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A NOTE ON A SELF INJECTIVE RING

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The purpose of this note is to give a direct proof of Koh's main theorem in [2, p. 31] applying well-known theorems of Faith and Utumi. While doing so, we shall mention a general theorem from which Koh's theorem may be obtained as a corollary.

For definitions and notation we refer the reader to [2].

THEOREM. *Let R be a semiprime ring which satisfies the maximum condition on annihilator right ideals and is right injective. Then R is semisimple and artinian.*

Proof. Suppose the Jacobson radical, $J(R)$ is nonzero. Since R is right injective, the right singular ideal of R is same as the Jacobson radical [1, p. 47] and hence is nonzero. Since R has maximum condition for annihilator right ideals, the right singular ideal is a nil ideal [1, p. 83]. Then by a lemma in [1, p. 82] R contains a nonzero nilpotent ideal, which contradicts that R is semiprime. Thus we have $J(R)=0$. This implies that R is a regular ring [3, Theorem 1]. Since principal right ideals in a regular ring are generated by idempotents they are certainly annihilator right ideals, for if eR is a principal right ideal generated by an idempotent e, then $(1-e)_r=eR$. Because of the maximum condition on annihilator right ideals R has the maximum condition on principal right ideals. So R is semisimple and artinian [1, p. 76].

COROLLARY [Koh]. *Let R be a prime ring which satisfies the maximum condition on annihilator right ideals and is right injective. Then R is a simple ring with minimum condition on right ideals.*

Proof. Since R is prime, R has no central idempotents other than 0 and 1, for if e is a central idempotent and if $e\neq 0$ and 1 then $eR(1-e)R=0$ which implies $e=0$ or $1-e=0$, since R is prime. Now, by virtue of the previous theorem R is semisimple and artinian and hence every ideal in R is generated by central idempotent. So R has no proper ideals. Thus the result follows.

An analogue of the above theorem can also be proved by weakening the hypothesis of right injectivity by divisibility and by strengthening the maximum condition by Goldie's maximum conditions. A ring R is said to be divisible if every regular element (nonzero divisor) is a right unit. This implies every regular element is a unit. Right injective rings are certainly divisible. Since divisible rings are their own two-sided quotient rings, by virtue of a classical result of Goldie we have

THEOREM *Let R be a divisible ring and satisfy Goldie's right or left maximum conditions. Then R is semisimple and artinian if R is semiprime.*

References

1. C. Faith, *Lectures on injective modules and quotient rings*, *Lecture notes in mathematics*, No. 49, Springer-Verlag, New York, 1967.

2. K. Koh, *A note on a self-injective ring*, Canad. Math. Bull. **8** (1965), 29–32.

3. Y. Utumi, *On continuous regular rings and semi-simple self-injective rings*, Canad. J. Math. **12** (1960), 597–605.

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