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FREE PRODUCTS OF TOPOLOGICAL GROUPS WITH CENTRAL AMALGAMATION

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In 1950 M.I. Graev introduced the concept of a free product of topological groups and proved the following significant result.

THEOREM. *The free product* $G \amalg H$ *of topological groups* G *and* H *is Hausdorff if and only if* G *and* H *are Hausdorff.*

Since then much work has been done on analyzing the structure of $G \amalg H$.

In this thesis, the notion of amalgamated free product of topological groups is introduced, and free products with central amalgamation are studied. The most important result proved is the

THEOREM. *Let* G *and* H *be Hausdorff topological groups with a common closed central subgroup* A . *Then the free product* $G \amalg_A H$ *of* G *and* H *with amalgamated subgroup* A *is Hausdorff.*

In order to prove this theorem we find it necessary to first develop a theory of amalgamated direct products of topological groups. This topic is of interest in itself and of vital importance for amalgamated free products. The other important ingredient in the proof of the above result is a nice representation of elements in $G \amalg_A H$. This representation only works in the case that A is central and hence our restriction to central amalgamations. We show that outside the central case a different technique would be needed.

Having settled the existence and Hausdorffness of $G \amalg_A H$ we

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proceed to describe its topology investigating such properties as local compactness. It is surprising to find that, unlike the situation for free products of topological groups, $G \coprod_A H$ can be a non-discrete locally compact group. We find necessary and sufficient conditions for $G \coprod_A H$ to be a Lie group, a complete metric group and a maximally almost periodic group.

A full description of the topology of $G \coprod_A H$ is given when G, H and A are k_ω-groups. In particular it is shown that for k_ω-groups G and H, the free product $G \coprod_A H$ with central amalgamation, is homeomorphic to $(G \times_A H) \times F(G/A \wedge H/A)$, where $G \times_A H$ is the direct product of G and H with A amalgamated, and $F(G/A \wedge H/A)$ is the free topological group on the smash product of G and H.

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