

# Dark Matter and the Tully–Fisher Law

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**Abstract:** I discuss the origin of the Tully–Fisher law in the context of the observed scaling laws for dark halos.

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## 1 The Tully–Fisher Law

Simple centrifugal equilibrium arguments for a self-gravitating disk (i.e. ignoring the contribution of the dark matter) give

$$L \propto V^4 / [I_o(M/L)^2]$$

where  $L$  is the luminosity of the disk and  $V$  its rotational velocity, and the central surface brightness  $I_o$  and the mass-to-light ratio ( $M/L$ ) are roughly constant from galaxy to galaxy. Observationally the observed exponent of  $V$  varies from about 3.2 in the  $B$ -band to about 4.5 at  $H$  (Sakai et al. 1999; see Figure 1). This probably reflects some dependence of  $I_o$  and  $M/L$  on luminosity (as for the tilt of the fundamental plane for elliptical galaxies).

The zero point of the Tully–Fisher (TF) law needs explaining. For example,

$$M_I = -10.00(\log W_{50} - 2.5) - 21.32$$

so a galaxy with  $M_I = -21.32$  has an inclination-corrected HI profile width  $W_{50} = 316 \text{ km s}^{-1}$ , and not some other quite different value. For a self-gravitating disk alone (i.e. ignoring dark matter), the zero point depends on the quantity  $I_o(M/L)^2$ . The  $M/L$  ratio depends on the stellar population of the disk. The central surface density  $\Sigma_o = I_o(M/L)$  depends on the mass  $M$  and angular momentum  $J$  for the disk:  $\Sigma_o \sim M^7/J^4$  (see for example Freeman 1970). The  $J(M)$  relation is defined by the dynamics of galaxy formation and evolution; it determines the zero-point of the TF law. Because of the well-known conspiracy for disks of normal surface brightness (i.e. the

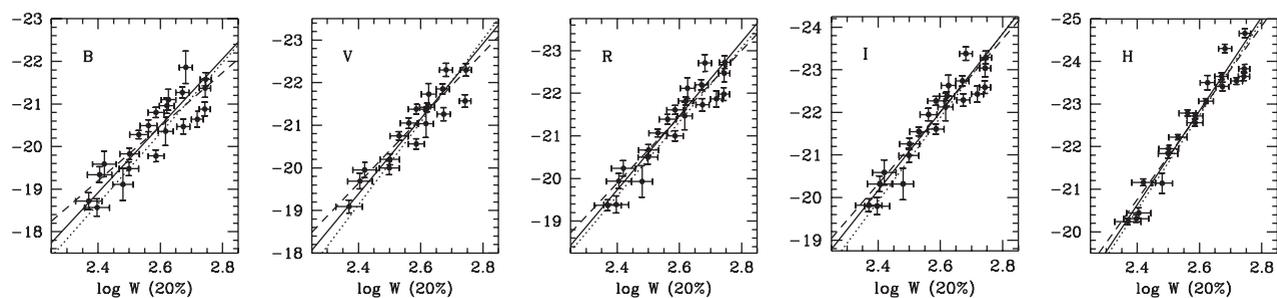
observation that the contributions to the rotation curve from the stellar disk and the dark halo are similar in magnitude), this argument is not changed much by the presence of the dark halo.

Now consider the low surface brightness (LSB) disk galaxies. In these systems, the gravitational field is believed to be dominated by the dark halo everywhere. But the observed TF law for LSB galaxies lies very close to the TF law for the high surface brightness galaxies, in slope and zero point (Zwaan et al. 1995; see Figure 2). This is a remarkable observation. In the LSB galaxies, the structure of the dark halo is primarily responsible for determining the velocity profile width  $W_{50}$ , while the baryons are entirely responsible for the observed stellar luminosity. Therefore, the baryon mass is related to the halo dynamics.

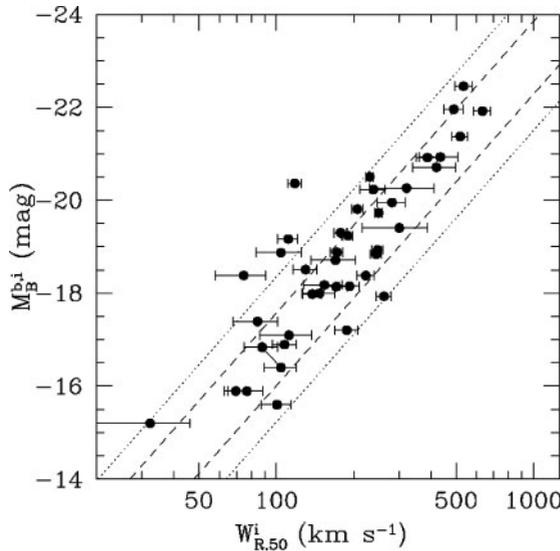
Why should this be? It is this relation between baryons and dark matter which makes the TF more interesting and complicated than the corresponding fundamental plane scaling law for ellipticals. The fundamental plane is an entirely baryonic phenomenon and is not really much more complicated than the virial theorem (plus some dependence of stellar population on the age and luminosity of the galaxy). It seems likely that the scaling laws for dark halos (e.g. Kormendy & Freeman 2004, hereafter KF04) are important for understanding the TF relation.

## 2 Dark Halo Scaling Laws

These scaling laws are the relationship of dark halo properties, such as the characteristic central density  $\rho_o$ , core



**Figure 1** TF data in the  $B$ - to  $H$ -bands, from Sakai et al. (1999). The slope of the relation steepens and the scatter diminishes from the blue to the NIR. The ordinate shows the absolute magnitude and the abscissa the log of the HI profile width ( $\text{km s}^{-1}$ ) at the 20% level.

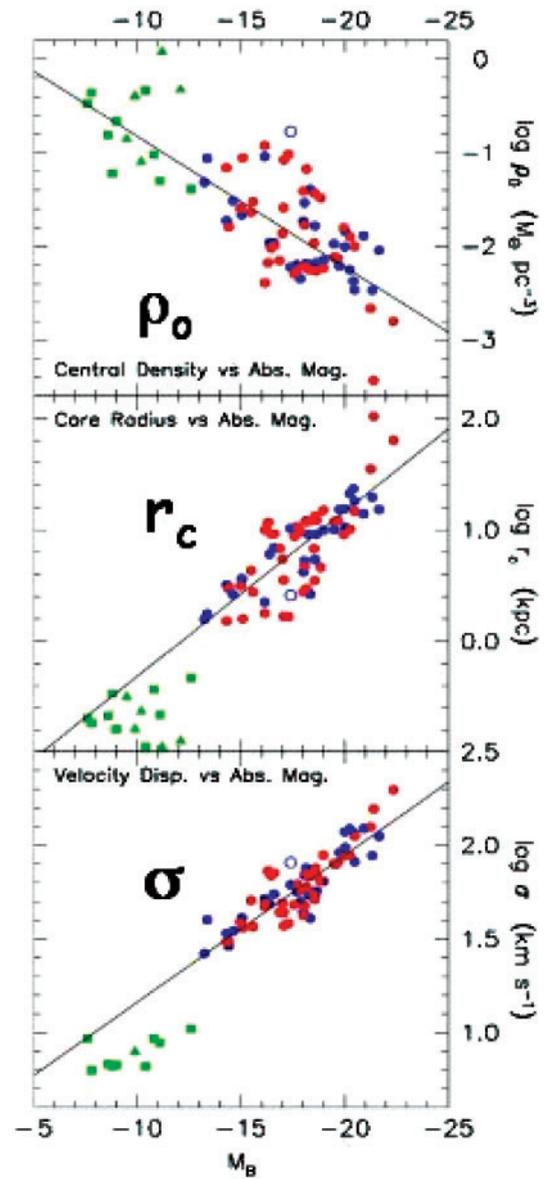


**Figure 2** The TF relation for a sample of LSB galaxies. The long and short dashes represent the  $1\sigma$  and  $2\sigma$  range of the TF fit to a sample of high surface brightness galaxies (from Zwaan et al. 1995).

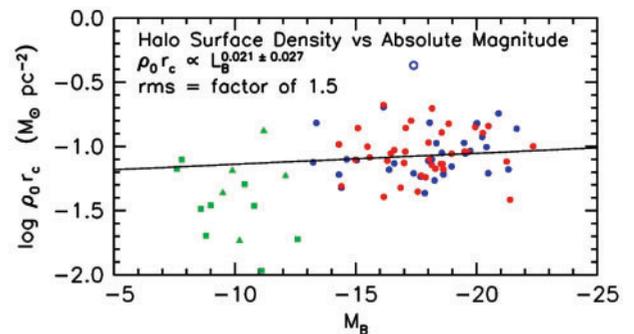
radius  $r_c$ , and velocity dispersion  $\sigma$ , with the luminosity of the parent galaxy. KF04 compiled a consistent sample of rotation curve data for Sc–Im galaxies. They modelled the dark halos as nonsingular isothermal spheres and adopted maximum disk decompositions to estimate the contribution of the disk to the observed rotation curve and then to derive the halo parameters  $\rho_o$ ,  $r_c$ , and  $\sigma$ . They also included some low-luminosity dIrr and dSph galaxies which are in pressure equilibrium, rather than centrifugal equilibrium, and for which only the halo density  $\rho_o$  can be derived.

The galaxies in the KF04 sample span about 15 mag in luminosity and show well-defined scaling laws in  $\rho_o$  and  $r_c$ . The central density decreases with luminosity as  $\rho_o \sim L_B^{-0.35}$  and the core radius increases with luminosity as  $r_c \sim L_B^{0.37}$  (Figure 3). The faintest galaxies have very dense dark halos, with central densities approaching  $1 M_\odot \text{pc}^{-3}$ . These faint dwarf galaxies are more numerous and more dominated by dark matter. They suggest the possibility that there is a large population of low-mass, completely dark objects. Such empty halos are likely to be small and dense, darker versions of the Draco and Ursa Minor dSph galaxies. Their high halo density would imply that they virialised early in the life of the universe, possibly before or during reionisation, and may have lost or never captured their share of baryons because the baryons were too hot at the time to be captured by these shallow potential wells.

The observed scaling laws for the dark halos are related to the slope of the power spectrum of the initial density fluctuations in the universe (e.g. Djorgovski 1992; Fall 2002). For understanding the TF law, probably the most significant aspect of the dark halo scaling relations is that their characteristic surface density is almost independent of luminosity:  $\rho_o r_c \sim L_B^{0.02 \pm 0.03}$  (see Figure 4). Simple equilibrium arguments show that systems whose surface density is independent of mass follow a Faber–Jackson



**Figure 3** The scaling laws for the characteristic central density  $\rho_o$ , core radius  $r_c$ , and velocity dispersion  $\sigma$  of dark halos, from KF04. See the text for further explanation. The different colors denote different sources of data.



**Figure 4** The scaling law for the characteristic surface density of dark halos, from KF04. The parameters  $\rho_o$ , and  $r_c$  are the core density and radius for the dark halos. See the text for further explanation. The different colors denote different sources of data.

relation of the form  $M \sim \sigma^4$  where  $\sigma$  is the characteristic velocity dispersion of the system.

### 3 Baryons and Dark Matter

So we now have a Faber–Jackson law for the dark halos, which has its origin in the slope fluctuation spectrum of the early universe. The law has the form

$$M_{\text{halo}} \propto \sigma^4 \propto V_{\text{rot,halo}}^4$$

where  $V_{\text{rot,halo}}^4$  is the characteristic circular velocity in the potential well of the dark halo. Then, if the ratio of baryon mass to dark mass is similar from galaxy to galaxy, a TF law with a slope of about four follows between the mass of baryons and the circular velocity in the halo potential. It is still necessary to get the zero point of the TF law right.

Some very gas-rich galaxies appear under-luminous for their HI line widths. For example, NGC 2915 and DDO 154 have dark to gas to stellar masses in the ratio (to order of magnitude) 100 : 10 : 1 and lie two to three magnitudes below the TF relation. If their gas content is notionally converted into stars with an  $M/L$  ratio of about 1, they rise to the TF relation (e.g. Freeman 1999). McGaugh et al. (2000) assembled a much larger sample of gas-rich galaxies and found that the relation between total *baryonic* mass and rotational velocity is a power law with a slope of about four, over about 4 dex in the baryonic mass. A similar relationship between *stellar* mass and rotational velocity follows the power law only over the upper 2 dex in stellar mass.

So the indications are that the TF law is a relationship between the total baryon content of disk galaxies and the

circular velocity of their dark halo. The conclusion at this stage is that the TF law comes about because of

- the power spectrum of the initial fluctuation spectrum which leads to the observed scaling laws for dark halos and hence to the Faber–Jackson law for the dark halos
- the approximate constancy of the baryon/dark matter ratio from galaxy to galaxy.

Very recent new results on the baryonic Tully–Fisher law are emerging from a study by Gurovich of low-luminosity gas-rich galaxies from the HIPASS survey (Gurovich 2004). His new data may modify the conclusions above.

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