An Infinite Product of Euler.

In a letter to Stirling, dated July 27, 1738, Euler mentions having happened on the infinite product, "satis notatu dignam," $\frac{3.5.7.11.13.17.19\ldots}{4.4.8.12.12.16.20\ldots}$, the numerators being the odd primes in their natural order, the denominators the multiples of 4 nearest to those primes. He says he can prove that the limit of this product is $\frac{\pi}{4}$. His proof was probably as follows:

$$\frac{3.5.7.11.13.17.19\ldots}{4.4.8.12.12.16.20\ldots}=\left(1+\frac{1}{3}\right)^{-1}\left(1-\frac{1}{5}\right)^{-1}\left(1+\frac{1}{7}\right)^{-1}\left(1+\frac{1}{11}\right)^{-1}\left(1-\frac{1}{13}\right)^{-1}\ldots$$

$$=\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\ldots\right)\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots\right)\left(1-\frac{1}{7}+\frac{1}{7^2}-\ldots\right)\left(1-\frac{1}{11}+\frac{1}{11^2}-\ldots\right)\left(\ldots\right)$$

$$=\sum_{n=0}^{\infty}(-)^n\frac{1}{2n+1}=\text{arc tan }1=\frac{\pi}{4}.$$

A. C. AITKEN.

An Elementary Proof of Girard's Theorem.

Girard enunciated in 1625 the following celebrated theorem, which is associated with the name of Fermat:

Every prime of the form $4m+1$ is the sum of two squares in one way only, and no prime of the form $4m-1$ is a factor of the sum of two squares that are not both multiples of that prime.

The theorem will be deduced here from an immediate corollary of the H.C.F. theorem, namely that if a is prime to b, we may find c and d such that $ad-bc=k$, where k is any assigned integer.

§ 1. If k is less than and prime to n, and q is given, only one number h less than n exists such that $kh\equiv q$ (mod. n).