

Linear and non linear study of a possible mechanism for the generation of stellar radio bursts: THE SYNCHROTRON MASER INSTABILITY

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Abstract: The emission mechanism studied here, the synchrotron maser instability is based on a gyroresonance of highly energetic electrons. Compared to the cyclotron maser, this mechanism has the great advantage to be a direct amplification process even if the ratio ω_p/ω_c is not very small; a situation relevant to stellar flares. The high efficiency of the process can be confirmed by the non linear study of its saturation.

I) INTRODUCTION:

The cyclotron maser instability is now widely acknowledged as being the appropriate mechanism for explaining planetary radio emissions. This instability, linked to the relativistic gyroresonance of energetic electrons (10 KeV) and fed by positive $\partial f/\partial v_{\perp}$ slopes in the electronic distribution function, as also been invoked for explaining certain solar and stellar radio bursts (Melrose and Dulk, 1982). This interpretation, however, suffers from serious difficulties (Sharma and Vlahos, 1984):

(i) In a low ω_p/ω_c plasma ($\omega_p/\omega_c < 0.3$), the most amplified waves are emitted on the X mode, at frequencies closed to the gyrofrequency. These waves are reabsorbed by the hot background plasma when, during their propagation they cross the layer where their frequency is twice the local gyrofrequency.

(ii) In a higher ω_p/ω_c plasma, the Z mode is the most amplified one. As this mode can not escape out of the plasma, a conversion mechanism is needed for explaining the emission. This conversion is however expected to greatly reduce the efficiency of the whole process.

The mechanism proposed here: the synchrotron maser, could constitute a solution to these problems. Indeed, this instability, based on a gyroresonance of highly energetic electrons (up to 1 MeV), predominantly amplifies the X mode, even if the ratio ω_p/ω_c is not very low. This result is presented in section II, devoted to the linear study of the instability. An estimation of the efficiency of the process is made in section III and finally, a discussion and the conclusion are proposed (section IV).

II) LINEAR STUDY OF THE INSTABILITY:

-Basic Remarks: The formal expression of the growth rate of the instability is the same as the cyclotron maser one. It can be written:

$$\delta = A(\omega_p, \omega_c, \omega) \sum_m \int d^3p J_m^2 \left(r \frac{k_{\perp} v_{\perp}}{\omega_c} \right) \cdot B \left(\frac{\partial \delta}{\partial v_{\perp}}, k, \omega \right) \cdot \delta \left(\omega - k_{\parallel} v_{\parallel} - m \frac{\omega_c}{\Gamma} \right)$$

(For an exact expression of the different coefficients, see Wu, 1985)

The growth rate of the instability is the result of a summation on the successive resonant pole contributions. The main differences between the cyclotron maser and the synchrotron maser instability come from this summation. In the cyclotron maser case, the electrons have low energies ($\Gamma \approx 1$); the resonance is then only fulfilled near the gyroharmonics and the growth rate is calculated by using an unic resonant pole. Conversely, in the synchrotron maser case, a wave with a given frequency ω can resonate with many different electrons having energies defined by: $\Gamma_m = m \cdot \omega_c / \omega$. The contributions of several resonant poles must then be taken into account in the calculation of the wave growth rate (Figure 1). See Louarn et al, 1986 for a more complete discussion.

-Calculation of the Growth Rate: The results concerning these calculations are described in a short article: Louarn et al 1987. The main conclusions are the following:

(i) The value of ω_p / ω_c for which the Z mode becomes predominant increases with the energy of the electrons. This value can reach 3 or 4 for an energy of 1 MeV (Figure 2).

(ii) The frequency bandwidth of the emission is large compared to the one obtained with the cyclotron maser (10-20% instead of 1%). In certain condition, pseudo harmonics can be obtained (Louarn et al 1986).

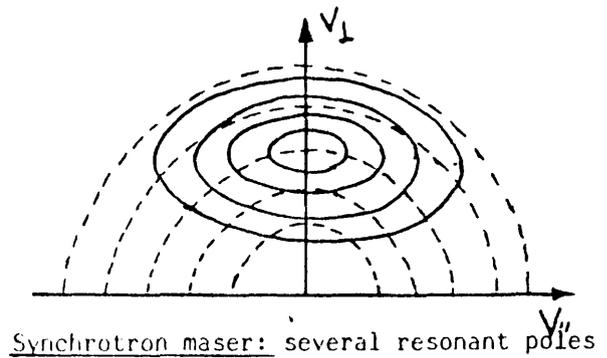
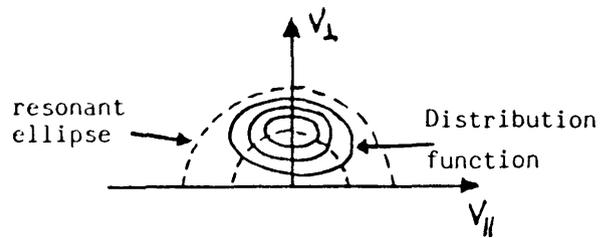


Figure 1.

The very interesting consequence of this linear study is that the synchrotron maser directly amplifies the X mode above the second or the third gyroharmonic in conditions where ω_p / ω_c is not very small. Then, during its propagation, the radiation is not severely reabsorbed by the background plasma.

An estimation of the efficiency of the process, obtained by the study of the non-linear saturation of the instability, is now presented.

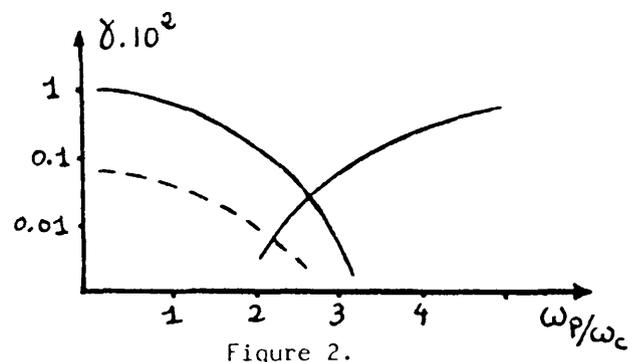


Figure 2.

Maximum growth rates of the 3 modes X, O and Z, as function of ω_p / ω_c (from Louarn et al., 1987).

III) NON LINEAR EVOLUTION OF THE INSTABILITY:

This study is made by using the quasi-linear diffusion theory. We only consider the most amplified waves (X mode in perpendicular propagation) and then the equations describing the evolution of the instability write:

$$\frac{\partial F}{\partial t} = \pi q^2 \frac{1}{P_{\perp}} \frac{\partial}{\partial P_{\perp}} \left[\int \frac{d^3 k}{(2\pi)^3} \sum_m P_{\perp} \delta(\omega - m \frac{\omega_c}{r}) \varepsilon_{k,m} \varepsilon_{k,m}^* \frac{\partial F}{\partial P_{\perp}} \right]$$

with $\varepsilon_{k,m} = \frac{1}{\sqrt{2}} \left[\xi_k^+ e^{i\varphi_{J_{m,1}}} \left(r \frac{k_{\perp} v_{\perp}}{\omega_c} \right) + \varepsilon_k^- e^{-i\varphi_{J_{m,1}}} \right]$

$$\frac{\partial W_{e,m}}{\partial t} = \pi^2 q^2 \int \frac{d^3 k}{(2\pi)^3} \int dP_{\perp} dP_{\parallel} \sum_m \frac{P_{\perp}^2}{r m_0} \delta(\omega - m \frac{\omega_c}{r}) \cdot \varepsilon_{k,m} \varepsilon_{k,m}^* \frac{\partial F}{\partial P_{\perp}}$$

These equations can be simplified by supposing that during its saturation, the spectral width of the emission does not change. The diffusion operator $D(p_{\perp})$ is then easily calculated; it can be approximated by a simple function: $D(p_{\perp}) = K p_{\perp}^{\alpha}$. The system of equations becomes:

$$\frac{\partial \xi}{\partial t} = -k_2 \left[\int d u_{\perp} \frac{1}{r^3} u_{\perp}^{\alpha} (\alpha u_{\perp}^2 + \alpha + 1) F(u_{\perp}) \right] \xi(t)$$

$$\frac{\partial F}{\partial t} = k_2 \frac{1}{u_{\perp}} \frac{\partial}{\partial u_{\perp}} \left[u_{\perp}^{\alpha} \xi(t) \frac{\partial F}{\partial u_{\perp}} \right]$$

$$u_{\perp} = P_{\perp} / m_0 c$$

For $\alpha = -1/2$, a value that relatively well fits $D(p_{\perp})$, a solution of this system is:

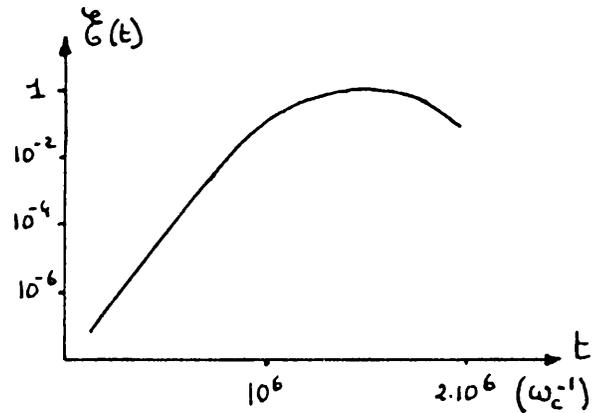
$$F(\tau, x) = A e^{-\left(\frac{x}{\xi}\right)^{7/4}} \left(1 - \Delta e^{-\left(\frac{x}{\xi}\right)^{7/4}} \right)$$

with $\xi = (\xi_0^{7/4} + \mathcal{E})^{4/7}$; $\mathcal{E} = \frac{\xi_0}{\xi_0} \left(\xi_0^{7/4} + \mathcal{E} + \frac{\mathcal{E}}{\xi_0^{7/4}} \right)$

and $\mathcal{E} = \int_0^{\tau} \frac{(7/4)^2}{4 K_2} \xi(\tau') d\tau'$

$$x = u_{\perp}^2 \quad \text{and} \quad \tau = \omega_c t$$

Evolution of the electromagnetic field



Evolution of the distribution

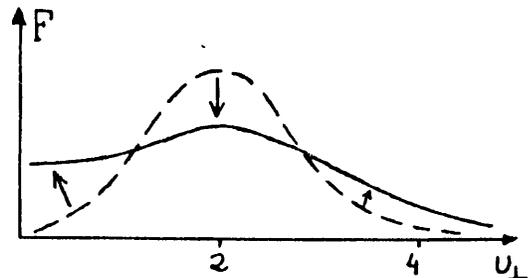


Figure 3. Evolution of the system for $\alpha = -1/2$

The time evolution of both the electromagnetic field and the electronic distribution function are shown on figure 3.

A typical exponentiation time of the instability is around: $10^{-2} n_c / n_h \omega_c^{-1}$ (where n_h/n_c is the ratio between the hot and the cold electron density). The efficiency of the process can reach 1%, a value similar to those obtained with the cyclotron maser. The saturation can be extremely rapid: starting with a small amount of electromagnetic energy compared to the kinetic one (10^{-8}), the saturation is reached after $\tau_{sat} = 10^2 n_c / n_h \omega_c^{-1}$. For $n_h/n_c = 10^{-4}$ and $\omega_c = 10^9$, this corresponds to 10^{-3} second.

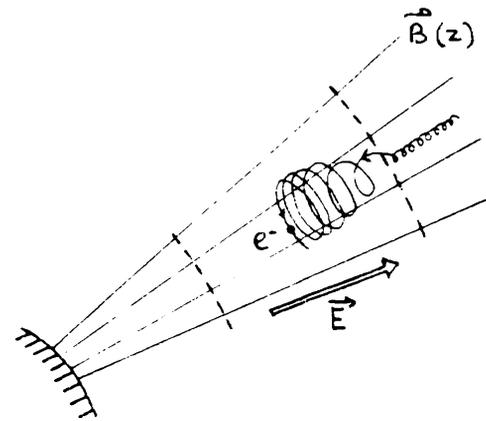
IV) DISCUSSION AND CONCLUSION:

From both the linear and the non linear study of the instability, a rough estimation of the brightness temperature of the emission can be obtained. Supposing that 1% of the energy of 1 MeV electrons contained in a volume L^3 (where L is $\tau_{\text{rot}} c$) is converted into a radiation having a spectral width of 10% and an angular aperture of 20° , the brightness temperature of the expected emission would be approximately $10^{24} \text{ Ph/n}_c \text{ K}^{\circ}$

The mechanism itself is consequently quite efficient, however, the existence of 1 MeV electrons presenting an inversion of population ($dN/dv_{\perp} > 0$) is still questionable. If such distributions are unlikely produced during solar flares, the situation could, however, be completely different in case of other objects (flares stars or binary systems) where extremely strong acceleration mechanisms certainly take place (Chanmugan, G., Dulk G. A., 1982).

Very simple models of acceleration regions can then be imagined (Figure 4) in which the combination of an acceleration due to a sufficiently high potential drop along a field line (a few 100 KeV) and the conservation of the adiabatic invariants of the electron motion in an inhomogeneous magnetic field lead to the formation of unstable distribution functions. In the presence of such distributions, a bright emission, with a relatively large spectrum and an high polarization, will be certainly produced by the synchrotron maser instability.

Model of the acceleration region :



Distribution obtained inside the acceleration region :

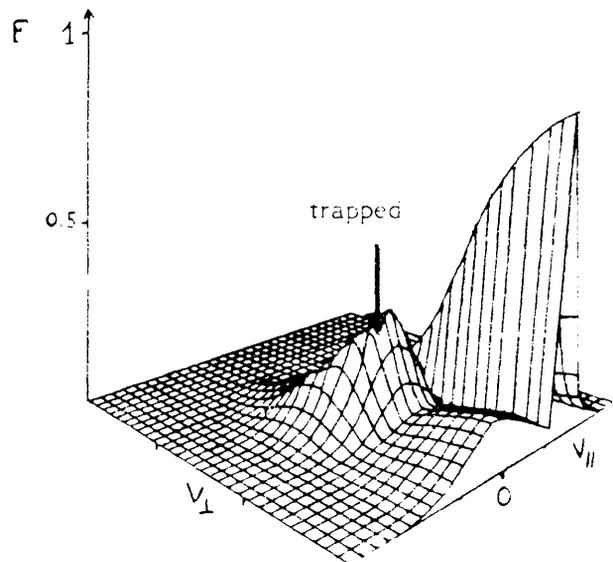


Figure 4.

References:

- Chanmugan, G., Dulk, G. A.: 1982, *Astrophys J.*, **57**, 285
- Louarn et al.: 1986, *Astron. Astrophys.*, **165**, 211
- Louarn et al.: 1987, *Solar Phys.*, **111**, 201
- Melrose D. B., Dulk G. A.: 1982, *Astrophys J.*, **259**, 844
- Sharma, R. R., Vlahos, L.: 1984, *Astrophys J.*, **280**, 405
- Wu, C. S.: 1985, *Space Sciences Rev.*, **41**, 215