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Spin-orbit coupling and chaotic rotation for eccentric coorbital bodies

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Abstract. The presence of a co-orbital companion induces the splitting of the well known Keplerian spin-orbit resonances. It leads to chaotic rotation when those resonances overlap.

Keywords. celestial mechanics, coorbitals, rotation, spin-orbit resonance

1. Introduction and Notations

Given an asymmetric body on a circular orbit, denoting θ its rotation angle in the plane with respect to the inertial frame, the only possible spin-orbit resonance is the synchronous one $\dot{\theta} = n$, n being the mean motion of the orbit. On an Keplerian eccentric orbit, Wisdom *et al.* (1984) showed that there is a whole family of spin-orbit eccentric resonances, the main ones being $\dot{\theta} = pn/2$ where p is an integer. In 2013, Correia and Robutel showed that in the circular case, the presence of a coorbital companion induced a splitting of the synchronous resonance, forming a family of co-orbital spin-orbit resonances of the form $\dot{\theta} = n \pm k\nu/2$, ν being the libration frequency in the coorbital resonance. Inside this resonance, the difference of the mean anomaly of the two coorbitals, denoted by ζ, librates around a value close to $\pm\pi/3$ (around the L4 or L5 Lagrangian equilibrium - tadpole configuration), around π (encompassing L3, L4 and L5 - horseshoe configuration) or 0 (quasi-satellite) configuration. We generalize the results of Correia and Robutel (2013) from the case of circular co-orbital orbits to eccentric ones.

2. Rotation

The rotation angle θ satisfies the differential equation:

$$\ddot{\theta} + \frac{\sigma^2}{2}\left(\frac{a}{r}\right)^3 \sin 2(\theta - f) = 0, \text{ with } \sigma = n\sqrt{\frac{3(B-A)}{C}}, \tag{2.1}$$

where $A < B < C$ are the internal momenta of the body, (r, f) the polar coordinates of the center of the studied body and a its instantaneous semi-major axis.

Let us consider that the orbit is quasi-periodic. As a consequence, the elliptic elements of the body can be expended in Fourier series whose frequencies are the fundamental frequencies of the planetary system. In other words the time-dependent quantity $\left(\frac{a}{r}\right)^3 e^{i2f}$ that appears in equation (2.1) reads:

$$\left(\frac{a}{r}\right)^3 e^{i2f} = \sum_{j \geqslant 0} \rho_j \; e^{(i\eta_j t + \phi_j)}. \tag{2.2}$$

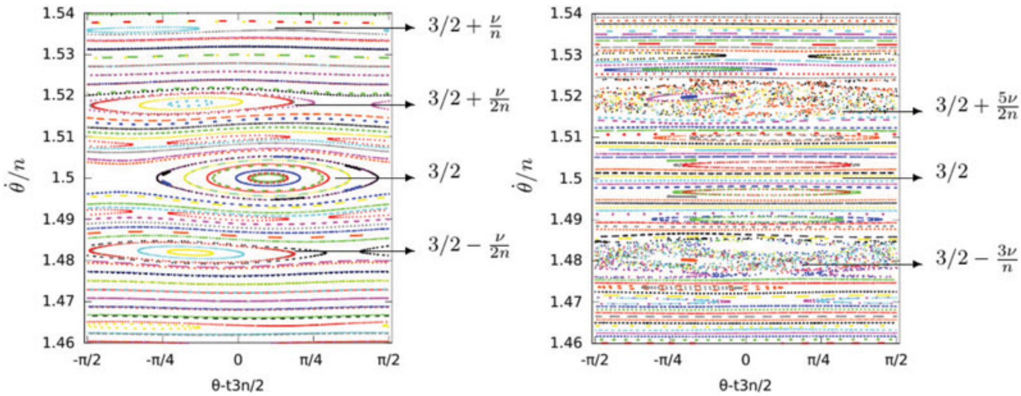

Figure 1. Poincaré surface of section in the plane $(\theta - t\frac{3n}{2}, \dot\theta/n)$ near the 3/2 spin-orbit eccentric resonance. (left): $\zeta_{max} - \zeta_{min} = 35°$ - tadpole configuration. (right): $\zeta_{max} - \zeta_{min} = 336°$ horseshoe configuration.

Where η_j are linear combinations with integer coefficients of the fundamental frequencies of the orbital motion (here n and ν) and ϕ_j their phases. Thus (2.1) becomes:

$$\ddot\theta = -\frac{\sigma^2}{2}\sum_{j\geqslant 0}\rho_j \sin\,(2\theta + \eta_j t + \phi_j). \tag{2.3}$$

For a Keplerian circular orbit, the only spin orbit resonance possible is the synchronous one, since $\rho_0 = 1$, $\eta_0 = 2n$, and $\rho_j = \eta_j = 0$ for $j > 0$. In the general Keplerian case we have the spin-orbit eccentric resonances, $\eta_j = pn$ and the ρ_j are the Hansen coefficients $X_p^{-3,2}(e)$ (see Wisdom *et al.*). For the circular coorbital case, Correia and Robutel (2013) showed that a whole family results from the splitting of the synchronous resonance of the form $\eta_j = 2n \pm k\nu$. For small amplitudes of libration around L4 or L5 (tadpole), the width of the resonant island decreases as k increases.

In the eccentric coorbital case, each eccentric spin-orbit resonance of the Keplerian case splits in resonant multiplets which are centred in $\dot\theta = pn/2 \pm k\nu/2$. For relatively low amplitude of libration of ζ, the width of the resonant island decreases as k increases, see Figure 1 (left). But for higher amplitude, especially for horseshoe orbit, the main resonant island may not be located at $k = 0$. In Figure 1 (right), the main islands are located at $\dot\theta = 3n/2 \pm 5\nu/2$ and $\dot\theta = 3n/2 \pm 6\nu/2$. These islands overlap, giving rise to chaotic motion for the spin, while the island located at $\dot\theta = 3n/2$ is much thinner.

3. Conclusion

The coorbital spin-orbit resonances populate the phase space between the eccentric resonances. Generalised chaotic rotation can be achieved when harmonics of co-orbital spin-orbit resonances overlap each other, which is a different mechanism than the one described by Wisdom *et al.* (1984), where the eccentricity harmonics overlap.

References

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