BINARY ASTEROIDS: SECULAR PERTURBATIONS

R. VILHENA DE MORAES and S. M. G. WINTER
Depto. de Mecânica do Vôo e Orbital
Instituto Tecnológico de Aeronáutica
Centro Técnico Aeroespacial
12225, São José dos Campos, SP, Brasil

Abstract. The motion of two small bodies orbiting each other whose barycenter is orbiting around a massive body is studied.The equations of motion are integrated considering the secular part of the disturbing function.

1. Introduction

Up to now a question without definitive answer is if there is or not double or multiple systems amidst the catalogued asteroids. A thorough analysis about this subject can be found in Chauvineau and Mignard (1990). The possibility of such objects to exist has been the incentive for the publication of several papers about the 2 + 2 body problem, from which binary asteroids is an application (Whipple and Szebehely (1984),Milani and Nobili(1988)).

In this work is studied the motion of two hypothetical asteroids of masses m_0 and m_1, orbiting each other, whose barycenter is in keplerian orbit around the Sun, m_2. The perturbation of a fourth body (Jupiter,m_3) orbiting the primary is also considered.

2. The Disturbing Function

The equations of motion are referred to a system centered at CM(m_0,m_1), the center of mass of the binary system. It is assumed as known the motion of m_2 relative to CM(m_0,m_1) and the motion of m_3 relative to m_2, both motions taken as unperturbed elliptical motions.

Legendre polynomials, up to order 2, can be used to express the disturbing function in terms of the orbital elements. Neglecting the inclinations, the perturbations F_{12} and F_{13} of m_1 by m_2 and m_3 respectively, can be expressed in the following forms(Winter(1990):

$$F_{12} = Gm_2 \left[\frac{a^2}{a'^3} \left(A_0 + e'A_1 + e'^2 A_2 + eA_3 + e^2 A_4 + ee'A_5 \right) \right] ,$$

$$\begin{aligned} F_{13} = Gm_3 \Bigg[& \frac{a^2}{a_{23}^3} \left(B_0 + e'B_1 + e'^2 B_2 + eB_3 + e^2 B_4 + ee'B_5 \right) \\ & + \frac{a^2 a'}{a_{23}^4} \left(C_0 + e'C_1 + e'^2 C_2 + eC_3 + e^2 C_4 + ee'C_5 \right) \\ & + \frac{a^2 a'^2}{a_{23}^5} \left(D_0 + e'D_1 + e'^2 D_2 + eD_3 + e^2 D_4 + ee'D_5 \right) \Bigg] , \end{aligned}$$

where A_i,B_i,C_i, and D_i $(i = 1, \ldots, 5)$ are functions of the eccentric anomalies, of the argument of pericenters and of the longitude of m_3. In those equations where

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 183–184.

considered only terms up to order 2 in the eccentricities and terms up to a_{23}^5 in the denominators . Moreover, a_{23} represents the semi major axis of the orbit of m_3, unprimed orbital elements are used to indicate the orbital elements of m_0 relative to m_1 and primes are used to represent the orbital elements of m_2.

3. Secular Perturbations

Using Bessel functions the equations of motion can be expressed in terms of the mean anomaly. Taking into account only the secular part F_s of the disturbing function, the following equations are obtained from the Lagrange equations in non-singular variables $h = e \sin \varpi$, $k = e \cos \varpi$:

$$\dot{h} = \frac{1}{na^2} \frac{\partial F_s}{\partial k} = (1,1)\, k \;,$$

$$\dot{k} = -\frac{1}{na^2} \frac{\partial F_s}{\partial h} = -(1,1)\, h \;,$$

where, considering terms up to the order of e^2,

$$(1,1) = \frac{3\,G}{4n} \left[\frac{m_2}{a'^3} + \frac{m_3}{a_{23}^3} \left(1 - \frac{3a'}{a_{23}} \right) \right] \;.$$

This system is easily integrable by analytical techniques. This result was applied to find the precessional motion of a hypothetical system of binary asteroids, and it is in accordance with the numerical experiments obtained by Pojman (1987).

4. Conclusions

An analytical solution was obtained to compute secular perturbations(planar case), due to the Sun and Jupiter, on binary asteroids. In general the perturbation due to Jupiter is about 10^4 smaller.

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