within reasonable limits is most desirable; and since the one suggested by Juvenis is obviously calculated to effect this object, it appears to me well deserving of our support, and I for one shall hope to see a constant succession of questions or cases put forward, and some agreement arrived at as to the principles in accordance with which they should be solved; for it is in the difference of the principles adopted for the solution, and not in the mere calculation, that the discrepancies most commonly arise—that is to say, the discrepancies are of a logical rather than of a mathematical kind. This will, I dare say, appear in the solutions to the question proposed by Juvenis, supposing more than one solution to be given.

It is not unlikely that the proposer contemplates a strictly mathematical solution, involving a minute investigation of the probabilities of survivorship amongst the lives he has enumerated. That would not be the view taken by an actuary before whom the case came in the ordinary way for an opinion. He would seek to discover what sum could be safely invested in the purchase of such a reversion, and what it would cost to assure against the contingencies affecting it; and he would find in that point of view that such a reversion was worthless. Hence we see that in this case, as in almost all others, more than one solution can be given; and our attention is directed to the importance of ascertaining what interpretations can be put upon a question, and of guarding against a solution intended for one of them being mistaken for another.

I am, Sir,

Your obedient servant,

A FELLOW OF THE INSTITUTE.

ON THE VALUE OF OPTIONS.

To the Editor of the Assurance Magazine.

DEAR SIR,—In the last Number of your *Journal* a letter by Mr. Makeham is inserted, in which the writer endeavours to prove that the method which I had previously given for finding the single premium for a deferred annuity, with the condition that the premium shall be returnable (without interest) at death or, at the option of the purchaser, at any time before the annuity becomes payable, is defective, inasmuch as in his opinion it provides only for the deferred annuity and the return of the premium in the event of death; which return he assumes that I have made payable, with one year's interest thereon, at the end of the year in which the life fails.

My present object is, in the first place, to show that Mr. Makeham has entirely overlooked the very point on which alone the interest in the problem may be supposed to rest, as he "naively," but erroneously, remarks, "*that in assurances of this description the value of the policy always exceeds the premium paid upon it—a circumstance which does not depend upon the mode of computing the premium, but arises from the nature of the contingency itself*"; and in the second place I wish to point out, that in the solution which I have given the sum returnable is P_x, and is made *at the time of*, or *prior to*, death, and is not $P_x(1+i)$; nor is it made at the end of the year in which the life fails, as Mr. Makeham has assumed.

It can hardly be necessary for me to refute the assertion that "*the* *value of the policy always exceeds the premium paid upon it,*" as it must be obvious to all your readers that when, from the failing health of a person who has purchased an ordinary deferred annuity on his life, death is presumably certain to take place within the term for which the annuity was originally deferred, the value of such an assurance becomes absolutely *nothing;* in fact, if this were not so, the premium usually charged in such cases would be utterly inadequate to provide for the benefit secured.

Now it is just at this point, and under these circumstances, that, in the solution which I have given, it is assumed that the "option" will be exercised. In short, it is assumed that it will be exercised whenever (were there no option) the value of the policy would be *less* than the premium paid upon it, and then only. But this would take place not only when there was a high probability that the assured would die within the term of n years, but it would also obtain whenever it became evident that, although the annuitant might survive the term, the value of the annuity about to be entered upon would, from his then shattered health, be *less* than P_x, at which price nevertheless the value of such an assurance would, under the optional clause, be "arbitrarily fixed," notwithstanding Mr. Makeham's statement that the assured "*would be entitled in addition to an allowance from the Office for the surrender of the deferred annuity secured by the annual interest.*"

Of course the value in the former instance would not, as a rule, be very much less than P_x, but it is easy to conceive a case, in which it might be morally certain that a person would survive the term but yet not live to receive even the first year's payment of the annuity, when the value of the policy would fall to zero. In dealing with this problem the only difficulty which has ever been felt has been in assigning a value to the probability of a person requiring the repayment of his premium in anticipation of death, or when, from his general state of health, he would be unable to pass the usual medical examination which is necessary before surrendering an ordinary deferred annuity policy.

The formula $\dfrac{N_{x+n}}{(N_{x-1}-N_{x+n-1})(1-v)+D_{x+n}}$, given by Mr. Makeham in your last Number, provides only for the deferred annuity or the return of P_x *at the end of the year in which the life fails*, whilst the formula which I have given, $\left(\dfrac{N_{x+n}}{(N_x-N_{x+n})i+D_{x+n}}\right)$, will provide for the annuity or the return of P_x *at the moment of death* or *at any time previously*, whenever, from failing health or other circumstances, such a course may be deemed desirable; the latter being greater than the former, proves that the option has a *positive* value—a *negative* value is impossible.

Of the very small difference which the option made I was fully aware before Mr. Makeham's letter appeared; but as my primary object was to show the sufficiency of the premiums given by me as contrasted with those charged by the Government, I avoided trespassing on your valuable space, further than appeared necessary at the time, into questions which did not tend to elucidate that point.

I am, dear Sir,
Yours truly,

London, 25th August, 1865. J. W. STEPHENSON.

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