30. A NUMERICAL ANALYSIS OF THE MOTION OF PERIODIC COMET BROOKS 2

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Abstract. Various sets of osculating elements of P/Brooks 2, derived by Dubyago, are introduced into an *N*-body integration programme and run from 1686 to 1976. Attempts are made to find a system of elements which links the apparitions before and after the close approach to Jupiter in 1922. The propagation of differential perturbations, and also nongravitational effects, is examined.

It is unavoidable that a set of osculating elements of a comet, determined from observations, contains finite errors. Consequently our knowledge of the orbit's evolution in phase space is represented by a 'tube' rather than by a 'thin line'. The coordinates of the centre of the tube, as a function of time, vary due to the combined effect of all planetary perturbations. The variation in shape and size of the tube cross-section is caused by differential perturbations among a group of massless bodies whose initial conditions correspond to the original orbital errors. If the error tube representing a first apparition, at the time of a second one, overlaps with the error tube of the latter to a certain extent, we would probably feel certain we have allowed for all the forces acting. However, if we find systematic separations between the tubes, we have to conclude that the motion of the comet was affected by additional forces. Therefore, the question of whether or not nongravitational forces influence a particular periodic comet depends very much on, among other things, its history of differential perturbations. A close approach to a planet, between two apparitions, not only changes the osculating elements of the comet but may also change the shape and size of the cross-section of the tube that represents the osculating elements (and their errors) of the first apparition. Without knowledge of these changes it would be difficult to compare the elements of the two apparitions.

I want to demonstrate these relations in the case of a comet whose motion is suitable for that purpose; P/Brooks 2, discovered in 1889, is such a case. Its period is seven years. Osculating elements have been derived from observations in 1889, 1896, 1903, 1911, 1925, 1932, 1939, and 1946 by Dubyago (1950, 1956), and these elements are classified as of highest quality in the catalogue of cometary orbits by Porter (1961). We know of four approaches to Jupiter; for brevity we denote them by $♃_1$, $♃_2$, $♃_3$ and $♃_4$. The first one took place three years before discovery, in 1886 (JD 2410108), with a minimum distance of less than 0.001 AU. The second one occurred in 1922 (JD 2423077), with a minimum distance of 0.086 AU. It is particularly the $♃_2$ approach which makes P/Brooks 2 such a suitable object for our purpose since there is probably no other comet known where two sets of observed and carefully evaluated apparitions are interrupted by such a close Jupiter encounter. The approaches $♃_3$ (1958, JD 2436280, minimum distance about 1.3 AU) and $♃_4$ (1969, JD 2440345, minimum distance about 1.6 AU) are of minor importance, compared with the first

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two spectacular ones. Dubyago found it impossible to represent the apparitions by one gravitational orbit both before and after $♃_2$. Therefore, for each series of apparitions, he included additional (nongravitational) terms in the orbit determination. His procedure, as seen from today's knowledge, may be somewhat doubtful, because he did not actually introduce nongravitational forces into his equations of motion. His results are nevertheless striking because they show a rather systematic nongravitational variation of all elements in periods both before and after $♃_2$. Assuming that Dubyago's elements are at least a reasonable first approach to the problem of the true motion of the comet, my numerical analysis may be valuable for a future orbit determination which includes a more modern treatment of the nongravitational forces. In addition, the analysis of a comet with both nongravitational effects and close Jupiter approaches might throw some light on the difficulties which, for instance, have been reported by Marsden (1969).

All perturbation calculations were carried out with the Heidelberg N-body programme (Schubart and Stumpff, 1966); the planets Venus to Neptune were taken into account throughout the whole investigation using the initial values in the original publication of the programme (l.c., Table VII). The results of the N-body calculations were then converted into heliocentric ecliptical osculating elements by a special programme. Since Dubyago uses the mean equinox 1890.0 for the definition of his first set of nongravitational effects, I have chosen the same equinox for the presentation of my results.

The osculating elements for the period 1889–1946 (Dubyago 1950, p. 25; 1956, pp. 26–27) were introduced into the N-body programme in the form of eight massless bodies and were integrated over the entire time interval 1887–1976. The steplength normally was two days, except during the time of the approach $♃_2$ when it had to be reduced to 0.5 days. I have attempted to find an orbit which gravitationally 'links' the two observed periods before and after $♃_2$, using a trial-and-error method based on the matrix of partial derivatives of the elements in the neighbourhood of the Jupiter approach. The conditions for the linking orbit were set so that a forward extrapolation of the nongravitational effects observed before $♃_2$, and a backward extrapolation of these effects observed after $♃_2$, would lead to approximately the same orbit. The forward extrapolation in itself was problematic because no elements were available for the 1918 apparition. A unique solution of the link problem is impossible with the methods I have applied. However, two of the solutions which I found may serve as a first approximation; they are represented by two bodies denoted by $L1$ and $L2$, respectively, $L1$ being somewhat better than $L2$.

In order to demonstrate the total perturbations caused by the planets, four plots are given in Figure 1 for the four osculating elements, $\tilde{\omega} = \Omega + \omega$,* i, φ, and n. During the approach, all elements oscillate heavily and reach maximum and minimum values which are not visible in Figure 1. To the left of $♃_2$, the thick line represents the 'tube' in phase space which contains the bodies 1889, 1896, 1903, and 1911. The

* This was chosen instead of a single representation for both Ω and ω, because these two elements are each changed due to the Jupiter perturbations by about 180° – an effect which, of course, is only a formal consequence of the usual convention for the orbital inclination.

strong differential perturbations during ♃$_2$ let them appear separately to the right of ♃$_2$. The direct numerical results indicate that the 'magnification factor' for the tube diameter is of the order of 30–300. To the right of ♃$_2$, the thick line represents the tube containing the bodies 1925, 1932, 1939 and 1946, and the differential perturbations (if we follow a backward calculation) split them into four different curves which are distinguishable in the left half of the plots; the magnification factors are here of the order of 80–600. That the bodies *L*1 and *L*2 are linking the two observational periods is clearly indicated by the fact that they are the only bodies which are contained in the thick line on both sides of ♃$_2$.

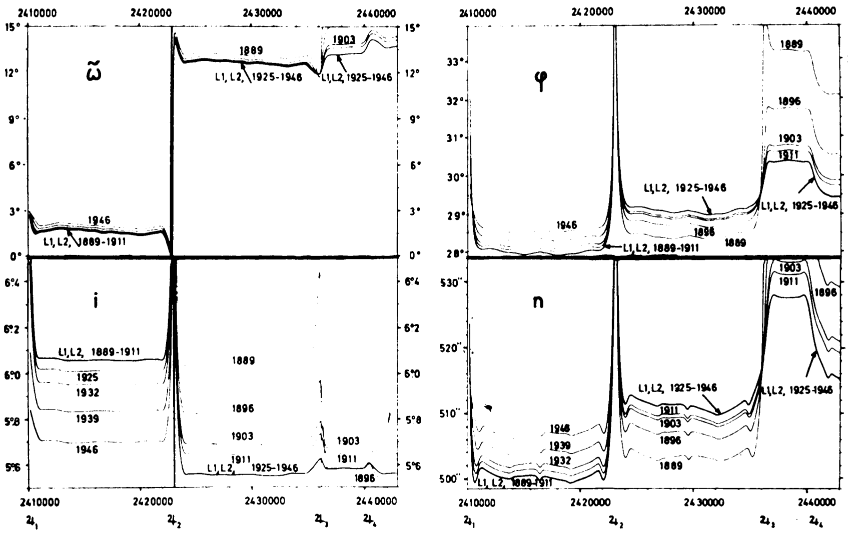

Fig. 1. Osculating elements between 1887 and 1976. Abscissa is the Julian Date. The curves are marked by the year of apparition. The times of Jupiter approaches are indicated by ♃$_1$ – ♃$_4$ on the bottom. *L*1 and *L*2 link the observations before and after ♃$_2$. Note that they coincide before ♃$_2$ with the apparitions 1889–1911 and after ♃$_2$ with the apparitions 1925–1946. Mean equinox 1890.0.

On the left edge of the plots, all the curves approach the singularity zone which corresponds to ♃$_1$; this will be discussed later. In the right half of the diagrams, one can see the effect of the ♃$_3$ and ♃$_4$ approaches. One should note that the order of magnitude of the total perturbations is similar for ♃$_2$, ♃$_3$ and ♃$_4$, whereas in the case of differential perturbations, ♃$_2$ has an effect which is tremendously large compared to ♃$_3$ and ♃$_4$.

Total perturbations by all planets are rather weak during the long time intervals from one close Jupiter approach to the next, and differential perturbations are negligible. The sharp minima in n near JD 2430000 and JD 2416000, and the corresponding maxima in φ, are caused by Jupiter (distance 3 AU).

We will now look in more detail at the differential perturbations and the non-gravitational effects. In Figures 2a and 2b, the differences of the osculating elements of the observed bodies relative to the osculating elements of our linking body, $L1$, are plotted over the entire time interval 1887–1976. Body $L1$ is represented by the zero line; note that by definition, this line crosses the ♃$_2$ singularity without any disturbances. The epochs of the apparitions are marked by arrows on the curves; the arrow immediately to the left of ♃$_2$ corresponds to the unobserved 1918 apparition. Let us particularly look at the plot of Δi, because its behaviour is ideally suited for the demonstration of nongravitational effects. Within both observing periods, all curves are parallel to the zero line and do not contain any disturbances. This means

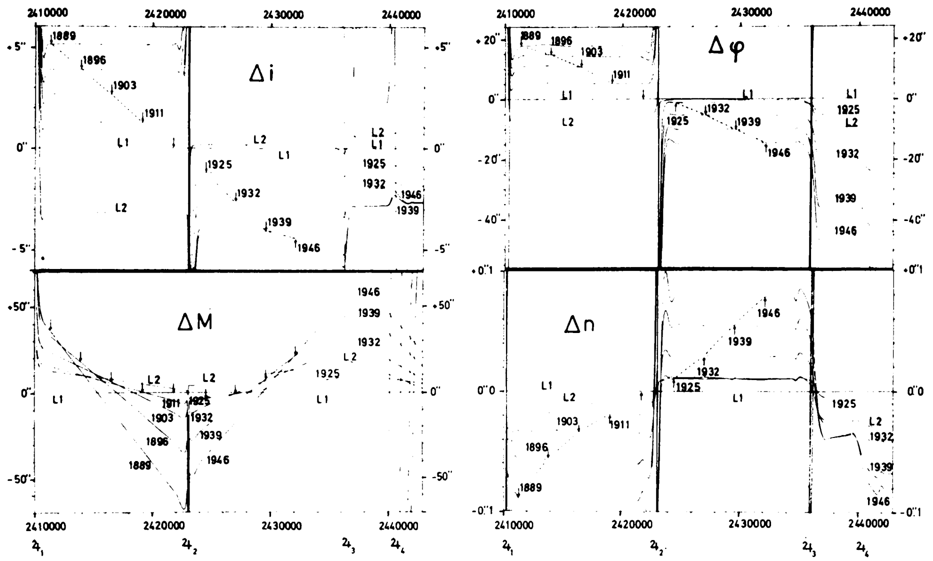

Fig. 2a. Differential evolution of inclination, eccentricity, mean daily motion, and mean anomaly between 1887 and 1976. Abscissa is the Julian Date. For each body, the difference of the elements relative to body $L1$ is plotted. The epochs of introduction of the bodies into the N-body programme (i.e., osculation epochs of Dubyago's elements) are marked by arrows on the curves. The arrow on the zero line left of ♃$_2$ corresponds to the perihelion time of the missing apparition. The broken line connecting the observed apparitions corresponds to the assumption that nongravitational effects are acting along the entire orbit.

that the orbital inclination was not affected by differential perturbations. The distance between each pair of consecutive curves corresponds to the total nongravitational effect per revolution. The complete absence of differential perturbations makes it possible to explain the action of nongravitational forces in many different ways. Firstly, these forces could have acted continuously along the entire orbit; in this case their effect on the inclination would then be represented by the broken line which connects the arrows. Secondly, they could have acted only during small time intervals near the perihelia, in which case a step function, jumping from one curve to the next, would demonstrate the effect in the orbital element. Finally, the distances between the curves could have been produced by a number of discrete actions along the orbit. This latter model is not very probable because it would be difficult to understand why a series of discrete events produces always the same total change per revolution.

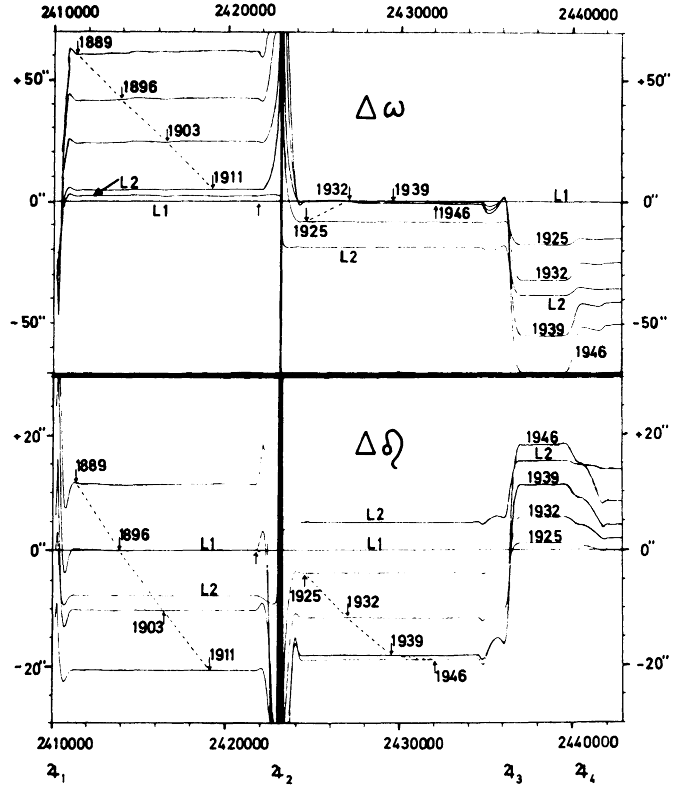

Fig. 2b. Differential evolution of longitudes of perihelion and ascending node between 1887 and 1976.

The diagrams for $\Delta\varphi$ and Δn are similar to the one for Δi, although we find here some differential perturbations caused by Earth and Mars at the times of the first three apparitions. In all three elements, our linking body allows us to approximate the nongravitational effects over the entire period by a straight line which is not disturbed by the ♃$_2$ singularity.

The small curvature in Δn before ♃$_2$ corresponds to a second-order term introduced by Dubyago in his evaluation of the observations; looking at the ΔM diagram, one finds it possible to approximate the envelope of the arrows by a parabola (which would correspond to a linear Δn), but it is questionable whether one can apply a more refined interpretation.

Differential perturbations are also absent in the case of $\Delta \Omega$ and $\Delta\omega$, as Figure 2b shows. However, the general behaviour of $\Delta \Omega$ and $\Delta\omega$ appears to be different from the behaviour of the other elements. In the case of $\Delta\omega$, this certainly reflects features of Dubyago's osculating elements which have nothing to do with the ♃$_2$ singularity or with the method I have used to analyse his data. In the case of $\Delta \Omega$, where on both sides of ♃$_2$ the nongravitational effect decreases linearly with a similar slope, I assume that it is my definition of the linking body, $L1$, which produces the strong discontinuity observed in Figure 2b. It is easy to understand that we would obtain a $\Delta\Omega$ representation similar to the diagrams in Figure 2a, if we were to use a linking body other than *L*1. Indeed, in my attempts to link the two observing periods, I have found such solutions. However, they always produced discontinuities in all other elements. One possible explanation for this difficulty would be that the observations were not treated correctly by Dubyago. The other possibility is that the nongravitational forces act along the entire orbit (and, therefore, also within the ♃$_2$ singularity zone). As I said before, this hypothesis cannot be proven in the observing periods which were free of differential perturbations. In Figures 2a and 2b, the curves enter the singularity zone at heliocentric distances of 3.3 AU (JD 2422000) and 3.7 AU (JD 2424000). Both these distances are outside the range of cometary activity proposed by Whipple (1950) in his comet model. If we assume that the nongravitational forces exist even at large distances from the Sun, the following will happen: changes will be continuously produced in the elements and will be continuously magnified by the differential perturbations – a feature not contained in my calculations but which might be extremely helpful in order to study possible mechanisms of cometary activity. If the numerical integration included those forces which correspond to the nongravitational effects observed in the osculating elements outside the singularity zone, a solution, free of discontinuities, for the linking body, might be found.

In the present investigation, it was not possible to carry out such an analysis. Therefore, I chose arbitrarily a solution for the linking body which left a discontinuity in $\Delta\Omega$ only but which permitted the nongravitational effects to be represented satisfactorily in all other elements. Even in this case, the discontinuity is only of the order of 40″, which is not very much if one takes into account the fact that one missing apparition and one very close Jupiter approach had to be linked.

Body *L*1 may be a useful starting point for future investigations. In particular, one could possibly try to use it to search for the comet during the missing 1918 apparition, if photographic plates should still exist somewhere. Table I gives a set of osculating elements defining body *L*1.

TABLE I

Osculating elements of Body *L*1 at Epoch JD 2421600.5

M =	354°49′11″.3	
ω =	343 36 25.7	1890.0
Ω =	17 53 15.6	1890.0
i =	6 03 41.7	1890.0
φ =	28 02 03.9	
n =	501″.267769	
T =	JD 2421637.7	

As was mentioned before, *L*2 is not as good as *L*1; the Figures 2a and 2b illustrate that it causes discontinuities in all elements except the mean daily motion.

The remaining features of Figure 2 are almost self-explanatory. The vertical lines near ♃$_3$ and ♃$_4$ correspond to some of the bodies 1889–1911 which, due to differential perturbations, cross the plots. The curve entering the plot of Δi from above, and reaching two minima near ♃$_3$ and ♃$_4$, belongs to body 1896.

From the rather systematic behaviour of the nongravitational effects within the entire interval 1889–1946, as indicated in Figures 2, I gained some confidence in the possibility of their backward extrapolation. This is indeed extremely interesting because the orbit before ♃$_1$ must have been completely different from the present orbit.

Dubyago (1950) himself had made such backward calculations, using his elements *A* (l.c, p. 25) and *B* (l.c., p. 26) as starting orbits, and following their development through the ♃$_1$ approach back until 1883. The comparison between these two bodies does not give a realistic picture of the influence of differential perturbations on the orbit, because their definition is not a measure for the uncertainties of the backward extrapolation of the nongravitational effects. In my own calculations, I am considering a group of eight bodies. The first one is body 1889 (Dubyago's elements *A*). Its backward integration was started at JD 2411284; at the date JD 2411000, a new body was introduced which corresponds to body 1889 plus the nongravitational effects accumulated from 2411284 until 2411000. This body is denoted here by *B*1. Similarly, at the date JD 2410800, a body *B*2 was introduced by adding to *B*1 the nongravitational effects accumulated since JD 2411000. In this way, a series of five bodies, *B*1–*B*5, was created which might be considered an approximation for all nongravitational effects back to JD 2410214, a date which is only 106 days away from ♃$_1$ and where the distances to the Sun and Jupiter were 5.3 and 0.3 AU, respectively. For this date, Dubyago gives osculating elements also, from his own calculations, and I have introduced these into my calculations for comparisons; the corresponding body is denoted here by *D*. The last body which I have considered is obtained by adding to body 1889, at the starting epoch JD 2411284, the total nongravitational effects to be expected for the interval between JD 2410214 and the starting epoch. This body is denoted by *T*; it demonstrates a model where half of the nongravitational effects per revolution are added to the osculating elements at perihelion. Since only the time from the starting epoch to JD 2410800 is completely free of differential perturbations, and since at the latter date the heliocentric distance was about 4 AU, we may also consider the body *T* as an approximation to Whipple's model where the nongravitational forces are acting continuously within 4 AU of the Sun. On the other hand, body *B*5 corresponds to a model where the forces have continuously acted within 5.3 AU of the Sun.

For Jupiter's flattened potential field, I have used a first-order term with the same coefficient, $J = 0.022273$, that Dubyago used. Dubyago did not include the perturbations by the other planets during the approach and treated the passage in jovicentric coordinates, whereas in my calculations heliocentric coordinates were kept (smallest steplength 0.001 day), and the influence of all planets was taken into account. These differences are probably responsible for the fact that bodies *B*5 and *D*, as will be seen below, are not identical. As far as the above definition of the eight test bodies is concerned, I feel that bodies *B*5, *D* and *T* are a reasonable approximation to the motion of the actual comet before and at ♃$_1$.

The results of the calculations are shown in Figure 3. Here, the differences of the osculating elements of all test bodies are plotted relative to the osculating elements of body 1889; the latter is then by definition represented by the zero line. The ratio between the 'tube' in phase space before and after ♃$_1$, as can be seen directly from Figure 3, is not as large as one might expect in the case of such a close approach to

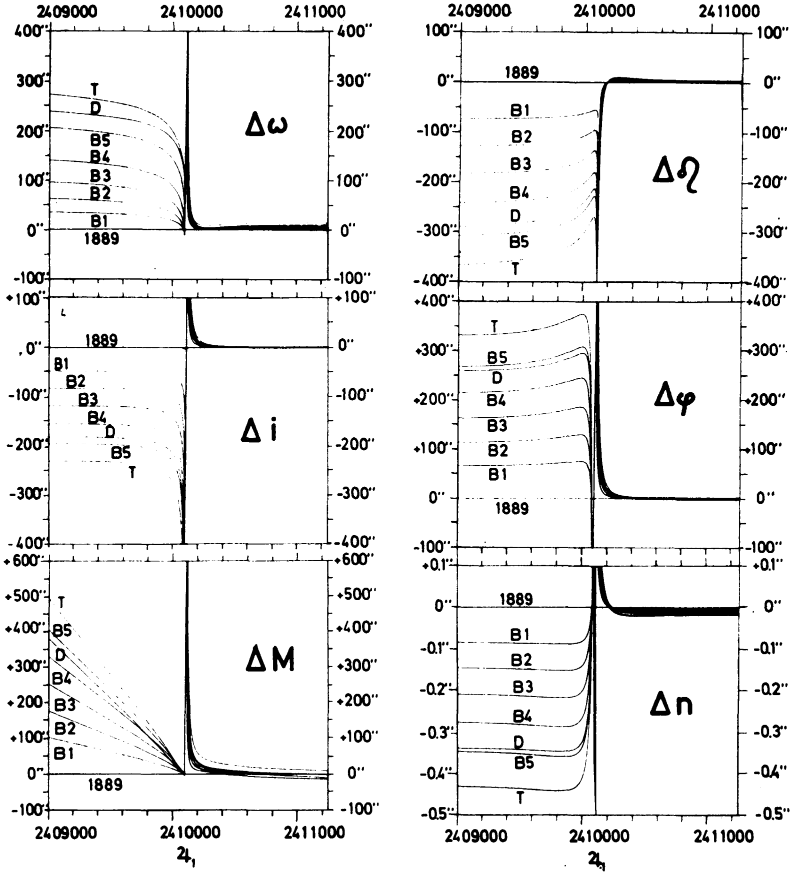

Fig. 3. Differential evolution of orbital elements near the ♃$_1$ approach. Abscissa is the Julian Date (1883–1889). For each of eight test bodies the element difference relative to body 1889 is plotted. The diameter of the 'tubes' near the right edge of the plots corresponds roughly to one half of the nongravitational effects per revolution. For the definition of the test bodies, see explanation in the text. The plot of Δn may be looked on as an illustration of celestial mechanics energy splitting due to strong perturbations.

Jupiter. The magnification factors for the various elements vary between 20 and 115, as the direct numerical results show.

The dates of closest approach to Jupiter, for the eight bodies considered here, are all within JD 2410108.51 and 2410108.78. The minimum distances from Jupiter's surface, in units of its equatorial radius (4.8×10^{-4} AU) are shown in Table II. The sequence of these numbers indicates that smaller distances would be obtained if

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TABLE II

Jupiter Approach 1886 ($♃_1$)

Body	Minimum distance from surface
1889	1.055
*B*1	1.040
*B*2	1.028
*B*3	1.016
*B*4	1.008
*B*5	0.995
D	1.001
T	0.987

stronger nongravitational effects were assumed. However, a more precise description of the potential field of Jupiter would also change these results, and Dubyago's remarks concerning the negligible influence of the satellites would have to be carefully studied before one can go any further with these speculations.

In order to investigate the history of the comet prior to $♃_1$, I have integrated the eight test bodies backward in time, ignoring nongravitational effects, to 1686. The results of this calculation are summarized in Table III. Between the first two dates

TABLE III

Osculating elements of bodies *B*5, *D*, and *T* before $♃_1$

Body	JD	1890.0			φ	n	q (AU)	Q (AU)
		ω	☊	i				
*B*5	2409000	2°21	186°43	6°51	26°83	112″55	5.48	14.48
	2357000	1.80	187.35	6.53	26.10	118.52	5.40	13.88
	2353000	354.91	190.50	6.83	22.30	140.93	5.33	11.85
	2337000	348.66	195.90	6.33	25.69	120.81	5.39	13.64
D	2409000	2.22	186.44	6.51	26.82	112.56	5.49	14.48
	2357000	1.82	187.35	6.54	26.10	118.49	5.40	13.89
	2353000	354.65	190.63	6.85	22.19	141.64	5.33	11.80
	2337000	342.66	201.14	5.78	27.13	113.26	5.41	14.47
T	2409000	2.23	186.42	6.50	26.84	112.47	5.48	14.49
	2357000	1.74	187.37	6.53	26.04	118.87	5.40	13.85
	2353000	357.51	189.30	6.68	23.38	133.98	5.36	12.41
	2337000	358.64	189.93	6.68	23.32	134.74	5.35	12.36

tabulated, all three orbits remain close to each other, and the mean orbit is quite stationary. Between the second and third dates, a perturbation by Jupiter occurs; the dates of the approach vary from JD 2354266 to JD 2354324 (Aug.–Oct. 1733) and the minimum distances are between 0.81 and 1.10 AU. All eight test bodies participate in this event, and it might be mentioned that body 1889 comes closest to Jupiter (0.31 AU). The orbit of body *T* remains rather stable from this encounter to the end of the calculations. The two other bodies, however, again pass close to Jupiter on JD 2344600 (March 1707), the minimum distances being 0.90 and 0.64 AU, respectively.

The mean anomalies at JD 2337000 are 95° (*B*5), 110° (*D*), and 79° (*T*). Thus the uncertainty in the position of the comet at the end of the calculations has become so large that it would be meaningless to follow the motion further back in the past. Just for curiosity it should be mentioned that body *B*2, which may not represent the actual comet, passes Saturn on JD 2343866 (1705) within 0.7 AU.

Acknowledgments

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Discussion

Yu. V. Evdokimov: Is there a nongravitational effect perpendicular to the orbital plane of P/Brooks 2?

P. Stumpff: If one believes Dubyago's orbit determinations, components in all directions must exist. If these forces were limited to the orbital plane, one would not expect the changes in Ω and i. However, I may not have made clear enough that in the case of Ω and ω the situation is more uncertain than in the case of the other elements, possibly due to the fact that i is so small.

S. K. Vsekhsvyatskij: P/Brooks 2 was observed at its first apparition in 1889 with a large number of satellites, indicating the comet's decay. Can one really therefore maintain that the comet made approaches to Jupiter in the eighteenth century?

P. Stumpff: After the discovery there was speculation that the comet was identical with P/Lexell, which had been observed in 1770, was known to have subsequently passed near Jupiter, but was never detected again. Dubyago had shown that the time between the approaches of the two comets to Jupiter was inconsistent with the revolution periods. I became interested in the pre-1886 orbit because it goes beyond Saturn's orbit, and I wanted to discover the role of Saturn in the comet's history. The backward calculations, showing possible earlier approaches to Jupiter and Saturn, are a reasonable extrapolation of the orbital information available to us, but of course no definite proof that these events actually took place.

E. I. Kazimirchak-Polonskaya: Did you consider that an inaccurate value for the mass of Jupiter could have affected the representation of the observations before and after the approach to Jupiter in 1922, or have you attributed the residuals entirely to nongravitational effects? Have you tried to use the close approach for determining the mass of Jupiter?

P. Stumpff: I wanted to define a strictly gravitational orbit – a sort of interpolation orbit, so to speak – that gives reasonable agreement with Dubyago's elements both before and after the approach. I found that varying Jupiter's mass by about $\pm 0.02\%$ within an interval of about 2000 days around the approach would produce differential effects in the elements which have a magnitude similar to the observed nongravitational effects. So the interpolation orbit will certainly depend on the value assumed for Jupiter's mass, but I doubt that one can in this case come to any conclusions about Jupiter's mass because the whole picture is so heavily disturbed by the existence of the nongravitational effects – or if you wish, by the existence of residuals which have not been explained properly.