THE ANGULAR SIZE - FLUX DENSITY RELATION

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The relation between angular size and flux density depends on the world model and more strongly on evolution in radio source properties with epoch. I shall consider here the simplest forms of evolution that explain the observed θ-S relations (Swarup 1975; Kapahi 1975a,b; Ekers, this symposium).

In order to calculate the expected N (S,θ) relations in any world model, one needs to know the generalized Luminosity Size Function, $\Phi(P,\ell,z)$, that gives the number of sources per unit volume at redshift z, in the luminosity range P to P + dP, with projected linear sizes in the range ℓ to $\ell + d\ell$. For simplicity let us assume that the local Φ can be factorized in terms of the Radio Luminosity Function (RLF) and the Radio Size Function (RSF), i.e.

$$\Phi(P,\ell)\ dP\ d\ell = \rho(P)\ dP.\ \psi(\ell)\ d\ell\ ; \qquad \text{where} \int_0^\infty \psi(\ell)\ d\ell = 1$$

This requires that P and ℓ be completely uncorrelated. The observational evidence is presented in Figure 1, where we have plotted P_{178} against ℓ(with H = 50 km/s/Mpc) for a complete sample of 87 3CR galaxies with $z < 0.25$ (redshift known for 84 and estimated from optical magnitudes for 3 galaxies). The low luminosity part of the figure has been filled in by plotting the B2 sample of radio sources identified with bright ellipticals (Colla et al. 1975) and a sample of giant ellipticals of low radio luminosity (Ekers, private communication). It is clear from Fig. 1 that for $P_{178} \gtrsim 10^{24}$ W/Hz/sr there is little correlation between P and ℓ. While the sizes do appear to be somewhat smaller at lower luminosities, it must be remembered that it is difficult to observe large sources of low luminosity. We shall therefore assume that P and ℓ are independent over the entire range of P, but will keep in mind the possibility that the low P sources may be physically smaller.

We now need to know the RLF and RSF. The local RLF is fairly well determined at low and intermediate P. If we scale the recent results of Perola et al. (this symposium) from 1400 MHz to 178 MHz (using α=-0.75)

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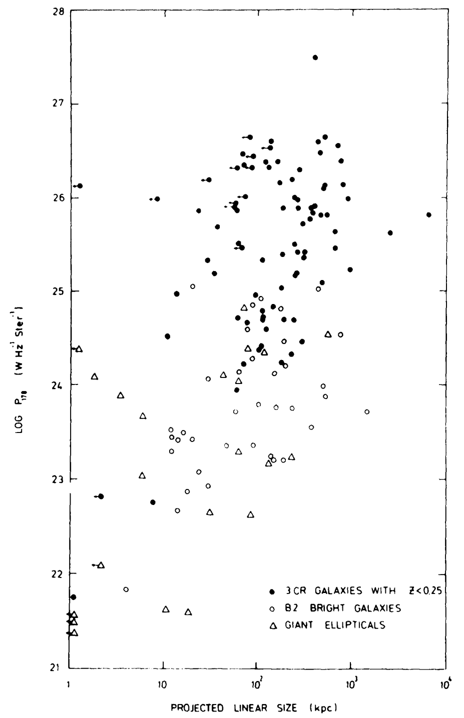

Figure 1. Luminosity-size diagram.

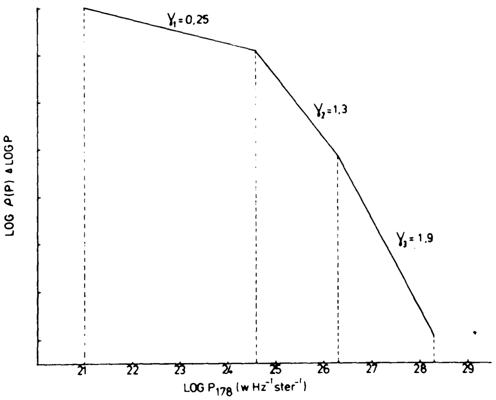

Figure 2. Radio Luminosity Function.

the RLF can be well approximated by two power laws of the form $\rho(P)\ dP \propto P^{-\gamma}\ dP$; with $\gamma_1 = 1.25$ in the range $10^{21} < P_{178} < 4 \times 10^{24}$ W/Hz/sr and $\gamma_2 = 2.3$ for $4 \times 10^{24} < P_{178} \leq 2.10^{26}$ W/Hz/sr, as shown in Fig. 2. It is not possible to determine the local RLF for $P \gtrsim 10^{26}$ W/Hz/sr independent of world model and evolution. There is evidence however that the slope of the RLF steepens considerably at higher P. The steepening is in fact required to explain the angular size data in terms of simple evolution (Kapahi 1975a,b). We shall take $\gamma_3 = 2.9$ for $P > 2.10^{26}$ W/Hz/sr and assume that no source has $P > 2.10^{28}$ W/Hz/sr.

The distribution of projected linear sizes for the sample of 87 3CR radio galaxies (shown by Dr. Ekers in the previous talk) is fitted reasonably well with an exponential. We therefore take the local RSF to be given by

$$\psi(\ell)\ d\ell = (1/\ell_o)\exp\ (-\ell/\ell_o)\ d\ell, \qquad \text{with } \ell_o = 300 \text{ kpc}.$$

Note that this gives a better fit to the observed distribution of sizes than the function used by us earlier (Kapahi 1975a,b).

The N(S,θ) relations are now readily computed. The expected relations between median values of θ and S in the Einstein-de Sitter Universe,

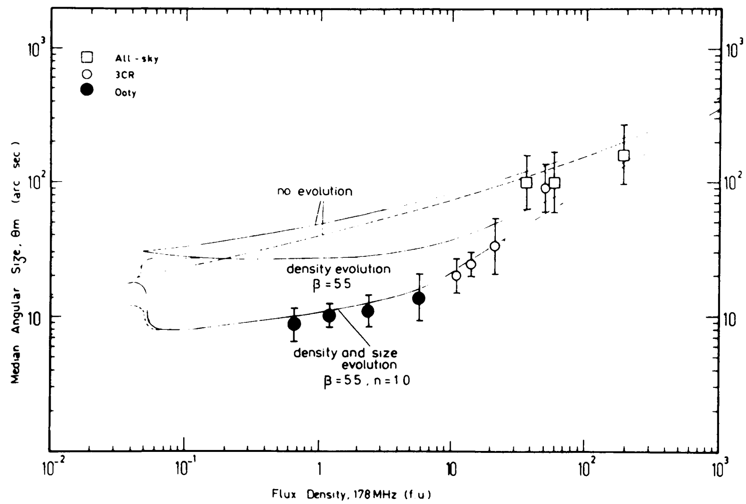

Figure 3. The observed $\theta_m(S)$ relation compared with predictions of evolutionary model.

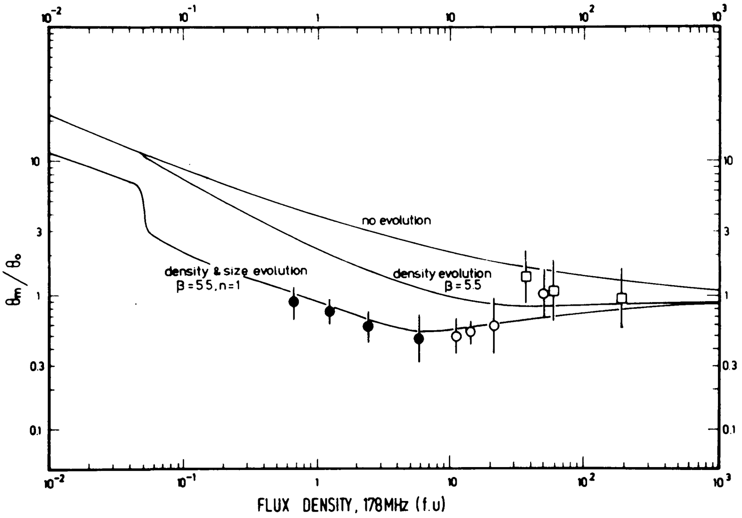

Figure 4. Same as Fig. 3, but with θm normalized with respect to the Euclidean expectation, θ_o.

with and without evolution, are compared with observations in Figure 3. This is done also in Figure 4, where θ_m has been normalized with respect to that expected in a static Euclidean Universe ($\theta \propto S^{\frac{1}{2}}$). A few remarks concerning the results in Fig. 3:

(a) The data cannot be explained without evolution because even at high flux densities one is seeing to appreciable z, and since the θ-z relation in most models is fairly flat at high z, going to lower flux levels (higher z) does not reduce θ_m very much.

(b) Values of θ_m at small S can be made smaller by having a larger contribution from more distant sources, as in density (or luminosity) evolution, or by requiring that distant sources have smaller physical sizes (size evolution). Either of the two types of evolution is not enough by itself; both are necessary to fit the data. By requiring a fit with the observed logN-logθ relation (Kapahi 1975b) for the 3CR sample in addition to the θ_m-S data, the amount of density and size evolution can both be determined. The simple forms of evolution assumed and the best fit parameters are listed below.

Density evolution:

$\rho(P,z)=\rho(P,z=0)(1+z)^{\beta}$ for $P_{178}>2.10^{26}$W/Hz/sr; $\beta = 5.5$

$\rho(P,z)=0$ for $z > z_c$; $z_c=3$

Size evolution:

$\psi(\ell,z) = \{1/\ell_o(z)\} \exp\{-\ell/\ell_o(z)\}\, d\ell$

$\ell_o(z) = \ell_o(z=o)\ (1+z)^{-n}$; $n = 1.0$

(c) The sharp discontinuity in the θ_m values at $S_{178} \simeq 5 \times 10^{-28}$Jy, seen in Figures 3 and 4 is somewhat artificial, resulting from limiting the density evolution to sources with $P>2.10^{26}$W/Hz/sr and from the cutoff in P and z. An extension of the θ_m-S relation to such low flux densities may throw light on the evolutionary properties of sources of low and intermediate luminosity.

(d) The effect of the low P sources having smaller physical sizes is shown by the dashed curves in Fig. 3, where we have assumed that the RSF for $P<4 \times 10^{24}$W/Hz/sr has the same exponential form as for higher luminosities but only one third as large an e folding size, i.e. $\ell_o=100$ kpc. Such a correlation of P and ℓ affects the θ_m values only at very low flux densities because low P sources make a negligible contribution at higher flux levels.

The predictions of the evolutionary model are compared in Fig. 5 with the distribution of angular sizes obtained by Katgert (1976) from Westerbork observations of the 5C2 region down to 13 mJy at 1400 MHz.

To summarize, it seems clear that the angular size data provide independent evidence of evolutionary effects similar to those inferred from the analysis of source counts, the V/V_m tests and the θ-z relation

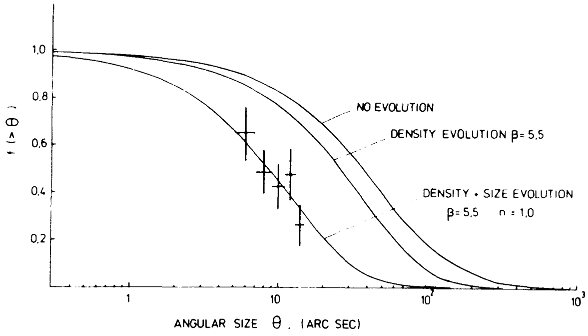

Figure 5. The fraction of sources with angular size >θ in the 5C2 region (Katgert 1976) compared with predictions of evolutionary model.

for QSOs. The models we have considered are admittedly oversimplified. It should be possible to refine these when detailed distributions of angular sizes specially at low flux levels become available.

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DISCUSSION

Conway: A sample of 72 sources from 4C has been mapped at Westerbork by Conway, Burn and Vallee. The distribution N(θ) of the component separations can be compared with the N(θ) for the Mackay 3CR sample. The agreement is excellent apart from the presence in the Mackay sample of 12% of sources with θ > 100", which is virtually absent from the 4C sample. We have shown that this is not a selection effect introduced by the WSRT, but it is quite consistent with the result of Caswell and Crowther that 4C is missing 15% of sources with large θ. If one corrects for this, there is no difference between 4C and 3CR in the statistic N(θ). (Flux range between 3.7 Jy and ∿ 12 Jy at 178 MHz.