COMMISSION 36: THEORY OF STELLAR ATMOSPHERES (THEORIE DES ATMOSPHERES STELLAIRES)

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I. COMMISSION ACTIVITY, SYMPOSIA

Commission 36 acts as a sponsor or co-sponsor at the following symposia and colloquia: IAU Colloquium No. 90 "Upper Main Sequence Stars with Anomalous Abundances", Crimea, USSR (May 1985), IAU Colloquium No. 89 "Radiation Hydrodynamics in Stars and Compact Objects", Copenhagen, Denmark (June 1985), IAU Symposium No. 120 "Astrochemistry", Goa, India (December 1985), IAU Colloquium No. 87 "Hydrogen Deficient Stars and Related Objects", Bangalore, India (December 1985).

The commission participates jointly with several other commissions in the organization of Joint Discussions at the XIXth General Assembly on the topics "Stellar Activity: Rotation and Magnetic Fields" and "Solar and Stellar Non-Radial Oscillations".

II. RECENT PROGRESS IN THE THEORY OF STELLAR ATMOSPHERES

As in earlier Reports on Astronomy, this report is not intended to be a comprehensive review of all work done in the field covered by our commission, nor does it contain a complete bibliography. (However, in Section III, below, an attempt has been made to list monographs, proceedings from symposia etc. of particular relevance for the activity of our commission.) Rather, the following pages focuses on a few areas in which significant progress has been made in the last three years. These areas represent different lines of direction that the study of stellar atmospheres now follows - the invention of new efficient numerical methods, the attempts to relate the atmospheric phenomena to global properties such as the rotation of the stars, the study of complex dynamical systems where mass flows are decisive such as Wolf-Rayet stars, the application of rather classical model-atmosphere techniques for more "exotic" stellar photospheres such as those of white dwarfs, and the use of stellar-atmosphere theory in the study of non-stellar objects, such as accretion discs.

It is obvious to any reader that new observations in new wavelength regions, or at higher spectral, temporal or spatial resolution, is of key importance today for the study of stellar atmospheres. In particular, one should here emphasize the fundamental value of solar studies. It is also clear that the development of new generations of fast computers has led and will lead to impressive progress in the theoretical simulation of stellar atmospheres and of phenomena in them. However, one must strongly stress the need for a deeper physical *understanding* of the interplay between the different complex phenomena that in interaction form the stellar atmospheres. An inquiry to some experienced astronomers in this field led to suggestions as to what problems are yet unsolved but would be worth attacking: The behaviour of radiation-pressure dominated plasmas; structure and physics of magnetized atmospheres, including interaction with motions in deeper layers; the problems of mass loss, coronae and extended layers; effects of dust and polyatomic molecules in cool stars; problems connected with partial redistribution; physical and numerical instabilities; atmospheres of accretion discs. The list could well be much longer - in fact, most problems beyond the classical treatment of plane-parallel atmospheres are still unsolved. Hopefully, it will be possible in future Reports of Astronomy to report on further significant progress in these respects.

1. Numerical Methods (Wolfgang Kalkofen)

Introduction. Much progress has been made in numerical radiative transfer during the last several years; the subject has been treated recently in some detail in a book entitled *"Methods in Radiative Transfer"*, published by Cambridge University Press, and in a workshop held in Trieste, for which the proceedings are to appear shortly by Reidel.

The principal numerical methods to emerge that are suitable for investigations of time-dependent media in which radiative transfer as well as gas dynamics play a role are based either on the probability of photon escape from a medium or on the perturbation of an operator. Both will be reviewed here. For the older methods which are suitable mainly for static media the reader is referred to these publications, as he is for two new ones that show great promise: one, by Anderson (1985), is based on a treatment of the frequency dependence of the radiation field in analogy to the variable Eddington factor for the angle dependence; it results in a significant reduction of the order of the system of coupled equations that must be solved; and the other, by Wehrse (1985), is a generalization of the formal integral of the scalar transfer equation to that of a matrix transfer equation.

This report reviews mainly the escape probability and operator perturbation methods, their derivation and the underlying principle; for applications the interested reader can be guided by the references to the basic papers. In the escape probability method (reviewed recently by Rybicki 1984, 1985), the mathematical description of the transfer process is much simplified; this results in a major economy, but also in errors over which one has only very limited control. In the operator perturbation method (reviewed by Kalkofen 1984, 1985a), the solution of the complete set of equations is speeded up by the use of an approximate integral operator in the calculation of correction terms; this still permits an accurate solution of the problem, but at the price of iterations.

The Escape Probability Method. In the Escape Probability Method one can distinguish between first- and second-order methods. In the former, the global relation between the radiation field and the source function which is ordinarily expressed either by the differential equation of transfer or its formal integral, $J(\tau) = \int K(\tau,\tau')S(\tau')d\tau'$, is replaced by a local relation, $J(\tau) = S(\tau) \int K(\tau,\tau')d\tau'$, for the mean integrated intensity, J, in a line or a continuum for a finite or semi-infinite plane-parallel atmosphere. This equation is obtained from the formal integral by assuming that the scale of variation of the source function, S, is large compared to that of the integral kernel, K. If this condition is satisfied the formal integral can be written in terms of the escape probability from the medium, $P(\tau) = 1 - \int K(\tau,\tau')d\tau'$, giving $J(\tau) = [1 - P(\tau)]S(\tau)$. Combining this relation with the equations of statistical equilibrium in the form of the source function equation, $S(\tau) = (1 - \varepsilon)J(\tau) + \varepsilon B$ (cf. Athay 1976), results in a local relation for the source function, $S(\tau) = \varepsilon B/[\varepsilon + (1-\varepsilon)P(\tau)]$. Thus, the source function and the mean integrated intensity are known throughout the atmosphere when the escape probability $P(\tau)$ is known; and $P(\tau)$ can be determined very rapidly by the use of asymptotic relations (cf. Canfield et al. 1984 who give asymptotic expressions for $P(\tau)$ for lines broadened by the Doppler, Voigt, Lorentz, or Stark effects, and for bound-free continua; cf. Ivanov 1973 for an approach to obtaining asymptotic results).

The first-order escape probability method has fair accuracy deep inside the medium (a factor of two or three for the net rate coefficient $\rho = 1 - J/S$ and hence for the energy loss rate from large depth). But even when the collision parameter ε and the Planck function B are constant the solution carries a large relative error near the surface of a medium (of order $2\sqrt{\varepsilon}$ for the source function of a line transition with complete redistribution in a semi-infinite static atmosphere), especially when scattering is important (i.e., when $\varepsilon \ll 1$). A modification by Frisch (1984) improves the solution near the surface and extends the method to the case of scattering by resonance lines described by the R_{II} redistribution function. - In contrast to static media, the first-order escape probability method works well in the Sobolev limit of a high velocity gradient. - A typical application of the method (for a static medium) is the discussion by Kwan and Krolik (1981) of the formation of emission lines in quasars.

The accuracy is improved significantly with the second-order escape probability method, often referred to as the probabilistic method. It originated in a suggestion by Athay (1972) to describe the radiative transfer in a spectral line by means of a first-order differential equation for the mean integrated intensity, with the escape probability as the independent variable. A rigorous derivation of this equation was given by Frisch and Frisch (1975); a derivation stating clearly the mathematical assumptions was given by Canfield et al. (1984); and one making explicit the physical assumptions, by Scharmer (1981, 1984).

A first-order differential equation, the probabilistic equation, describes the transport of energy and hence information in one direction only. Thus, in the normal course of the solution from the inside in the outward direction, the structure of the outer layers of a medium cannot influence the state of the gas in the deeper layers; a major shortcoming. Other assumptions that must be satisfied for the equation to be valid are that the profile function for the line absorptions be depth-independent, that the line be formed in complete redistribution (CRD), that differential velocities be negligible, and that the background continuum be weak. When these assumptions are satisfied, the equation yields the exact solution at the outer surface of a semi-infinite medium, a distinct improvement over the first-order method. Although the probabilistic equation appears to yield a fair accuracy throughout an atmosphere when these conditions are satisfied (a typical value is 20%), the problems that call for such fast methods tend to be those where these conditions are not met, resulting in larger errors that are difficult to estimate. A further drawback is that the method does not provide for a way to improve the numerical result; another method must be used if a higher accuracy is desired. - A modification of the probabilistic equation by Canfield et al. (1984) separates the equation into two first-order differential equations, for the inward and outward-directed mean integrated intensities, respectively. This allows the specification of two boundary conditions for the incident intensities in a finite medium.

The Operator Perturbation Method. The Operator Perturbation Method for the solution of the differential equation of radiative transfer subject to integral constraints was originally proposed by Cannon (1973). He applied it to partial redistribution (PRD) problems, perturbing the PRD operator about the isotropic CRD operator; to media with differential flow velocities, perturbing about the static case; and to media with spherical symmetry, perturbing about the plane-parallel case (for references to these and to related papers, cf. Cannon 1984).

Scharmer (1981, 1984) and Scharmer and Nordlund (1982) have expressed the operator perturbation in terms of integral equations. In their formulation, corrections to a solution are based on the error incurred in the conservation equation; the reference to a conserved quantity makes this powerful approach suitable also for non-linear problems.

The essence of the method is the separation of the calculation into two parts: the *approximate* calculation of corrections to a solution using an approximate integral operator, and the *accurate* calculation of the error with which the solution satisfies the conservation equation. The accuracy of the converged solution depends only on the accuracy of the error calculation.

In the integral equation formulation, the transfer problem is expressed in the form $\mathsf{L}S = \varphi$, where L is a linear integral operator, S is the source function or another variable for the state of the gas that is to be determined, and φ is the known inhomogeneous term of the problem; this equation may describe the line transfer of a two-level atom with complete (Cannon 1984, Scharmer 1981, 1984) or partial (Scharmer 1983) redistribution, a multi-level atom (Scharmer and Carlsson 1984), or a model atmosphere (Nordlund 1984, Kalkofen 1985b); and both the linear and the non-linear cases. The procedure by which the integral equation is solved is to expand the unknown function in a series $S^{(n)} = \mathrm{S} + \Sigma_{i=1}^{n} s^{(i)}$, $S = S^{(\infty)}$, where the initial estimate S is either the converged solution of the preceding time step in a time-dependent problem, or an estimate, usually S = 0, in a static problem. Given the solution in the n^{th} order, the solution in the $(n+1)^{th}$ order is obtained from the equation $L' s^{(n+1)} = \epsilon^{(n)}$, where L' is an approximate integral operator (approximating L) and $\epsilon^{(n)}$ is the error made by the n^{th} order solution in satisfying the conservation equation, $\epsilon^{(n)} = \varphi - LS^{(n)}$.

The speed of the operator perturbation method, giving it a distinct time advantage over the *direct* solution of the equations, depends on the speed with which the approximate integral operator L' can be constructed and with which the set of coupled equations for the corrections $s^{(n)}$ can be solved. Now, the operator L' can be constructed very rapidly with Scharmer's (1981, 1984) one-point quadrature formula for the intensity in terms of the source function, i.e., essentially the Eddington-Barbier relation applied to the interior of a medium. This results in a matrix of nearly triangular structure, leading to fast solution times (proportional to N^2, not N^3 as ordinarily, where N is the number of depth points for a single variable S, or the product of the number of depth points and the number of levels in a multi-level atom) for the corrections $s^{(n)}$ by means of an $l \times u$ decomposition of the matrix L'; this decomposition needs to be carried out only once in each interation cycle. - The error term $\epsilon^{(n)}$ is computed by solving the (scalar) differential equation of transfer for known source function S either as the Feautrier equation with second-order accuracy, or in Auer's (1976, 1984) formulation with fourth-order accuracy. Thus the approximate integral operator L' is the only matrix that needs to be constructed.

In line transfer, the large thermalization length due to scattering is contained in the approximate integral operator L'; the error calculation, since it solves the transfer equation for known source function separately for each angle and frequency point, contains only the short-range behavior of the monochromatic transfer equation (Kalkofen 1985a). The long-range behavior must be preserved when the matrix L' is simplified in order to speed up the solution. - The operator perturbation method can be adapted to computers with short word length by cancelling analytically intensity-dependent terms in the equations of statistical equilibrium (Scharmer 1984, Scharmer and Carlsson 1984), allowing the treatment of line transfer problems with very small scattering parameter ε.

The Core Saturation Method. One other approximate method that should be mentioned here is the Core Saturation Method (Rybicki 1972). It is not very fast since it may require a large number of iterations for convergence, but the essential physics on which it is based can be used to advantage in reducing the construction time for the approximate integral operator L' in the operator perturbation method (Kalkofen 1985a). In the core saturation method, the (scalar) transfer equation is solved by means of a Λ-iteration for known source function in the shallow layers, where the monochromatic optical depth is smaller than some value of order unity (for a modification that allows for structure inside the medium such as shocks, cf. Kalkofen and Ulmschneider 1984); at larger optical depth, the intensity is usually assumed to have the same value as the source function. The core saturation assumption permits a very general approach to complex problems; it has been used for media with cylindrical symmetry (Stenholm and Stenflo 1977), the transfer of polarized radiation (Stenholm and Stenflo 1978), and the co-moving frame description of the transfer in a moving medium (Stenholm 1980). The method can also be modified to yield the exact solution of a transfer problem (Rybicki 1984).

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2. Manifestations of the Interaction between Convection and Rotation in Stellar Atmospheres (Lee Hartmann)

Introduction. According to dynamo theories, the interaction between convection and rotation is responsible for the generation of magnetic fields in solar-type stars. Stellar observations can explore the ways the global parameters, like the surface rotation and internal structure, affect the production of magnetic fields. In this very brief review I wish to concentrate on recent progress in understanding the evolution of stellar rotation, and the dependence of stellar cycles and magnetic activity on rotation, internal structure, and age.

Stellar Rotation. Most of the progress that has been made in understanding the origins of magnetic activity derives from an explosion in the amount of stellar rotation data. The measurement of rotational modulation of the Ca II emission in slowly-rotating main sequence stars (Baliunas et al. 1983) has now provided a large sample of stars for which the rotational periods are accurately known. Modulation of broad-band photospheric light by starspots even makes it possible to determine accurate rotational periods for stars in the Hyades (Lockwood et al. 1983).

One of the biggest surprises in this area is the discovery that late-type dwarfs go through a phase of very rapid rotation (van Leeuwen and Alphenaar 1982; Soderblom, Jones and Walker 1983; Stauffer et al. 1984). Low-mass stars apparently spin up (by a factor of 10!) as they contract toward the main sequence (Stauffer et al. 1984). Furthermore, the observation of rapid rotation among G dwarfs in the α Per cluster (Stauffer et al. 1985), compared with the slow rotation of similar stars in the Pleiades cluster (cf. Benz, Mayor and Mermilliod 1984) shows that many solar-type stars go through a phase of rapid spin-down (from ~50 to ~10 km s^{-1}) between the ages of 5×10^7 and 7×10^7 years. While the implications of this behavior are not fully