ACCELERATION AND THE "CLOCK PARADOX".

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In the controversy which has been continuing over the "clock paradox", it has been claimed by supporters ([1]) of the orthodox interpretation that the difference in behaviour of clocks carried by observers in motion relative to each other can be traced to differences in the motion relative to the universe. Whilst A remains unaccelerated relative to the universe, B for a portion of his journey does not. Professor Dingle, in reply ([2]), remarked that he would "not dispute a possible slight effect" but could not understand why, if the acceleration of B were so important, it did not appear explicitly in the answer.

I would point out that the acceleration is important and does appear in the answer. There is no disparity between the readings of the clocks if neither observer is accelerated relative to the universe, whilst there is the full retardation, as usually predicted, only if B suffers an infinite acceleration at the outermost point of his journey.

Suppose that B passes A on his outward journey at a speed not greater than V, and he suffers a uniform retardation g for such a period that he returns past A with a speed not greater than V, and the total time, as recorded by A, that B is away is T. For simplicity suppose that the succession of events on the return journey is the reverse of those on the outward, so that only half the space-time path of B need be considered.

The space-time path of B_0 in A's coordinate system, is given parametrically by

$$t = \tau, \qquad x = V\tau, \qquad 0 < \tau < t_0$$

$$t = \tfrac{1}{2}T - \frac{c}{g}\sinh\sigma, \qquad x = X - \frac{c^2}{g}(\cosh\sigma - 1) \quad 0 < \sigma < \sigma_0.$$

The parameters t_0, σ_0, X are to be chosen so that the two portions join up smoothly. This will be so if $\tanh\sigma_0 = V/c$,

$$t_0 = \tfrac{1}{2}T - \frac{V}{g\sqrt{1-\dfrac{V^2}{c^2}}}, \quad \text{and } X = \tfrac{1}{2}VT - \frac{c^2}{g}\left(1 - \sqrt{1 - \frac{V^2}{c}}\right).$$

This parametric form holds so long as $t_0 > 0$, i.e. so long as $g > 2V/T\sqrt{1 - V^2/c^2}$. For $g < 2V/T\sqrt{1 - V^2/c^2}$, the second set of equations only are applicable, and σ_0 is now chosen to that $\frac{1}{2}T = c/g \sinh \sigma_0$, and

$$X = \frac{c^2}{g}(\cosh \sigma_0 - 1) = \frac{c^2}{g}\left(\sqrt{1 + \left(\frac{gT}{2c}\right)^2} - 1\right). \quad \text{For } g = \frac{2V}{T\sqrt{1 - \frac{V^2}{c^2}}},$$

the two forms of solution agree.

For the first solution, the time recorded by B is

$$T_B = \tfrac{1}{2}T\sqrt{1 - \frac{V^2}{c^2}} - \frac{V}{g} + \frac{c}{2g}\log\frac{c+V}{c-V} \to \tfrac{1}{2}T\sqrt{1 - \frac{V^2}{c^2}} \text{ as } g \to \infty.$$

For the second solution,

$$T_B = \frac{c}{g}\log\left[\frac{gT}{2c} + \sqrt{1 + \left(\frac{gT}{2c}\right)^2}\right]$$

and for g small $T_B = \frac{1}{2}T(1 - (gT/2c)^2 + \cdots) \to \frac{1}{2}T$ as $g \to 0$.

So if g is small, the effect is small, whilst if g is infinite the effect is merely finite. So one may say that the effect of the acceleration is smaller, rather than larger, than expected.

It is not therefore surprising that the clock retardation is absolute because it depends not only on the *relative velocity* of A and B, but also on the *absolute acceleration* of B. In this context, "absolute" implies reference to the universe. The other formulation of the paradox with three unaccelerated observers has an asymmetry built in at the beginning, and in that case, a difference in clock readings may be expected.

References

[1] Crawford, F. S. Nature 179, 1071 (1957).
[2] Dingle, H. Nature 179, 865, 1129, 1242 (1957).

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