements are often determined to one part in 10^7, whereas the radial data are sometimes known only to one part in ten or one part in a hundred. Thus the weights can easily vary by ten orders of magnitude. In combining data of unequal weight, a factor of two error in the relative weights can lead to useless results. The weights can be iteratively determined, but then one must form a data base of all parallax and radial velocity measurements made over the last century. One must also become intimately familiar with the various systematic errors of these data.

The situation remains that the best methods of computing parallaxes and radial velocities are the conventional ones. In computing the mean position and proper motions, therefore, one should use modern, accurate values for the parallax and radial velocity and assume them to be exact. In the event that either value is unavailable, one should set it equal to zero.

Rigorous formulae for expressing the two spherical coordinates in terms of the initial position, proper motions, parallax and radial velocity under the assumption of uniform rectilinear motion are given by Eichhorn and Rust (1970) and will not be repeated here.

With the longer time span and greater accuracy of future observations, one can conceive of solving for positions, proper motions, parallaxes, radial velocities, galactic rotation, the speed of light, etc. in a single adjustment. The size and complexity of data adjustments will continue to grow, but this growth will require the matching of computational ability with theoretical knowledge and, as always, careful analysis of the data.

4. PRACTICAL CONSIDERATIONS

There are several interesting consequences of solving for both coordinates in a single adjustment. The error sums of squares and degrees of freedom are pooled, resulting in more degrees of freedom when making tests of hypotheses such as the determination of outliers. Also, the variance of the mean squared error is reduced, which gives rise to fewer cases of estimates of zero error. The possibility of estimates of zero error is the leading argument for estimating errors from catalog weights rather than from residuals.

The combined adjustment also results in a single mean (or central) epoch. Thus all four star parameters are correlated with each other. But this only tells us in a quantitative way what we already know: one coordinate is not independent of the proper motion in the other coordinate. But, as we also know, these correlations are very small except in special situations.

Since the sum of the residuals of both coordinates is being minimized, the error estimates of the coordinates are very nearly equal given similar numbers of observations and weights. One could argue that agreement in one coordinate would be adversely affected by scatter in the other. But it is generally agreed that very small error estimates are artificial. Also, a better job of outlier rejection is done with the combined error sum of squares and degrees of freedom.

Several caveats are now in order. The right ascension residuals must be scaled to the declination residuals by multiplication by the cosine of the declination. The declination used to scale the right ascension should be a fixed, reference declination and not allowed to vary in the adjustment. Care must be taken with units. Angular data must be expressed in radians and parallax and radial velocity must be appropriately scaled. Along with having to estimate the relative quality of the various input catalogs, the investigator must now also be concerned with the relative quality of the two angular measures within a catalog.

Two methods of adjustment have been tried, both using commercially available software. One method (IMSL 1987) uses a nonlinear regression routine which approximates the partial derivatives using finite differences and a second method (REDUCE 1987) uses an algebraic manipulator to give an exact representation of the partial derivatives. In comparing these methods with conventional ones, all results agree within the estimated errors.

5. SUMMARY

In solving for mean positions and proper motions, the model used is uniform, linear motion in 3-dimensional space. Positions and proper motions are solved for using fixed values for the parallaxes and radial velocities. This uniform rectilinear motion model leads to non-linear condition equations which require linearization and initial estimates of the star parameters. Both coordinates are solved for in a single adjustment, resulting in a single mean epoch and a pooling of the error sums of squares and degrees of freedom.

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