THEORETICAL REVIEW OF SECULAR INSTABILITIES IN THE SUN

M. GABRIEL
Institut d'Astrophysique de l'Université de Liège

INTRODUCTION

In this review we discuss the problems raised by the discovery that the sun was, in the past, unstable towards non-radial oscillations.

STABILITY OF THE SUN TO NON-RADIAL OSCILLATIONS

In 1972, Fowler (1972), in an attempt to explain the low-neutrino flux neasured in Davis' experiment (now 1.6 snu, while the standard solar model predicts 4.4 snu) suggested that the sun could have undergone, some 10^7 years ago, a change in structure because of sudden mixing of the inner core. During the same year Dilke and Gough (1972) suggested the sun is unstable to low-order gravity modes (g^+ modes) of non-radial oscillation and that the mixing is triggered when the amplitude of the oscillation becomes large enough.

This work was criticized by several authors - Ulrich and Rood (1973), Unno (1975), Dziembowski and Sienkiewicz (1973). In fact, the theory failed to predict the spherical harmonic ℓ of the unstable modes and the age of the unstable models. However, a more thorough attempt by Unno (1975) also failed because it is very difficult to predict the run of the eigenfunctions for low-order modes of low spherical harmonic order as these modes are not trapped in one region of the sun. Even if this result was a lucky one, it had the merit of initiating calculations of the stability of the sun to non-radial oscillations. Calculations were made by Dziembowsky *et al.* (1973), Christensen-Dalsgaard *et al.* (1974, 1975), Noels *et al.* (1975), Shibahashi *et al.* (1975) and Boury *et al.* (1975). In all cases, the vibrational stability was computed from the adiabatic solution by a perturbation technique. This means that effects such as the κ-mechanism, important in the outer layers, could not be taken properly into account. These calculations differ only in the way that they treat the interaction between pulsation and convection. They all agree that the sun was unstable in the past, when it was roughly

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between 2 x 10^8 and 3 x 10^9 years of age. The unstable modes are the g_1 and g_2 modes of $\ell = 1$. The nuclear reactions drive these instabilities, but this driving is large enough to de-stabilize some solar models only because of a very peculiar behaviour of the eigenfunctions.

While $\delta r/r$ shows nothing striking, $\delta p/p$ shows for the unstable models large values, of the same order as near the surface, in the region of large nuclear energy production. This characteristic of $\delta p/p$ and $\delta T/T$ magnifies the destabilizing influence of the nuclear energy generation. Because this was unexpected and because $\delta T/T$ goes to zero at the center for non-radial oscillations, it had always been argued before that these modes would show small perturbations of the energy generation rate in the central core and as a result would be stable.

Also important in the discussion of the stability is a property of the p - p chain that I will explain using its simplified form:

$$2\left({}^1H(p,e^+\nu)\ {}^2H\ \ (p,\gamma)\,{}^3He \right),\quad {}^3He\ ({}^3He,\ 2p)\,{}^4He$$

The mean lifetimes of ^1H, D and He are respectively of the order of 5 x 10^9 yr., 3 sec., and 5 x 10^5 yr. Since the time-scale of the evolution is much longer than 10^6 yr., ^3He and ^2H keep constantly their equilibrium value, although the abundance of ^3He does not change. As a result, the sensitivity of the energy generation rate ε to the temperature, $\partial ln\varepsilon/\partial lnT$ is equal to that of the p - p reaction, i.e.,

$$\partial ln\varepsilon/\partial lnT = \nu_{11} \simeq 4$$

During the oscillations, conditions change on a time-scale of the order of one hour and ^2H still keeps its equilibrium value, though the abundance of ^3He does not change. As a result, the energy-production by $^3He(^3He,2p)^4He$, ε_{33}, changes now with the temperature as

$$\partial ln\varepsilon_{33}/\partial lnT = \nu_{33} \simeq 16$$

and ε changes as $\nu_e = \dfrac{\partial ln\varepsilon}{\partial lnT} = \dfrac{(\varepsilon - \varepsilon_{33})\nu_{11} + \varepsilon_{33}\nu_{33}}{\varepsilon}$.

If ^3He has its equilibrium value, this gives $\nu_e \simeq 1/2(\nu_{11}+\nu_{33}) \simeq 10$, a value larger than the static one. If the static value were used, no instability would be found. In unstable models, the $He(^3He,p)^4He$ reaction must produce about half the energy and the ^3He abundance must be close to its equilibrium value in standard models. This impies a ^3He gradient in the inner core. It can sometimes be found in the literature that this ^3He gradient has a strong de-stabilizing influence. This is not correct. What matters is the value of ν_e and therefore the ratio $\varepsilon_{33}/\varepsilon$. If, for instance, the core of a solar model is suddenly mixed, lots of He is brought near to the center, increasing ε_{33} and also the instability, though the He gradient is destroyed.

As I said before, the treatment of the outer layers was very inaccurate in these calculations. Since then, Goldreich and Keeley (1977) have computed with a non-adiabatic code the stability of radial oscillations for the present sun. They found a strong de-stabilization in the hydrogen ionization zone and in the H^- opacity region. These driving mechanisms

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are very likely also important for low-order g^+ and p modes of non-radial oscillations. Therefore all the calculations should be re-done with a non-adiabatic code before we can give significant numbers for the e-folding times of the instabilities and for the time-interval during which models are unstable.

Another problem needs also to be solved. It is that of the interaction between convection and pulsation. The problem is very difficult and no completely satisfactory solution exists. Also, very often people choose to neglect the interaction. However, this is not a clear statement since one can neglect either the Lagrangian or the Eulerian perturbation of the convective flux, or of its divergence; and other possibilities exist. All these hypotheses lead to very different results, as can be seen by comparing the results of the Japanese and the Cambridge groups. So far, the only theories I know of for non-radial oscillation are those developed in Liège (Gabriel *et al.*, 1974,1975) by Unno (1976) and by Xiong (1978), but only the first one has been used in numerical calculations.

It would be useful to have other theories and to compare their predictions. Moreover, the non-linear effects of convection could be important if, as argued by Goldreich and Keeley (1977), they are able to limit the amplitudes to very small values. Again this is a very difficult problem, but one which should be kept in mind when considering the possible consequences of these instabilities on the evolution of the sun. The instabilities will influence the evolution if they lead to some kind of mixing when they reach finite amplitude.

I see two ways of producing mixing without resorting to rotation or magnetic fields. First, one mode excites also its harmonics - i.e., spherical harmonic orders with $k < 2\ell$. Among them there is a time-independent distortion which will lead to meridional circulation. In the particular case of the unstable modes discussed here, the distortion will have $k = 2$. Ulrich (1974) discussed this problem in a rather crude way for the present sun. He concluded that, because of the observational limits on the pulsational velocity, mixing cannot occur in the present sun. Roxburgh (1978) finds that the circulation travels half the solar radius in 10^9 yr if $\overline{\delta T/T} \simeq 7 \times 10^{-3}$. I would rather suggest that 10^9yr are required for the circulation to travel 0.1 $R_\odot$ if $\overline{\delta T/T} \simeq 10^{-2}$. These figures are only rough orders of magnitude and more accurate calculations should be done. Even if circulation is inefficient now, it could have been present in the past when the eigenfunctions were very different and when the sun was peobably more unstable.

A second way of producing mixing is for one mode to excite all the other overtones. This could produce some mixing, but on a local scale, and it could lead to diffusion of H and ^{3}He towards the center. The hydrogen profile will be modified, but the ^{3}He profile will be changed only where the mixing time is larger than its mean lifetime. If ^{3}He is burned in regions where $\delta T/T$ is large, the instability will be reinforced.

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Complete mixing could only occur if the mixing process becomes unstable at some critical level. I know of no work on this problem and presently nobody really knows if mixing will occur and, if it does, how it will modify the chemical composition in the solar core.

The study of the consequences of the solar vibrational instability is important not only for our understanding of the sun but also because all non-fully convective stars less massive than the sun go through the same instability (Noels *et al.*, 1974, 1976) during some fraction of their main-sequence lifetime.

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