CHARACTERISTICS OF PERIODIC ORBITS IN ELLIPTICAL GALAXIES

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Abstract. The properties of the characteristic curves of several families of periodic orbits, in a conservative dynamical system of two degrees of freedom, symmetric with respect to both axes, are reviewed. The two main types of families are presented. One sees that the pattern of the characteristics in the exact resonance case is similar to that of the near resonance case except for the basic characteristic. The form of the characteristics can be found theoretically by means of the second integral.

1. INTRODUCTION

The motion of a star on the plane of symmetry of a non axisymmetric galaxy can be described by the Hamiltonian

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \omega_1^2 x^2 + \omega_2^2 y^2) + \text{higher order terms} \tag{1}$$

Several investigators have used the Hamiltonian (1) in order to study the families of periodic orbits and successive bifurcations, considering only one mixed third order term (see, e.g., Contopoulos, 1970a; Contopoulos and Michaelides, 1980; Contopoulos and Zikides, 1980; Contopoulos, 1981).

In the following we shall deal with the Hamiltonian

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \omega_1^2 x^2 + \omega_2^2 y^2) - \varepsilon(a_1 x^4 + 2a_2 x^2 y^2 + a_3 y^4) \tag{2}$$

where $\omega_1, \omega_2, a_1, a_2, a_3$ are positive constants and ε is the perturbation. The Hamiltonian (2) may be considered to describe the motion of a star on the plane of symmetry of a non rotating elliptical galaxy.

This choice is justified by the fact that the dynamics of elliptical galaxies have gained a considerable interest in the last few years. Now our understanding of the shapes and ways to form elliptical galaxies is clearly different from that of a few years ago. Elliptical galaxies were then thought to be rotationally flattened oblate systems with isotropic velocity dispersions (e.g., Freeman 1975). Observations and detailed dynamical studies have recently shown that most elliptical galaxies are not flattened by rotation but instead they appear to be triaxial systems with anisotropic velocity dispersions (Binney 1978, Illingworth 1981).

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As a consequence of the above thoughts a study of periodic orbits, in a galaxy described by the Hamiltonian (2) seems to be of interest because these orbits are the main landmarks in exploring the totality of orbits.

2. DESCRIPTION OF THE FAMILIES

It is well known that, for any given value of ε one can find periodic orbits making n oscillations along the x-axis and m oscillations along the y-axis whenever the ratio of the unperturbed frequencies ω_1/ω_2 is near a rational number n/m. We study periodic orbits intersecting the x-axis perpendicularly at a point x for a constant value of the energy h. For a given value of n and m the value of x varies by varying the perturbation ε. Then the curve $x = x(\varepsilon)$ is called the characteristic of the corresponding family of periodic orbits.

In general, two types of families are present: a) Regular families which are connected with the main families directly or through bifurcations. b) Irregular families not connected with the above families. These families appear at relatively large perturbations and seem to be connected with the dissolution of the invariant curves of non periodic orbits (Contopoulos 1970a).

Figure 1 shows the characteristic curves of several families of periodic orbits when $\omega_1^2 = 0.4$, $\omega_2^2 = 0.1$ $(\omega_1/\omega_2 = 2/1)$ $a_1 = 0.1$, $a_2 = 0.5$, $a_3 = 0.02$. Solid line indicate stable orbits while dashed lines indicate unstable orbits. We can see three groups of regular families:

i) In the first group belong the basic characteristic and its bifurcations. The basic characteristic starts from the $\bar{x}$-axis $(\bar{x} = \omega_1 x)$ in the exact resonance cases while in the near resonance cases starts always from the boundary line given by the equation

$$2\varepsilon a_1' \bar{x}^4 - \bar{x}^2 + 2h = 0, \tag{3}$$

where $a_1' = a_1/\omega_1^4$.

ii) In the second group belong the characteristics which bifurcate from the boundary line.

iii) Finally, in the third group belong the characteristic which bifurcate from the $\bar{x} = 0$ axis.

We can find approximately the form of the characteristics for small values of ε using the formula

$$r = -\frac{\omega_1}{\omega_2}\{1 - \varepsilon[(\frac{3a_1}{\omega_1^4} - \frac{4a_2}{\omega_1^2\omega_2^2} + \frac{3a_3}{\omega_2^4})\Phi_{10} + (\frac{2a_2}{\omega_1^2\omega_2^2} - \frac{3a_3}{\omega_2^4})h]\} \tag{4}$$

where r is the rotation number, h is the total energy and Φ_{10} is the second integral (Caranicolas 1982). This formula is not valid when ω_1/ω_2 is near 1. In this case we can find the form of the basic characteristic using a special form of the second integral (Caranicolas and Barbanis 1982).

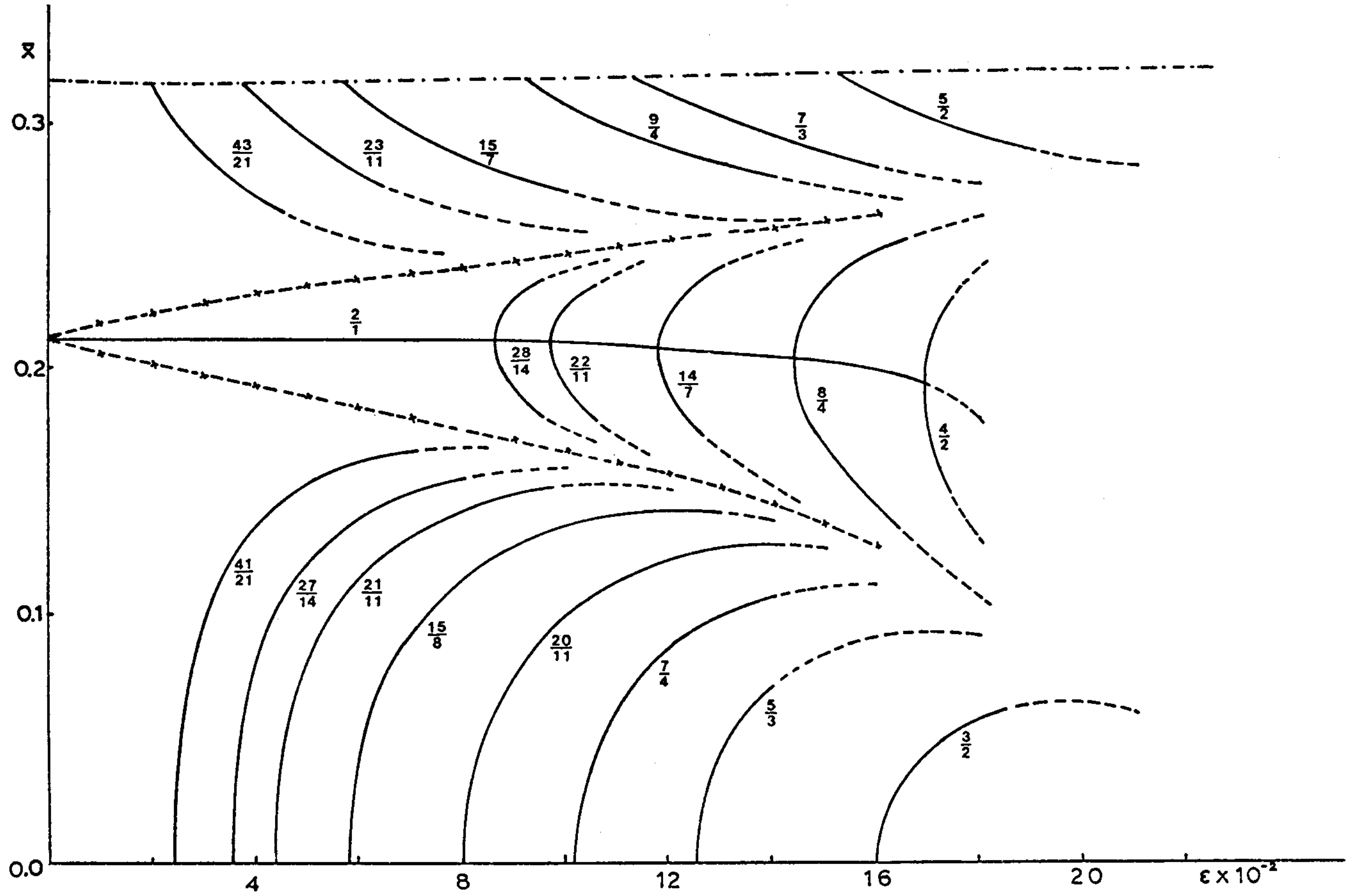

Fig. 1. Characteristics of families of periodic orbits closing after n oscillations along the x-axis and m oscillations along the y-axis when $\omega_1^2 = 0.4$, $\omega_2^2 = 0.1$, $a_1 = 0.1$, $a_2 = 0.5$, $a_3 = 0.02$, $h = 0.05$. The values n/m are marked near each family.

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REFERENCES

1. Binney, J.: 1978, Comment Ap., 8, pp. 2-
2. Caranicolas, N.: 1982, Astrophys. Space Sci. 87, 237.
3. Caranicolas, N., and Barbanis, B.: 1982, Astron. Astrophys. (in press).
4. Contopoulos, G.: 1970a, Astron. J. 75, pp. 96-107.
5. Contopoulos, G., and Zikides, M.: 1980, Astron. Astrophys. 90, pp. 198-205.
6. Contopoulos, G., and Michaelidis, P.: 1980, Celest. Mech. 22, pp. 403-413.
7. Contopoulos, G.: 1981, Lett. Nuovo Cimento 28, pp. 498-502.
8. Freeman K.C.: 1975, in Galaxies and the Universe, Stars and Stellar Systems, Vol. IX, ed. A. Sandage and J. Kristian (Chicago,University of Chicago Press), pp. 406-507.
9. Illingworth, G.: 1981, in the Structure and Evolution of Normal Galaxies, ed. S.M. Fall and D. Lynden-Bell (London: Cambridge University), pp. 27-41.