12
Conclusions, present status and outlook

In this book we have attempted to present in a structured fashion the various aspects of the use of loops in the quantization of gauge theories and gravitation. The discussion mixed historical and current developments and we rewrote many results in a more modern language. In this chapter we would like to concentrate on the outlook arising from the material presented and focus on current developments and on possible future avenues of work. We will divide the discussion into gauge theories and gravity, since the kinds of developments in these two fields follow naturally somewhat disjoint categories.

12.1 Gauge theories

Overall, the picture which emerges is satisfying in the sense that the bulk of the techniques developed can be applied systematically to the construction of loop representations for almost any theory based on a connection as the main canonical variable, either free or interacting with various forms of matter. In this respect we must emphasize the developments listed in chapters 1, 2 and 3 which are the main mathematical framework that we used to understand the physical applications. Many of these aspects, as we have mentioned, have been studied with mathematical rigor by various authors in spite of the fact that the presentation we have followed here is oriented towards physicists.

The main conclusion to be drawn from this book is that loop techniques are at present a practical tool for the analysis of the quantum mechanics of gauge theories. There are three main lines of attack that are worthwhile discussing separately:

• Quantization of gauge theories in the continuum. Even though the loop representation has very appealing features such as the gauge invariance and its geometrical content, there has not been great improvement

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over other computational methods up to now. Although as we saw in chapter 5 there is a very compact description of the theory, no exact solutions of the non-perturbative Hamiltonian eigenvalue equation known. The loop equations need to be renormalized, but we do not know how to introduce a non-perturbative renormalization and therefore the strategy has been to try to solve the regularized equations and renormalize at the end.

At present it seems that a significant revision of these issues could come from the new developments in loop techniques we mentioned in chapters 2 and 3 The use of rigorous methods to define the loop transform and the measure could shed new light on many issues in non-perturbative Yang–Mills theories such as the issue of confinement. The expectation is that with the introduction of a rigorous measure one could compute the functional integral involved in the expectation value of the Wilson loop that we discussed in chapter 6, and therefore provide an exact expression for all the Green functions of the theory. A less ambitious hope would be to at least obtain a rigorous characterization of the asymptotic behaviors of these expressions and therefore elucidate the issue of confinement in a solid mathematical framework. On the other hand, the use of extended loops could prove a powerful tool for producing solutions to the quantum Hamiltonian eigenvalue problem and set a better stage to address the non-perturbative regularization problem.

Historically there was a fair amount of activity in this area in the early 1980s, prior to the introduction of extended loops and the rigorous transform but then the level of activity decayed. The expectation is that in the future the old results will be revised in the light of the new techniques and the hope is that many exciting new developments will take place. Some results are already starting to appear such as the ones mentioned for Maxwell theory [72, 65] and also some non-Abelian results [212, 96, 211, 102, 214] (see also reference [213] for a review), but a greater increase of activity is needed in this area in order to exploit in full the possibilities offered by the new techniques.

An appealing aspect of the use of loop variables in gauge theories is that if one considers interactions with matter fields, the loop variables are naturally adapted to the physical degrees of freedom of the theory. In particular the confined lines of force give rise naturally to the physical excitations in the confinement phase. For instance, in QCD the physical state space is defined in terms of loops and open paths with two or three quarks at the end points. These variables are respectively associated with the physical excitations, gluons, mesons, and barions.

• Gauge theories on the lattice. In this area there has been sustained activity over the last decade. The main obstruction has always been the overcompleteness of the basis of loops. The proliferation of loops when

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one considers larger lattices and higher dimensions completely washes out the advantages provided by the formalism. Although this difficulty can be remedied in part by the use of cluster expansions, the fact that the approximations involved are uncontrolled discourages in part a systematic use of them for tackling more realistic problems in $3 + 1$ dimensions. It is still the case that the use of loops offers overall advantages over other Hamiltonian methods. Monte Carlo techniques are, however, at present more efficient overall. Again, the use of extended loops motivated the introduction of classical loop actions for the lattice (as discussed in references [72, 215, 216]). These actions allow the use of Monte Carlo techniques in terms of loops and have a very simple expression, being proportional to the quadratic area associated with the world-sheet defined by the evolution of the loop. This may even suggest some connections with string theory.

• Topological field theories. Although topological field theories are a particular case of gauge theories, many special techniques and approaches have been developed for their study and a significant activity has taken place in this field in recent years. The main driving force is that, as we saw in chapter 10, topological field theories can be a powerful practical tool for the study of problems in mathematics. As an example of the recent results in this area, apart from the well known results of Witten we have mentioned in chapter 10, one can cite many results on triangulations (see [220] for references). For collected sets of articles see the books by Baez [217], Yetter [218] and Baadhio and Kauffman [219].

In summary, there are great opportunities for future developments in the application of loops to gauge theories with the possibility of obtaining concrete practical results that cannot be obtained by other methods. The future years will tell us if these expectations are fulfilled.

12.2 Quantum gravity

The impact of the introduction of loop techniques in quantum gravity has been quite great since their inception in the late 1980s. As opposed to gauge theories, where loop techniques are a minor part of the overall effort, in quantum gravity they have become one of the main approaches to the problem. In fact, they have significantly changed the perspective on many of the central issues of the field. The late 1980s and early 1990s has been a period of great excitement for the loop approach to quantum gravity and many formal, indicative results have been established: the use of knot theory to solve the diffeomorphism constraint, the observation that one could find solutions to the Wheeler–DeWitt equation, the connection with Chern–Simons theory and the Jones polynomial, the realization that

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one can define rigorously loop transforms and the discovery of a set of operators that are well defined without a renormalization, which led to the idea of weaves. It seems that present activity is concentrating on putting many of these results on a more solid footing and laying out the basis for a solid rigorous theory of quantum gravity. Current efforts are also being directed towards tackling the main problems of the field: those of observables and time, through different kinds of approximations. The activity in the field at present is channeled into three main approaches, which we would like to comment upon separately. Dividing any field in a set of efforts is usually only a partial characterization and we do so here only to order ideas in some way.

• The definition of a physical Hamiltonian and the use of the spin network basis. This approach involves choosing a matter time clock and the introduction of a topological Hamiltonian in the space of knots. One of the open issues at present is to provide a well defined characterization of the action of the Hamiltonian in the space of knots. Also checking the consistency of the constraints is a difficult task that has yet to be completed. The main effort is currently being directed towards the computation of possible topological Feynman rules arising from the proposed Hamiltonian coupled to matter. The main challenge once these technical issues have been settled is to connect these approximation techniques effectively with the situations in which there is interest in exploring quantum gravity effects, for instance, Hawking evaporation. A subject we have barely had any chance to discuss in this book due to its recent nature is the use of spin network states to construct a basis of independent loop states (free of Mandelstam identities) that could also help to diagonalize many physical operators in the theory. It is clear that these states may have implications for physical predictions of the theory. They may find use in Yang–Mills theory as well.

• The use of a rigorous measure to compute the loop transform and an inner product. This direction of research could potentially lead to a revision of several sections of this book. It is expected that the material we developed here will provide the basis on which to deal with the potential new expressions for constraints and wavefunctions that the introduction of non-trivial rigorous measures in the loop transform could produce. At present there is great excitement due to the possibility of incorporating in the measure the reality conditions through the results of Hall which we discussed in chapter 3. Not only would this allow us to define the transform but it would also allow us to introduce a physical inner product into the theory, which would be a major achievement in the case of quantum gravity and would allow us among other things to decide which solutions to the constraints are normalizable. The main challenge of this approach is to ensure that the measures introduced produce physical theories with the

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expected properties. Introducing measures in infinite-dimensional spaces is, as we discussed, not a trivial task and therefore one has to ensure that what one constructs has a non-trivial content. This approach may allow a rigorous definition of the constraints of quantum gravity and also of the functional integrals of interest to knot theorists and particle physicists. It may also allow us to establish to what extent the connection and loop representations are "functional duals" of each other.

• The use of extended loops and the extended representation. A powerful machinery has been set up to represent quantum gravity through extended loops. It is not clear at present if this machinery will simply be a calculational tool to perform computations in terms of loops or if extended loops are genuinely needed to represent quantum gravity. As a calculational tool it has proved powerful to generate solutions to the Wheeler–DeWitt equation and their regularization. It has introduced a systematic way to operate with loops, which is a valuable achievement in itself. In spite of the fact that there are several indications, as we commented in chapter 10, that loops may not be enough to represent gauge theories there is also evidence that extended loops are "too big" to represent a quantum theory. It has been pointed out [222] that simple examples can be constructed in which the use of extended loops leads to loss of gauge invariance and other serious pathologies. The root of these difficulties always lies in convergence problems of the extended expressions, which we have largely ignored in this book. It was inevitable that this should happen, since one is trying to build a "functional dual" of the space of connections modulo gauge transformations and it is therefore unavoidable to delimit proper domains of convergence. Is the proper domain given by just the ordinary loops or do we inevitably need some of the extended elements? This is the main challenge of this approach at present: to find a subset of extended loops (usually referred to as "thickened out loops") that is large enough to capture all information needed from the gauge theory in question and yet is not so large as to run into convergence problems. There are several proposals currently under study for such objects [21].

• Other ideas. Great possibilities also lie in the integration of all these efforts. For instance, at present it is not clear how to introduce rigorous measures for the extended loop transform. The root of this problem lies in the notion of independent loops that was crucial to introduce the cylindrical measure, which it is not clear how to generalize to extended loops. It appears that this difficulty is in no way fatal and some suitable generalization is likely to be found. Mixing the rigorous transforms with the extended loops could provide a powerful tool for addressing the convergence problems of extended loops. Another avenue is to try to fuse together the ideas used to define the finite diffeomorphism invariant observables and the notion of weaves with the extended representation. There does not seem to be any real obstruction to doing this and it seems to be a problem ready to be tackled. In particular the ideas of spin networks that serve naturally to diagonalize some of the operators are very likely to have a counterpart in algebraic properties of the multitensors of the extended representation. Finally, exhausting the permutations of the above points, one could apply the rigorous measure to give a proper definition of the physical Hamiltonian constraint in the space of knots that is being proposed using the square-root techniques. This would be an important point since the ambiguities introduced by the addition of small loops in the space of knots pervade all the loop formalism, not just this last approach.

• Open problems. The main open problems in canonical quantum gravity have for many years been the problems of time and of the interpretation in quantum gravity. Suppose we succeed in finding a large space of solutions to the Wheeler–DeWitt equation. Suppose we find non-local quantities that commute with the constraints. Suppose an inner product implementing the reality conditions is found. What next? Some authors hold the view that this is not the way to make physical sense of quantum gravity but that the proper way lies in identifying an "internal time" in the theory in terms of which to write it as a Schrödinger equation. There have been several attempts to do this, and even for simplified midi-superspace models [223] it has proved an elusive issue. The usual answer to these objections is that in order to obtain a notion of time from quantum gravity one should have recourse to the "complexity" and "many degrees of freedom" of the theory and therefore simplified models are too "frozen" to be a good arena to test these issues. Loops provide a framework in which "complexity" can be tabulated in a particular way, by specifying the degree of knottiness. As a consequence, a notion of time can be associated with the increase of complexity of the knottings that appear in the wavefunctions. This has been explored in some detail [221] with encouraging results, leading to a notion of evolving Hilbert spaces. In general, the problem of identifying an internal clock in a system is the problem of choosing a suitable set of variables to describe it in terms of which such a clock is manifest. It may be that the new variables are better suited for this problem than the geometrodynamical ones, as suggested by the results of Ashtekar [56], or that loops may present a better picture, as suggested by the evolving Hilbert space construction.

All in all we can say that several new avenues have been opened by the use of loops to represent quantum gauge theories and quantum gravity. There are many issues to be tackled before a final word can be given on the usefulness or otherwise of this approach. The new perspectives introduced in this process and the insights to several results that have been provided are probably going to be a lasting contribution to physics and mathematics regardless of what the final theory is.