FLUX TUBES AND DYNAMOS

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Abstract. The structure of solar surface magnetic fields, the way they erupt from the the convection zone below, and processes like flux expulsion and fragmentation instabilities support the view that magnetic flux in a stellar convection zone is in an intermittent, fragmented state which can be described as an ensemble of magnetic flux tubes. Depending on size and field strength, the dynamics of magnetic flux tubes can strongly differ from the behavior of a passive, diffuse field which is often assumed in conventional mean-field dynamo theory. Observed properties of active regions like emergence in low latitudes, Hale's polarity rules, tilt angles, and the process of sunspot formation from smaller fragments, together with theoretical considerations of the dynamics of buoyant flux tubes indicate that the magnetic structures which erupt in an emerging active region are not passive to convection and originate in a source region (presumably an overshoot layer below the convection zone proper) with a field strength of at least 10^5 G, far beyond the equipartition field strength with respect to convective flows. We discuss the consequences of such a situation for dynamo theory of the solar cycle and consider the possibility of dynamo models on the basis of flux tubes. A simple, illustrative example of a flux tube dynamo is presented.

Key words: Flux tubes – Solar/stellar magnetic fields – Convection zone

1. Introduction

Magnetic flux tubes are ubiquitous in the solar atmosphere. The observed surface fields of the Sun form a hierarchy of structures with strong fields ranging from large sunspots down to small-scale magnetic elements (e.g. Zwaan 1987, Stenflo 1989). The surface flux emerges in active regions from the convection zone below and apparently does so in a dynamically active way, not being dominated by convective flow patterns. For instance, large sunspots form out of fragments (McIntosh 1981, Garcia de la Rosa 1984) and the initial polarity mix in an emerging active region rapidly disentangles to form a bipolar structure. This is in accordance with the 'rising tree' picture (Zwaan 1978, 1992) of a partially fragmented, rising magnetic flux tube. Hale's polarity rules for active regions and sunspot groups are nearly strictly obeyed (Howard 1989) and the tilt angles of active regions show a systematic dependence on latitude (Hale et al. 1919, Wang & Sheeley 1991) which indicates that the emerging flux and the basic system of toroidal magnetic field from which it originates are not passively carried and distorted by convective flows and thus cannot be treated in the kinematic approximation. Only later, after the initial stage of flux emergence, the surface fields come progressively under the influence of convective flow patterns (granulation, supergranulation).

What is the origin of the strong magnetic flux tubes which erupt from the solar convection zone ? One possibility is flux expulsion by flows within the convection zone itself (Galloway & Weiss 1981). Numerical simulations by Nordlund et al. (1992) clearly show flux expulsion and flux tube formation in the case of non-stationary, three-dimensional convection. However, under the conditions prevailing in the solar convection zone the field strength of flux concentrations formed in this way does not exceed the limit of energy equipartition with respect to the generating convective flows so that they are unlikely to decouple dynamically from these flows.

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Time and length scales as well as orientation of the generated flux concentrations are then determined by the perpetually changing flow patterns, in contrast to the large-scale order indicated by the polarity rules and inclination angles of active regions. Furthermore, as shown by the results of helioseismology (e.g. Christensen-Dalsgaard 1992), differential rotation within the convection zone does not dominate over convective motions and thus cannot impose a preferred toroidal orientation on magnetic flux concentrations formed by flux expulsion.

On the other hand, if the fields are stronger than equipartition with convective flows, the magnetic buoyancy problem (Parker 1975) is further aggravated and, together with a number of other arguments (see Schmitt, this volume), casts doubt upon a convection zone dynamo as main source for the solar activity cycle. A weakly turbulent and stably stratified overshoot region with dominating differential rotation, on the other hand, could provide the proper environment for the formation of a strong toroidal flux system as source for the active regions at the surface (Spiegel & Weiss 1980, Galloway & Weiss 1981, Schüssler 1983). The magnetic Rayleigh-Taylor instability (e.g. Acheson 1978, Cattaneo & Hughes 1988, Hughes 1992) may lead to the formation of flux tubes in a natural way while the undulatory tube instability (Spruit & van Ballegooijen 1982, Moreno-Insertis 1986, Ferriz-Mas & Schüssler 1993) causes rapid eruption of flux loops towards the surface.

2. Flux Tube Dynamics

The dynamics of concentrated magnetic structures can be described with aid of the flux tube concept. In ideal MHD we define a flux tube as a bundle of magnetic field lines (constant magnetic flux) which is separated from its non-magnetic environment by a tangential discontinuity (surface current). As a consequence, the coupling between the tube and its environment becomes purely hydrodynamic, mediated by pressure forces, so that the flux tube can move relatively to a perfectly conducting surrounding plasma. This is different from a diffuse field which has to follow all plasma motions because of the flux freezing condition.

If the diameter of the flux tube is small compared to all other relevant length scales (scale heights, wavelengths, radius of curvature, etc) the *thin flux tube approximation* can be employed, a quasi-1D description which greatly simplifies the mathematical treatment (Spruit 1981, Ferriz-Mas & Schüssler 1993). In what follows we shall assume that this approximation is valid. For a simple estimate of the relative importance of the various forces acting on a thin flux tube let us assume a toroidal tube (flux ring encircling the axis of rotation in axial distance R) with circular cross section of radius a which is in temperature equilibrium with its environment. The most important forces (per unit length) perpendicular to the tube axis are then:

Buoyancy force : $$F_{\mathrm{B}} \doteq \frac{B^2 a^2}{8H_p} \qquad (1)$$

Curvature force : $$F_{\mathrm{C}} = \frac{B^2 a^2}{4R} \qquad (2)$$

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$$\text{Coriolis force:} \qquad F_\Omega = 2\varrho v \Omega \pi a^2 \tag{3}$$

$$\text{Drag force:} \qquad F_\text{D} = C_D \varrho_e v_e^2 a \tag{4}$$

where B is the field strength, H_p the pressure scale height, ϱ the density inside the tube, ϱ_e the external density, v the velocity component of the tube perpendicular to the axis of rotation, Ω the angular velocity, v_e the external velocity component perpendicular to the tube axis, and C_D the drag coefficient (of order unity). The relative magnitudes and directions of these forces determine the dynamics of a flux tube (for a more detailed discussion see Schüssler 1984, 1987). In the following section we shall apply the thin flux tube concept to emerging solar active regions and show that quite large field strengths at the bottom of the convection zone are implied by the observed properties of active regions.

3. A case for strong fields

A number of dynamical properties of magnetic flux tubes provide evidence that the parent toroidal flux system from which solar active regions originate must have a field strength which exceeds the equipartition value (of $\simeq 10^4$ G in the lower part of the solar convection zone, cf. Spruit 1977) by at least on order of magnitude.

3.1. No dominance of convective drag forces

Hale's polarity rules, the tilt angles of active regions, and the observed features of flux emergence and sunspot formation (for an overview see Zwaan 1992) indicate that convective flows do not dominate the dynamics of magnetic structures during the early phases of active region development. In later stages this is no longer the case, probably due to the progressive fragmentation and 'shredding' of large magnetic structures.

Decoupling from convective flows means that the drag forces (4) exerted by them do not dominate over the other forces, particularly the buoyancy force, on a rising flux tube. Introducing the equipartition field strength by $B_\text{eq} = v_c\sqrt{4\pi\varrho_e}$ where v_c is the convective velocity we find from equations (1) and (4) that the ratio of buoyancy to drag force exerted by convective flows ($v_e = v_c$) is given by (ignoring numerical factors of order unity):

$$\frac{F_\text{B}}{F_\text{D}} = \left(\frac{B}{B_\text{eq}}\right)^2 \left(\frac{a}{H_p}\right) . \tag{5}$$

Similar relations can be found for the other forces. They show that the drag forces due to convective flows dominate for sufficiently small tube diameter: very small tubes inevitably become passive with respect to convection. We see from equation (5) that for $B \simeq B_\text{eq}$, the drag force dominates unless $a > H_p$ (which would violate the condition for validity of the thin flux tube approximation). In the lower part of the solar convection zone we have $H_p \simeq 5 \cdot 10^4$ km, i.e. only huge tubes which fill a significant part of the whole convection zone could decouple dynamically from convective flows if the field strength is of the order of the equipartition field ($\simeq 10^4$ G). Only one of these tubes would already comprise 10^{24} mx, a major part

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of the total magnetic flux which erupts during a whole 11-year cycle. A tube with 10^{23} mx (already corresponding to a very large active region) and $F_{\rm B}/F_{\rm D} = 10$ (buoyancy dominates) requires $B \simeq 10^5$ G and $a/H_p \simeq 0.1$ which gives a radius of about 5,000 km. Consequently, a field strength significantly in excess of $B_{\rm eq}$ is required in order to avoid the dominance of convective drag forces.

3.2. The effect of rotation and polar escape

In a rotating star the Coriolis force tends to suppress any motion which changes the distance of a mass element from the axis of rotation. Assume an axisymmetric, toroidal flux tube (a flux ring) which encircles the axis of rotation and expands in the (cylindrically) radial direction. The expansion leads to an azimuthal Coriolis force which drives an azimuthal flow against the direction of rotation (angular momentum conservation). This flow, on the other hand, causes a Coriolis force directed *inward*, i.e. against the expansion. A corresponding process acts against a (cylindrically) radial contraction of the flux ring. If the flux ring is under the influence of a *spherically* radial buoyancy force, the Coriolis force tends to suppress its expansion perpendicular to the axis of rotation while motion under the influence of the axial component of the buoyancy force (which is non-zero if the ring is located outside the equatorial plane) is unaffected. Consequently, buoyant flux tubes tend to rise parallel to the axis of rotation and emerge in high latitudes if the Coriolis force dominates over the buoyancy force. Using equations (1) and (3) we find for the ratio of buoyancy to Coriolis force

$$\frac{F_{\rm B}}{F_{\rm C}} = \left(\frac{B}{B_{\rm eq}}\right)^2 \left(\frac{v_{\rm c}}{v}\right) \left(\frac{\rm Ro}{2}\right) , \tag{6}$$

where Ro is the *Rossby number* defined by $\mathrm{Ro} \equiv v_c/(2H_p\Omega)$. The Rossby number essentially is the ratio between the period of rotation and the convective turnover time and thus measures the degree of rotational influence on the convective motions. Since a rough estimate yields that the velocity of buoyant rise is of the order of the Alfvén velocity $v_A = B/\sqrt{4\pi\varrho_e}$ (Parker 1975) we may write $v_c/v = B_{\rm eq}/B$ and obtain

$$\frac{F_{\rm B}}{F_{\rm C}} = \left(\frac{B}{B_{\rm eq}}\right) \left(\frac{\rm Ro}{2}\right) = \frac{\rm Ro_m}{2} , \tag{7}$$

where $\mathrm{Ro_m} \equiv v_A/(2H_p\Omega)$ defines a 'magnetic Rossby number'. At the bottom of the solar convection zone we have $\mathrm{Ro} \approx 0.2$, so that for equipartition fields of the order of 10^4 G the right-hand side of equation (7) has a value of about 0.1 and the Coriolis force dominates. Consequently, flux tubes with equipartition field strength have to move parallel to the rotation axis and would emerge at high latitudes, in contradiction to the observed characteristics of solar activity. Motion perpendicular to the axis of rotation is suppressed or, more precisely, transformed into inertial oscillations with a frequency $\omega = 2\Omega\cos\theta$ where θ is the colatitude (Moreno-Insertis et al. 1992). The amplitude of these oscillations increases with the field strength. Simulations of Choudhuri & Gilman (1987, see also Choudhuri 1989)

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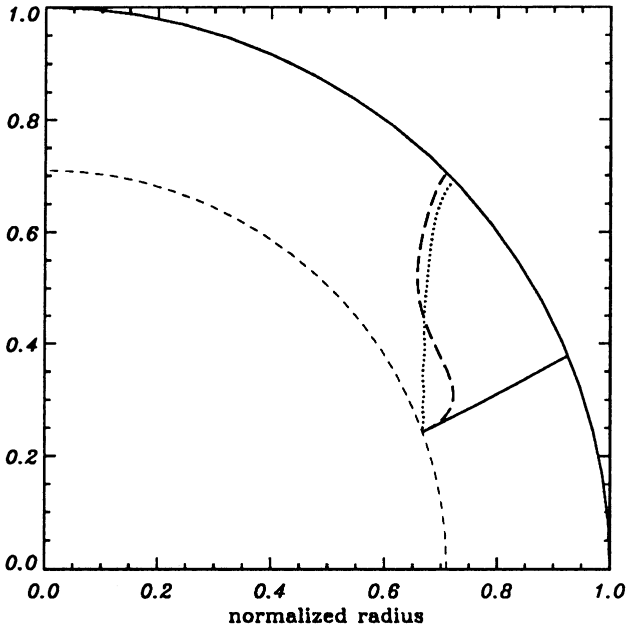

Fig. 1. Motion of toroidal flux tubes (flux rings) starting at a latitude of 20° at the bottom of the solar convection zone. The curves show the trajectories of the intersection of axisymmetric flux rings with a meridional plane which contains the (vertical) axis of rotation. The dotted curve is for initial field strength of 10^4 G (equipartition field), the dashed curve for $5 \cdot 10^4$ G, and the full curve for $2 \cdot 10^5$ G. While the tubes with weaker fields emerge at high latitudes, the tube with an initial field of $2 \cdot 10^5$ G rises radially.

have demonstrated the dominance of Coriolis forces for large equipartition flux tubes in the solar convection zone. The effects of turbulence and Kelvin-Helmholtz instability are unlikely to modify this conclusion since they lead to suppression of the polar escape of equipartition fields only for tiny tubes with diameters below a few hundred km, much smaller than the sizes of sunspots (D'Silva & Choudhuri, 1991). Consequently, we may conclude from Equation (7) that in the case of the Sun the initial field strength of the erupting tubes at the bottom of the solar convection zone must be at least an order of magnitude larger than the equipartition value of 10^4 G in order to avoid emergence in high latitudes.

Fig. 1 illustrates the dependence of the path taken by a rising flux tube on its (initial) field strength: the trajectories of rising axisymmetric flux rings starting at 20° latitude at the lower boundary of the solar convection zone is given for three values of the initial field strength, namely 10^4 G (equipartition value, dotted curve), $5 \cdot 10^4$ G (dashed curve), and $2 \cdot 10^5$ G (full curve). The trajectories which have been calculated by numerical integration (see Moreno-Insertis et al. 1992) are depicted as curves in a meridional plane containing the (vertical) rotation axis. The flux tubes pierce this plane perpendicularly and stay toroidal during the whole evolution.

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In accordance with the simple estimate given in equation (7), a tube with 10^4 G is constrained to move parallel to the axis of rotation and erupts at high latitudes (about 50° in this case). The inertial oscillation which is superposed over the axial motion is well visible for the tube with $5 \cdot 10^4$ G. Buoyancy dominates only if the field strength exceeds 10^5 G and forces a radial path of the rising flux tube and an emergence at low latitudes, as observed in the case of the Sun.

As a side remark, let us mention that there are observations of very active cool stars with large spots which rotate much more rapid than the Sun. A polar emergence of buoyant flux tubes cannot be avoided for these stars since the necessary initial field strengths would be much larger than the critical fields for the onset of non-axisymmetric tube instabilities (Schüssler & Solanki 1992). Indeed, many of these stars show prominent polar spots (Byrne 1992).

3.3. Tilt angles of active regions

It is well known (Hale et al. 1919) that bipolar active regions are always inclined with respect to the East-West direction: the preceding (p) polarity is closer to the equator than the following (f) polarity. The angle between the line connecting both parts and the E-W direction (the tilt angle γ) varies linearly with latitude λ ("Joy's law"):

$$\sin\gamma = 0.48\sin\lambda + 0.03 \tag{8}$$

(Wang & Sheeley 1991). The tilt can be understood in terms of the Coriolis force acting on a rising flux loop (Schmidt 1968): Matter flowing downward along the legs of a rising loop experiences a Coriolis force which leads to the correct sense of the tilt. D'Silva and Choudhuri (1993) have performed numerical simulations of erupting flux loops and showed that Joy's law provides a stringent constraint for the initial field strength of rising tubes. They find that fields below about $5 \cdot 10^4$ G show tilt angles which disagree with the observations; for the equipartition value of 10^4 G even negative tilt angles appear. Very strong fields, on the other hand, lead to insignificant tilts: the magnetic tension resists the twisting Coriolis force. The best fit to the observed tilts is obtained for field strengths around 10^5 G which reproduce the observed relation (8) very well. Although this value may somewhat depend on the initial conditions used in the calculation, in any case field strengths far beyond equipartition are required at the bottom of the solar convection zone.

3.4. Expansion of a rising tube

Since pressure and density decrease strongly towards the solar surface a rising tube must expand significantly in order to keep pressure equilibrium with its environment. Unless the initial field is very strong this has the consequence that along the rise the field strength sooner or later falls below the local equipartition value. At that point, the tube cannot resist being fragmented and shredded by convective motions. If this happens already deep within the convection zone, a later formation of sunspots and of a coherent active region is unlikely, let alone subtleties like Joy's law. Moreno-Insertis (1992) has shown that for initial fields of the order 10^4 G rising

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tubes fall short of the equipartition value almost immediately after their start from the bottom of the convection zone. For a field of 10^5 G he finds $B < B_{\rm eq}$ only at depths smaller than about 10^4 km. Some fragmentation at that late stage would probably not disrupt the coherence of the forming active region while a certain degree of fragmentation seems in fact to be required by the observed characteristics of active region and sunspot formation. Initial field strengths of the order of 10^5 G are required in order to avoid sub-equipartition fields early along the rise of an erupting flux tube.

3.5. Flux storage, adiabatic loops, and instability conditions

The amount of magnetic flux erupting in complexes of activity would fill a large part of the underlying convection zone if it is stored with equipartition or even smaller field strength (Parker, 1987a). These storage requirements are drastically alleviated if the field strength is significantly larger. The magnetic flux which can be stored in the latitude interval ±30° within an overshoot layer of thickness d is given by

$$\Phi_{\rm mag} \simeq 5 \cdot 10^{24}\, \alpha\, B_5\, d_9 \text{ mx} \tag{9}$$

where α is the filling factor, $B_5 = B/10^5$ G and $d_9 = d/10^9$ cm. Taking the 'fiducial' value of 10^{24} mx for the flux emerging during a solar cycle (Howard 1974), d_9 between about 2 and 5 for the width of the subadiabatic storage region (which is somewhat larger than the overshoot region since it extends into the convection zone proper, cf. Pidatella & Stix 1986), and B_5 between 1 and 5 for the stored field we find a range of filling factors between 0.01 and 0.1, i.e. the flux can easily be stored and also a significant degree of intermittency (flux tube structure) is permitted.

Another point in favor of large field strength at the bottom of the solar convection zone is provided by the adiabatic flux loop models of van Ballegooijen (1982). Such loops are the natural end product of erupting tubes whose footpoints stay anchored in a stably stratified overshoot layer. For reasonable field strength in the upper layers the resulting loop models show fields between 10^5 G and 10^6 G at the bottom of the convection zone.

Finally, if the flux is stored in a subadiabatic overshoot layer in the form of flux tubes, it requires a certain minimum field strength to form an erupting loop by non-axisymmetric (undulatory) instability. For reasonable values of the subadiabaticity, this minimum field strength again is of the order of 10^5 G (Ferriz-Mas & Schüssler 1993). Tubes with equipartition fields of 10^4 G are stable in an overshoot region and would not erupt at all.

3.6. Consequences of strong fields

We have provided a number of arguments which indicate that the field strength of toroidal flux tubes at the beginning of their eruption from the bottom of the convection zone must be about an order of magnitude larger than the equipartition value of 10^4 G. Upper limits may be provided either by stability considerations (Ferriz-Mas & Schüssler 1993), by the poleward slip motion of toroidal flux

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tubes (Moreno-Insertis et al. 1992, see also this volume), or by the tilt angles of active regions which seem to exclude fields of the order of 10^6 G (D'Silva &Choudhuri 1993). More detailed considerations are necessary here, however, since a tube which becomes unstable within the overshoot layer may lose a significant part of its buoyancy during its initial rise within the stably stratified subadiabatic region. Therefore, fields well in excess of 10^5 G prior to eruption could possibly be required in order to maintain the dominance of buoyancy. We may note in this context that Dziembowski & Goode (1989) have estimated from oscillation data that a magnetic field of the order of $10^{6\pm1}$ G might reside near the bottom of the solar convection zone.

How could such large field strength be achieved ? Flux tube stretching by differential rotation is a possible source (Petrovay 1991, Fisher et al. 1991) although no dynamically consistent calculation has been published so far. Super-equipartition field strength must not necessarily cause fundamental problems even for the operation of an $\alpha\Omega$-type dynamo if sufficiently large energy input into differential rotation is guaranteed (Durney et al. 1990). In any case, fields of 10^5 G or even 10^6 G cannot be stored in the convection zone proper since there they are extremely buoyant and unstable – even the invocation of thermal shadows (Parker 1987b) would not help in this case. The only possible storage location seems to be a stably stratified overshoot region or even the outer layers of the radiative core. Clearly, the large field strength excludes any attempts to treat the dynamo problem in the kinematic approximation; a 'dynamic dynamo' on the basis of strong fields is required.

4. Flux Tube Dynamo: Beyond Cartoons ?

The flux tube structure of solar surface fields and its possible extension throughout the whole convection zone raises the question whether a dynamo mechanism could operate on the basis of flux tubes (e.g. Schüssler 1980). An example of a flux tube dynamo is the STFW-dynamo ('Stretch-Twist-Fold-Wait') of Vainshtein & Zeldovich (1972, see also Zeldovich et al. 1983) which, however, has not been considered in detail so far. Processes reminiscent of STFW have been identified in the dynamo simulations of Nordlund et al. (1992). It is presently unclear, however, how this kind of mechanism could provide field reversals or a butterfly diagram.

Given the arguments for very strong fields prior to eruption presented in the preceding section it seems improbable that they result from a flux tube dynamo mechanism of any kind within the convection zone proper since there is no way to store tubes of 10^5 G or more within a superadiabatically stratified region for times of the order of the solar cycle period. Even if that could be achieved, instabilities would shred the magnetic structures until eventually the resulting fragments become passively coupled to convection by drag forces (Schüssler, 1984, 1987). While such a system of passive fibrils may be described well by conventional kinematic mean-field dynamo theory (Parker 1982) it shares, on the other hand, the problems of this theory regarding the dynamical properties of emerging active regions.

In a subadiabatic overshoot region the situation is different. We have already seen in Sec. 3.5 that the total flux emerging during one activity cycle fills only a small fraction of the available volume of the overshoot layer if the field strength is

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larger than 10^5 G. Consequently, flux tube structure of this field cannot be excluded. The stable stratification provides the possibility to store such strong fields until they eventually become unstable with respect to non-axisymmetric, undulatory instabilities and erupt to form active regions.

Which kind of dynamo mechanism could operate on the basis of an ensemble of strong-field flux tubes ? One possibility is to consider a mean-field theory with an α-effect based on flux tube physics. It is well known (e.g. Schmitt 1987) that non-axisymmetric instabilities of toroidal magnetic fields under the influence of rotation lead to growing helical waves which provide an α-effect. Similarly, the undulatory flux tube instability or the resonant excitation of flux tube eigenmodes leads to a an α-effect which, together with differential rotation, could form the basis for dynamo of the $\alpha\Omega$ type which avoids the problems of the turbulent, kinematic dynamos of this type. Together with considerations of fragmentation/coalescence processes (e.g. Bogdan 1985, Bogdan & Lerche 1985) such an approach could provide the basis for a flux tube dynamo in an overshoot layer.

In what follows we shall briefly describe a related, but somewhat different, model which may serve as an illustration of how a cartoon-like idea can be given some mathematical elaboration. This model has been developed jointly with T. Bogdan (High Altitude Observatory, Boulder). Let us start with a cartoon which schematically presents the basic dynamo process. Assume a toroidal flux tube situated in a plane parallel to the solar equator. An undulation develops as consequence of non-axisymmetric instability with a downflow along the legs:

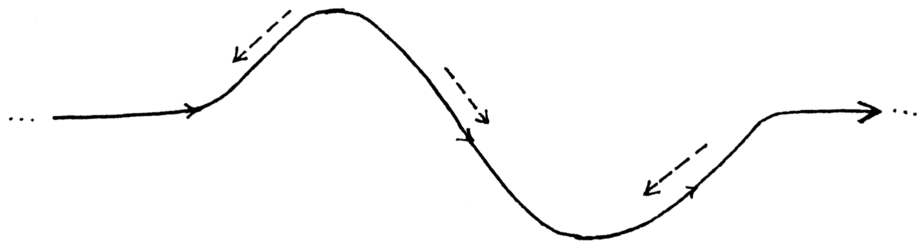

Fig. 2. Loop formation by undulatory instability

The undulation is twisted out of the plane by the Coriolis force acting on the downflow and reconnects, thus forming a closed loop:

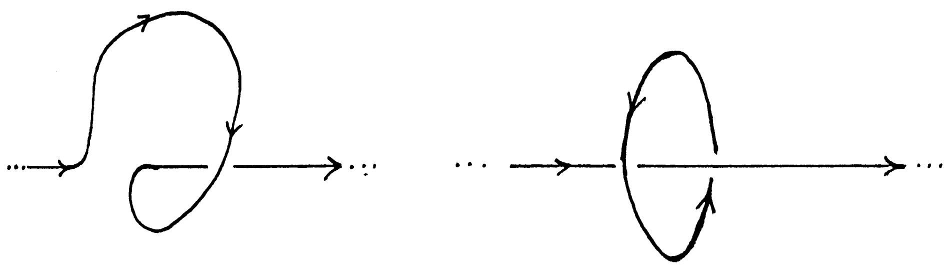

Fig. 3. Twisting by Coriolis force and reconnection

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Differential rotation now stretches the loop until the "ends" meet again and reconnect. This creates a pair of alternately directed flux tubes which are (anti)parallel to the initial tube:

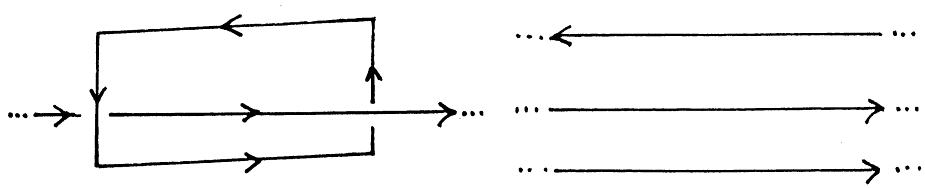

Fig. 4. Stretching by differential rotation

The closed loop is recovered in this reconnection process and could, in principle, be stretched again to form another pair of tubes. The result of this process (which is a variation of the now classical Parker mechanism) is the pair production of toroidal tubes in the vicinity of a given tube.

Certainly, this is a highly idealized picture and we do not claim that it represents correctly the flux tube dynamics in the solar overshoot region. In fact, the stretching by differential rotation is treated in a purely kinematical way which surely is not justified for large field strength. Let us take the model just as an illustration for the kind of mechanisms which might provide the basis of a flux tube dynamo.

Our mathematical treatment of the flux tube dynamo process is a local, spatially one-dimensional, statistical model for straight, parallel flux tubes. We define a local 'tube density' $N(x,t)$ as the sum of the densities for tubes of positive (N^+) and negative (N^-) polarity, viz.

$$N(x,t) = N^+(x,t) + N^-(x,t)\,. \tag{10}$$

The coordinate x, perpendicular to the tube orientation, would correspond to latitude in a spherical model while the tubes would then become toroidal. The local net flux is given by

$$D(x,t) = N^+(x,t) - N^-(x,t)\,. \tag{11}$$

with $|D| \leq N$. In this simple model we assume that all tubes are identical, except for their polarity. We now consider the following processes which determine $N(x,t)$ and $D(x,t)$:

1. The flux tube locations fluctuate through random processes (e.g., turbulence) which are described by a diffusion term with effective diffusivity η.
2. Flux tubes of different polarity at the same (statistical) location annihilate with a rate σN^+N^- where σ represents an efficiency parameter.
3. The dynamo process described above creates pairs of tubes, which appear symmetrically to an existing tube in a distance $\pm\Delta$ and with a production rate ϵ. Since the process of loop formation, stretching, and reconnection takes some time, a time delay τ is introduced.

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By considering these processes we obtain the following pair of equations for N^+ and N^-:

$$\frac{\partial N^+}{\partial t} - \eta\frac{\partial^2 N^+}{\partial x^2} = -\sigma N^+ N^- + \epsilon N^+(x-\Delta, t-\delta) + \epsilon N^-(x+\Delta, t-\delta)\,, \quad (12)$$

$$\frac{\partial N^-}{\partial t} - \eta\frac{\partial^2 N^-}{\partial x^2} = -\sigma N^+ N^- + \epsilon N^-(x-\Delta, t-\delta) + \epsilon N^+(x+\Delta, t-\delta)\,. \quad (13)$$

We now assume that Δ is small compared to the system dimension L (solar radius) and that τ is small compared to the cycle length (or small compared to the diffusion time L^2/η) so that we may expand and write

$$N^+(x \pm \Delta, t-\delta) \simeq N^+(x,t) \pm \Delta\frac{\partial N^+}{\partial x} - \delta\frac{\partial N^+}{\partial t} \mp \Delta\delta\frac{\partial^2 N^+}{\partial x \partial t}\,, \quad (14)$$

and similar for N^-. Adding and subtracting the resulting equations for N^+ and N^- leads to the following system of equations for N and D:

$$(1+2\delta\epsilon)\frac{\partial N}{\partial t} - \eta\frac{\partial^2 N}{\partial x^2} = -\frac{\sigma}{2}(N^2 - D^2) + 2\epsilon N\,, \quad (15)$$

$$\frac{\partial D}{\partial t} - \eta\frac{\partial^2 D}{\partial x^2} = -2\epsilon\Delta\frac{\partial D}{\partial x} + 2\epsilon\Delta\delta\frac{\partial^2 D}{\partial x \partial t}\,. \quad (16)$$

Note that the equation for the net flux D is linear and decoupled from the other equation. In the simplest case of an infinite interval the solution is given by

$$D = \hat{D}\exp(i\omega t + ikx)\,, \quad (17)$$

where ω is the (complex) growth rate, k the (real) spatial wave number, and $\hat{D}$ = const. If we insert (17) into (16) we find for the real and imaginary parts of ω:

$$\Re(\omega) = -\frac{2\epsilon\Delta k(\delta\eta k^2 + 1)}{(2\epsilon\Delta k)^2 + 1}\,, \quad (18)$$

$$\Im(\omega) = \frac{\eta k^2 - (2\epsilon\Delta k)^2\delta}{(2\epsilon\Delta k)^2 + 1}\,. \quad (19)$$

Consequently, we obtain an oscillatory dynamo with propagating dynamo waves. The dynamo is excited if

$$\frac{(2\epsilon\Delta)^2\delta}{\eta} > 1\,. \quad (20)$$

In the marginal case [$\Im(\omega) = 0$] we find for the oscillation frequency

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$$\Re(\omega) = -\left(\frac{\eta k^2}{\delta}\right)^{1/2} = -(\tau_d\, \delta)^{-1/2}, \tag{21}$$

where $\tau_d \equiv 1/(\eta k^2)$ represents the diffusion time over one wavelength. Hence, the oscillation period is of the order of the harmonic mean of the time scales of diffusion and tube pair production. As we see from (20) non-vanishing spatial (Δ) and temporal (τ) delays for the production of tube pairs are crucial for the excitation of the dynamo. As in all dynamo models, the excitation must exceed a certain level which depends on the value of the diffusivity.

We may determine the required values for η and for ϵ, the rate of pair production, in the solar case by inserting reasonable numbers for the other parameters. We take a value of $8 \cdot 10^{10}$ cm for the wavelength of the dynamo wave and assume a loop formation/stretching timescale of 10^8 s (about 3 years). By requiring that the period should match the solar cycle period of 22 years we obtain from (21) a value for the diffusivity: $\eta = 1.3 \cdot 10^{12}$ $\text{cm}^2{\cdot}\text{s}^{-1}$. This value is compatible with weak turbulence (velocities of a few m/s) in a layer of a few times 10^9 cm thickness. If we take $\delta = 2 \cdot 10^9$ cm (width of the overshoot layer) we obtain from the excitation condition (20) that $\epsilon > 3 \cdot 10^{-8}$ s^{-1} which means that the required rate of pair production for a given tube is of the order of once in 400 days. This is not a prohibitive number and we might consider it possible that a dynamo of this kind operates in the solar overshoot layer.

The approach presented here can be extended in various ways. On the one hand, nonlinearities like flux loss by buoyancy can be easily introduced in order to limit the growth of the net flux to a finite value. On the other hand, one can consider a distribution of flux tube sizes and include fragmentation and coalescence processes. In any case, a dynamical treatment of the flux tube stretching by differential rotation and the twisting by the Coriolis force must be incorporated in order to fulfill the claim for a 'dynamic dynamo'.

References

Acheson, D.J.: 1978, *Solar. Phys.* **62**, 23

Bogdan, T.J.: 1985, *Astrophys. J.* **299**, 510

Bogdan, T.J., Lerche, I.: 1985, *Astrophys. J.* **296**, 719

Byrne, P.B.: 1992, in J.H. Thomas and N.O. Weiss, ed(s)., *Sunspots: Theory and Observations*, Kluwer: Dordrecht, 63

Cattaneo, F., Hughes, D.W.: 1988, *J. Fluid Mech.* **196**, 323

Choudhuri, A.R.: 1989, *Solar. Phys.* **123**, 217

Choudhuri, A.R., Gilman, P.A.: 1987, *Astrophys. J.* **316**, 788

Christensen-Dalsgaard, J.: 1992, in J.T. Schmelz & J.C. Brown, ed(s)., *The Sun: A Laboratory for Astrophysics*, Kluwer:Dordrecht, 29

D'Silva, S., Choudhuri, A.R.: 1991, *Solar. Phys.* **136**, 201

D'Silva, S., Choudhuri, A.R.: 1993, *Astron. Astrophys.* , in press

Durney, B.R., De Young, D.S., Passot, Th.P.: 1990, *Astrophys. J.* **362**, 709

Dziembowski, W.A., Goode, P.R.: 1989, *Astrophys. J.* **347**, 540

Ferriz-Mas, A., Schüssler, M.: 1993, *Geophys. Astrophys. Fluid Dyn.* , in press

Fisher, G.H., McClymont, N., Chou, D.-Y.: 1991, *Astrophys. J.* **374**, 766

Galloway, D.J., Weiss, N.O.: 1981, *Astrophys. J.* **243**, 945

Garcia de la Rosa, J.I.: 1984, *Solar Phys.* **92**, 161

Hale, G.E., Ellerman, F., Nicholson, S.B., Joy, A.H.: 1919, *Astrophys. J.* **49**, 153

https://doi.org/10.1017/S0074180900173826 Published online by Cambridge University Press

Howard, R.: 1989, *Solar. Phys.* **123**, 271
Howard, R.: 1974, *Solar. Phys.* **38**, 283
Hughes, D.W.: 1992, in J.H. Thomas and N.O. Weiss, ed(s)., *Sunspots: Theory and Observations*, Kluwer: Dordrecht, 371
McIntosh, P.S.: 1981, in L.E. Cram & J.H. Thomas, ed(s)., *The Physics of Sunspots*, Sacramento Peak Observatory: Sunspot, New Mexico/USA, 7
Moreno-Insertis, F.: 1986, *Astron. Astrophys.* **166**, 291
Moreno-Insertis, F.: 1992, in J.H. Thomas and N.O. Weiss, ed(s)., *Sunspots: Theory and Observations*, Kluwer: Dordrecht, 385
Moreno-Insertis, F., Schüssler, M., Ferriz-Mas, A.: 1992, *Astron. Astrophys.* **264**, 686
Nordlund, Å., Brandenburg, A., Jennings, R.L., Rieutord, M., Ruokolainen, J., Stein, R.F., Tuominen, I.: 1992, *Astrophys. J.* **392**, 647
Parker, E.N.: 1975, *Astrophys. J.* **198**, 205
Parker, E.N.: 1982, *Astrophys. J.* **256**, 302
Parker, E.N.: 1987a, *Astrophys. J.* **312**, 868
Parker, E.N.: 1987b, *Astrophys. J.* **321**, 984
Petrovay, K.: 1991, *Solar. Phys.* **134**, 407
Pidatella, R.M., Stix, M.: 1986, *Astron. Astrophys.* **157**, 338
Schüssler, M.: 1980, *Nature* **288**, 150
Schüssler, M.: 1983, in J.O. Stenflo, ed(s)., *Solar and Stellar Magnetic Fields: Origins and Coronal Effects*, Reidel: Dordrecht, 213
Schüssler, M.: 1984, in T.D. Guyenne & J.J. Hunt, ed(s)., *The Hydromagnetics of the Sun*, ESA SP–220, 67
Schüssler, M.: 1987, in B.R. Durney & S. Sofia, ed(s)., *The Internal Solar Angular Velocity*, Reidel:Dordrecht, 303
Schüssler, M., Solanki, S.K.: 1992, *Astron. Astrophys.* **264**, L13
Schmidt, H.U.: 1968, in K.O. Kiepenheuer, ed(s)., *Structure and Development of Solar Active Regions*, Reidel:Dordrecht, 95
Schmitt, D.: 1987, *Astron. Astrophys.* **174**, 281
Spiegel, E.A., Weiss, N.O.: 1980, *Nature* **287**, 616
Spruit, H.C.: 1977, thesis, University of Utrecht
Spruit, H.C.: 1981, *Astron. Astrophys.* **102**, 129
Spruit, H.C., van Ballegooijen, A.A.: 1982, *Astron. Astrophys.* **106**, 58
Stenflo, J.O.: 1989, *Astron. Astrophys. Rev.* **1**, 3
Vainshtein, S.I., Zeldovich, Ya.B.: 1972, *Sov. Phys. Usp.* **15**, 159
Van Ballegooijen, A.A.: 1982, *Astron. Astrophys.* **106**, 43
Wang, Y.-M., Sheeley, N.R.: 1991, *Astrophys. J.* **375**, 761
Zeldovich, Ya.B., Ruzmaikin, A.A., Sokoloff, D.D.: 1983, *Magnetic Fields in Astrophysics*, Gordon&Breach: New York
Zwaan, C.: 1978, *Solar. Phys.* **60**, 213
Zwaan, C.: 1987, *Ann. Rev. Astron. Astrophys.* **25**, 83
Zwaan, C.: 1992, in J.H. Thomas and N.O. Weiss, ed(s)., *Sunspots: Theory and Observations*, Kluwer: Dordrecht, 75

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