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Letter to the Editor

Envelope solitons at a plasma–vacuum interface

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Abstract. It is shown that a nonlinearsurface plasma wave at a plasma–vacuum interface can propagate in the form of a dark/grey envelope soliton. The latter is associated with a subsonic density cavity, which traps the complex surface wave electric field.

Surface waves [1–6], which are ubiquitous wave phenomena, have been studied in diverse areas of physics, such as acoustics, plasmas, material sciences and ocean physics, as well as biological sciences. Surface plasmons (SPs) propagate along the surface of a conductor [7, 8]. SPs have been explored recently in view of their potential applications in magneto-optic data storage, microscopy and Solar cells, as well as for constructing sensors for detecting biologically interacting molecules. Surface wave solitons occurring at the interface between a dielectric medium (air) and a nonlinear material are also observed in laboratory experiments [9, 10].

Surface plasma waves (SPWs) [11–16] propagating at a plasma-dielectric/vacuum interface are intriguing phenomena in plasmas. The localization of SPWs at the plasma surface is caused by the nonlinear effects involving the ponderomotive force of the SPW, which drives electron density perturbations at a slow time scale. About 30 years ago, Yu and Zhelyazkov [17] considered the nonlinear coupling between negative group dispersive SPWs and low-frequency ion-acoustic perturbations, and reported the possibility of supersonic bright SPW envelope solitons. The latter are composed of a bell-shaped electric-field envelope of the SPWs and an associated density hump created by the ponderomotive force of the SPWs. Thus, the features of the bright SPW envelope soliton are different (similar) from (to) the Langmuir/electromagnetic envelope solitons [18–21] (upper-hybrid envelope solitons [22–24]) in bulk plasmas.

Our objective here is to point out the possibility of subsonic dark and grey type envelope solitons [25–29] of SPWs. A negative group dispersive subsonic dark/grey surface plasma wave envelope soliton is composed of a complex SPW electric-field envelope, which is trapped in a self-created density hole at the plasma–dielectric interface.

Let us consider the amplitude modulation of a short wavelength (in comparison with the electron skin depth c/ω_{pe}, where c is the speed of light in vacuum and ω_{pe}

is the electron plasma frequency) SPW with the frequency [17]

$$\omega \approx \frac{\omega_{\rm pe}}{\sqrt{2}}\left(1-\frac{\omega_{\rm pe}^2}{8k^2c^2}\right), \tag{1}$$

where k is the wave number (along the direction of the plasma–dielectric interface). The equation governing the slowly varying envelope of short-wavelength SPWs in the presence of low-frequency (in comparison with the ion plasma frequency) and long wavelength (in comparison with the electron Debye length) density perturbations is [17]

$$i\left(\frac{\partial}{\partial\tau}+V_{\rm g}\frac{\partial}{\partial\xi}\right)E_z - P\frac{\partial^2 E_z}{\partial\xi^2} - \frac{\omega_{\rm p}}{2\sqrt{2}}\frac{n_{\rm e1}}{n_0}E_z = 0, \tag{2}$$

where $\partial E_z/\partial\tau \ll \omega E_z$, $\partial E_z/\partial\xi \ll kE_z$, E_z is the wave electric field along the surface (directed along the ξ direction), $V_{\rm g} \approx \omega_{\rm p}^3/4\sqrt{2}k^3c^2$ is the group velocity of the SPW, $P \approx 3\omega_{\rm p}^3/8\sqrt{2}k^4c^2$ represents the group dispersion, $\omega_{\rm p} = (4\pi n_0 e^2/m_{\rm e})^{1/2}$ is the unperturbed plasma frequency, n_0 is the equilibrium electron number density, e is the magnitude of the electron charge and $m_{\rm e}$ is the electron mass. The slowly varying time and space (along the $z-$ direction at the plasma surface) variables in (2) are denoted by τ and ξ, respectively. Furthermore, the electron number density perturbation associated with the low-phase speed (in comparison with the electron thermal speed) ion-acoustic perturbation (IAP) is denoted by $n_{\rm e1}(\ll n_0)$. It is obtained from the inertialess electron equation of motion

$$\frac{e^2}{2m_{\rm e}\omega_{\rm p}^2}\frac{\partial |E_z|^2\exp(-2kx)}{\partial\xi} = e\frac{\partial\varphi}{\partial\xi} - \frac{k_{\rm B}T_{\rm e}}{n_0}\frac{\partial n_{\rm e1}}{\partial\xi}, \tag{3}$$

where x is the direction normal to the surface, φ is the electrostatic potential associated with the IAP, $k_{\rm B}$ is the Boltzmann constant and $T_{\rm e}$ is the electron temperature. The left-hand side of (3) represents the ponderomotive force of the SPW.

The electrons are coupled to the ions via the electrostatic potential. The equations governing the ion dynamics supporting IAP are

$$\frac{\partial n_{\rm i1}}{\partial\tau} + n_0\frac{\partial u}{\partial\xi} = 0, \tag{4}$$

and

$$m_{\rm i}\frac{\partial u}{\partial\tau} = -e\frac{\partial\varphi}{\partial\xi} - \frac{k_{\rm B}T_{\rm i}}{n_0}\frac{\partial n_{\rm i1}}{\partial\xi}, \tag{5}$$

where $n_{\rm i1}$ $(\ll n_0)$ is the ion number density perturbation, u is the ion fluid velocity, $m_{\rm i}$ is the ion mass and $T_{\rm i}$ is the ion temperature. The ponderomotive force of the SPWs acting on the ion fluid is smaller by a factor $m_{\rm e}/m_{\rm i}$ (in comparison with that on the electrons), and therefore ignored in (5).

We now combine (3)–(5) by using the quasi-neutrality condition $n_{\rm e1} = n_{\rm i1}$, to obtain the driven ion-acoustic wave equation

$$\left(\frac{\partial^2}{\partial\tau^2} - V_{\rm s}^2\frac{\partial^2}{\partial\xi^2}\right)\frac{n_{\rm e1}}{n_0} = \frac{\exp(-2kx)}{4\pi n_0 m_{\rm i}}\frac{\partial^2|E_z|^2}{\partial\xi^2}, \tag{6}$$

where $V_{\rm s} = [k_{\rm B}(T_{\rm e}+T_{\rm i})/m_{\rm i}]^{1/2}$ is the effective ion-sound speed. Equation (6) generalizes (18) of [17] to include the finite ion temperature effect. The latter is very important in a quasi-stationary limit in which the ion inertia is ignored, and the electric force ($-e\partial\varphi/\partial\xi$) on the ion fluid is balanced by the ion pressure gradient along the surface. In fact, for $\partial^2 n_{e1}/\partial\tau^2 \ll V_s^2 \partial^2 n_{e1}/\partial\xi^2$, we have from (6)

$$n_{e1} = -\frac{\exp(-2kx)|E_z|^2}{4\pi k_B T}, \tag{7}$$

where $T = T_e + T_i$. The expression (7) shows how the electron density cavity is created by the ponderomotive force of the SPW.

We can now seek stationary solutions of (2) and (6) by supposing that $n_{e1} = n_1(\eta)$ and $E_z = E(\eta)\exp[-i\theta(\tau) + i\psi(\xi))$, where $\eta = \xi - V_g\tau$ and $\theta(\tau)$ and $\psi(\xi)$ are slowly varying functions of time and space, respectively. Hence, we have from (6)

$$n_{e1} = -\frac{\exp(-2kx)|E|^2}{4\pi k_B T(1 - M^2)}, \tag{8}$$

where $M = V_g/V_s$ is the Mach number. For $M \ll 1$, the expression for n_{e1} from (8) can be inserted into (2) to obtain the cubic nonlinear Schrödinger equation

$$P\frac{\partial^2 E}{\partial\eta^2} - \lambda E - Q|E|^2 E = 0, \tag{9}$$

where $\lambda = \partial\theta/\partial\tau$ is the frequency shift and $Q = \omega_p \exp(-2kx)/8\sqrt{2}\pi n_0 k_B T$. Since the third term in the left-hand side of (9) is negative, (9) admits both dark and grey type envelope surface wave solitons consisting of a density dip in which the complex SPW envelope is trapped. Explicit analytical solutions for the dark and grey envelope solitons are presented in, e.g., [25, 27–29].

To summarize, we have pointed out the possibility of subsonic/standing dark and grey SPW envelope solitons at a plasma-dielectric/vacuum interface. The present envelope soliton is composed of a density cavity, which traps the modulated wave envelope of the SPW. The nonlinear structure, as reported here, may be useful for the transport of localized plasmonic energy along the plasma surface.

References

[1] Agranovich, V. M. 1975 *Sov. Phys. Usp.* **18**, 99.
[2] Agranovich, V. M., Babichenko, V. S. and Chernyak, V. Ya. 1981 *JETP Lett.* **32**, 512.
[3] Agranovich, V. M. and Mills, D. L. 1982 *Surface Polaritons*. Amsterdam: North-Holland.
[4] Raether, H. 1988 *Surface Plasmons on Smooth and Rough Surfaces and on Gratings (Springer Tracts in Modern Physics, 11)*. New York: Springer.
[5] Vukovic, S. 1996 *Surface Waves in Plasmas and Solids*. Singapore: World-Scientific.
[6] Vladimirov, S. V., Yu, M. Y. and Tsytovich, V. N. 1994 *Phys. Rep.* **241**, 1.
[7] Barnes, W. L., Dereux, A. and Ebbesen, T. W. 2003 *Nature (London)* **424**, 824.
[8] Diaconsu, B. et al. 2007 *Nature (London)* **448**, 57.
[9] Wang, X. et al. 2007 *Phys. Rev. Lett.* **98**, 12 3903.
[10] Alfassi, B. et al. 2007 *Phys. Rev. Lett.* **98**, 21 3901.
[11] Ritchie, R. H. 1963 *Prog. Theor. Phys. (Kyoto)* **29**, 607.
[12] Kaw, P. K. and McBride, J. B. 1970 *Phys. Fluids* **13**, 1784.
[13] Gradov, O. M. and Stenflo, L. 1983 *Phys. Rep.* **94**, 111.
[14] Brodin, G. and Gradov, O. M. 1991 *J. Plasma Phys.* **46**, 459.
[15] Lundberg, J. and Brodin, G. 1995 *Phys. Rev. Lett.* **74**, 1994.
[16] Kumar, G. and Tripathi, V. K. 2007 *Appl. Phys. Lett.* **91**, 161 503.
[17] Yu, M. Y. and Zhelyazkov, I. 1978 *J. Plasma Phys.* **20**, 183.

[18] Schamel, H., Yu, M. Y. and Shukla, P. K. 1977 *Phys. Fluids* **20**, 1286.
[19] Yu, M. Y. and Shukla, P. K. 1977 *Plasma Phys.* **19**, 889.
[20] Sharma, R. P. and Shukla, P. K. 1983 *Phys. Fluids* **26**, 87.
[21] Shukla, P. K. and Stenflo, L. 1984 *Phys. Rev.* A **30**, 2119.
[22] Kaufman, A. N. and Stenflo, L. 1975 *Phys. Scr.* **11**, 269.
[23] Stenflo, L. 1982 *Phys. Rev. Lett.* **48**, 1441.
[24] Shukla, P. K. and Yu, M. Y. 1982 *Phys. Rev. Lett.* **49**, 696.
[25] Hasegawa, A. and Tappert, F. 1973 *Appl. Phys. Lett.* **23**, 112.
[26] Hasegawa, A. 1990 *Optical Solitons in Fibers*. Berlin: Springer.
[27] Fedele, R. and Schamel, H. 2002 *Eur. Phys. J.* B **27**, 313.
[28] Fedele, R. 2002 *Phys. Scr.* **65**, 502.
[29] Kourakis, I., Lazarides, N. and Tsironis, G. P. 2007 *Phys. Rev.* E **75**, 067601.