Convection in Astrophysics
Proceedings IAU Symposium No. 239, 2006
F. Kupka, I. W. Roxburgh & K. L. Chan, eds.

doi:10.1017/S1743921307000154

Coherent structures in granulation convection and their importance for higher order closure models

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Abstract. We use numerical simulations of granulation in the Sun and a K dwarf to study the effects of coherent structures on higher order moments. The latter need to be calculated in non-local Reynolds stress models of turbulent convection. Models that explicitly account for the asymmetry between up- and downflows as well as hot and cold drafts provide a substantial improvement over traditional ones, such as the quasi-normal approximation, which is only able to provide order of magnitude estimates for this type of flow.

Keywords. Convection, turbulence, Sun: atmosphere, interiors, stars: atmospheres, interiors

1. Introduction and motivation

Observations of solar surface granules and numerical simulations of stellar convection have shown that convection overshoots into stably stratified layers and is inhomogeneous and anisotropic, particularly in transition regions between stable and unstable stratification. None of these features are accounted for in local convection models. To overcome these difficulties non-local models of stellar convection have been proposed. They predict lower order moments of the ensemble averaged fluctuations of velocity, temperature, and density around their mean values as a function of location and time. The non-linearities in the hydrodynamical equations imply that the hierarchy of ensemble averaged equations is unclosed. Closure approximations truncate such moment expansions at third or fourth order. To avoid diffusion type approximations (cf. Kupka & Muthsam 2007) it has been suggested to consider the full dynamical equations up to third order (Canuto 1992, 1993, 1997; Grossman & Narayan 1993; Xiong *et al.* 1997). This requires knowledge of the fourth order moments of velocity and temperature fluctuations, which are usually assumed to obey a Gaussian distribution. Lower order moments may differ, hence the name quasi-normal approximation. This assumption can contradict physical realisability. Eddy-damping methods have been used to re-establish this property (Canuto 1992, 1993, 1997). More recently, models have been suggested which instead explicitly account for the role of coherent, large scale structures in heat and momentum transport.

2. Numerical simulations of the Sun and a K dwarf

As a benchmark for the closure models we used 3D numerical simulations (large eddy simulations, LES) of surface granulation in the Sun and a K dwarf. The solar simulation is case D discussed in Robinson *et al.* (2003). It assumes a box of

2900 km $\times$ 2900 km $\times$ 3000 km on a grid with 58 $\times$ 58 $\times$ 170 points (z-coordinate denotes the vertical component). Realistic microphysics is employed (AF94 opacities combined with OPAL96 data) and hydrogen and metal abundances are taken to be $(X, Z) = (0.7385, 0.0181)$ as in the Yale Standard Solar Model. A $T_{\rm eff}$ of 5777 K and a $\log(g)$ of 4.4377 are prescribed. Horizontal boundary conditions are periodic, vertical ones closed and stress-free with a constant energy flux imposed at the bottom. Radiative transfer is treated in the grey approximation. For the K dwarf a $T_{\rm eff}$ of 4609 K and a $\log(g)$ of 4.6523 are assumed based on an evolutionary track for a star of $M = 0.7 M_\odot$ at an age of $t \sim 6$ Gyr. Computations were done with a grid of 58 $\times$ 58 $\times$ 180 points corresponding to a spatial domain of 2700 km $\times$ 2700 km $\times$ 1350 km and $(X, Z) = (0.7, 0.018)$. Boundary conditions and microphysics are otherwise taken as in the solar case.

After the simulations had thermally relaxed, horizontal averages were computed for the lower order moments of velocity and temperature fields. The horizontal averages were averaged over time. Typically 10 to 20 turn-over (granule life) times were required to achieve convergence to a statistically steady state (i.e., averages are independent of time). Successful comparisons with simulations (Stein & Nordlund 1998) and helioseismological data were discussed in Robinson *et al.* (2003) for the solar case. Sufficiently away from the differently treated upper and lower boundaries agreement between our solar case and a simulation with the code of Stein & Nordlund (1998) (see Belkacem *et al.* 2007) is found in the range of 10% to 20% for mean values and lower order moments.

3. Models for third and fourth order moments

Guided by their measurements of velocity and temperature in the convective planetary boundary layer of the atmosphere of the Earth, Gryanik & Hartmann (2002, GH2002) suggested that two-scale mass flux averages can be used to derive scaling relations between higher order moments of velocity and temperature. This way one explicitly accounts for the coherent, large scale structures of purely convective flows which appear as up- and downdrafts as well as hot and cold drafts covering generally different horizontal area fractions. If fluctuations among and within individual drafts are neglected, only contributions from the velocity averages over all up- and downdrafts as well as hot and cold streams are accounted for. For this case a number of analytical relations exists between higher order moments. As they were found to scale very well with Reynolds averages, GH2002 and Gryanik *et al.* (2005) suggested to replace the quasi-normal (QN) model with a new set of closures (see Kupka & Robinson 2007 for a discussion).

4. Discussion of results and conclusions

The GH2002/2005 model, motivated by the very good scaling of two-scale mass flux averages with Reynolds stress averages for terrestrial atmospheric convection, provides improvements over the traditionally used QN approximation by up to an order of magnitude for both the solar and the K dwarf simulation (see Fig. 1 to compare the relative performance of $\overline{w^2\theta^2} = \overline{w^2}\,\overline{\theta^2} + 2\overline{w\theta}^2 + (\overline{w^3}/\overline{w^2})(\overline{\theta^3}/\overline{\theta^2})\overline{w\theta}$ by GH2005 with $\overline{w^2\theta^2} = \overline{w^2}\,\overline{\theta^2} + 2\overline{w\theta}^2$, i.e. the QN model). Similar to the geophysical case, the improvements are mostly found for the interior (quasi-adiabatic) part of the convection zone. The origin of these improvements is the boost provided by an asymmetric distribution of velocity and temperature fluctuations stemming from the coherent, large scale flow structures. The super-adiabatic part of the convection zone is characterised by efficient radiative cooling and re-heating of gas and rapid expansion of upflows. This type of boundary effect has not been accounted for in the new model, hence little of an improvement is found for the surface layers.

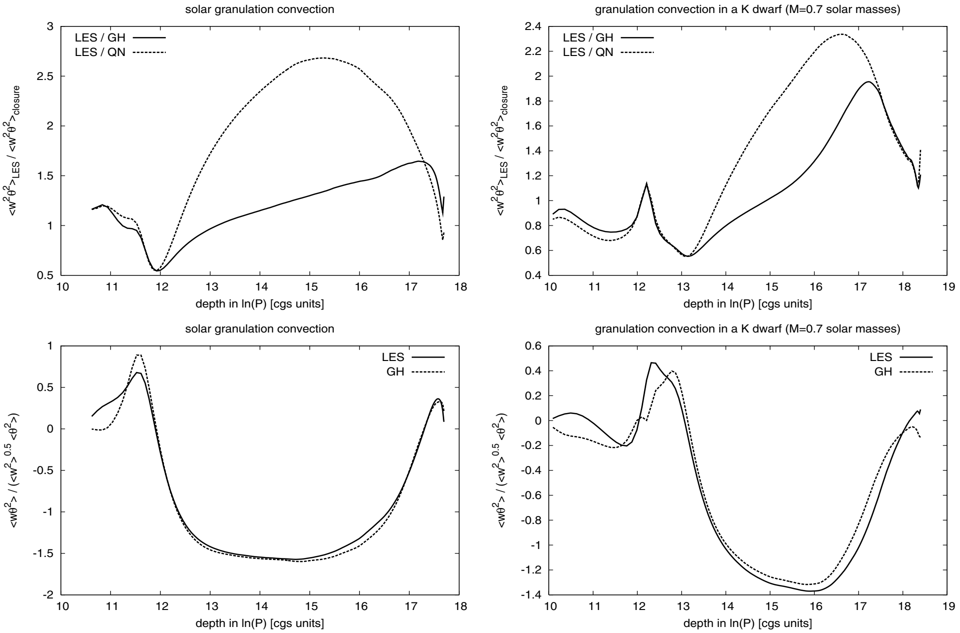

Figure 1. The top two panels show the ratio of the directly computed moment to the GH2005 and QN closures for $\langle w^2\theta^2\rangle$ (left panel: Sun, right panel: K dwarf). The closer the ratio is to 1 the better the model. The bottom two panels (left one: Sun, right one: K dwarf) show the GH 2002 closure for $\langle w\theta^2\rangle = \overline{w\theta^2} = (\overline{\theta^3}/\overline{\theta^2})\,\overline{w\theta}$ (dashed) and the direct computation (solid).

For similar reasons neither is there much improvement near the base of the simulation domain with its stress-free but impenetrable boundary (this limitation is irrelevant for normal stars). A detailed study (Kupka & Robinson 2007) demonstrates the large range of validity of these results by an intercomparison with simulations with idealised microphysics and geophysical cases. The new closures can be used to improve the modelling of p-mode excitation in solar type stars (see Belkacem *et al.* 2007). This may also be expected for non-local Reynolds stress models of convection provided further improvements concerning the physics of the boundary layers of convection zones are made.

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