57. ON THE STABILITY OF THE OORT CLOUD

E. M. NEZHINSKIJ
Institute for Theoretical Astronomy, Leningrad, U.S.S.R.

Abstract. An attempt has been made to estimate the rate of destruction of the Oort cloud by stars passing through it. Two different mechanisms of cloud dispersion have been investigated. Numerical estimates show that the cumulative dispersing mechanism plays the leading role. The lower limit for the half-life of the cloud is 1.1×10^9 yr.

1. Introduction

Recently there have appeared works (Sekanina, 1968; Vsekhsvyatskij, 1967; Zal'kalne, 1969) investigating the problem of destruction of the Oort cloud by stars passing through it. In these works it has been usual to discuss a model in which the total influence of the Galaxy is not taken into account; the cloud is considered to be a local formation whose properties and behaviour are fully determined by the Sun and passing stars.

New possibilities for research appear when the Oort cloud is regarded as the total collection of comets in the vicinity of the Sun within Hill's sphere, i.e., when the Jacobi integral is used for the system Sun-Galaxy centre-comet (Chebotarev, 1964, 1966; Antonov and Latyshev, 1972). With such an approach the stability of the cloud may be investigated without knowledge of the cometary orbits themselves. Trajectories of particles in the cloud are crucial for the investigation of the cloud's evolution, although (1) they are not known, and (2) in the general case, at least in the cloud's periphery, they differ significantly from conics (Chebotarev, 1964, 1966).

The need for statistical analysis of variations in cometary motion and of destruction of the cloud by the random passage of stars naturally evolves directly from the formulation of the problem. However, in stellar astronomy similar problems have been formulated and studied with sufficient profundity.

In this work an attempt is made to study the stability of the Oort cloud by the methods of stellar astronomy.

2. Model for the Oort Cloud

The following simplified model is discussed. The centre of the Galaxy and the Sun are taken to be two material points describing circular orbits about their common centre of mass (Ogorodnikov and Latyshev, 1969), at which point is located the origin of the rotating system of coordinates xyz. The xy plane is taken to be that of the motion of these two points, while the x-axis is directed along the line connecting them. The coordinates of the Sun and Galaxy centre are $x=d_1$, $y=z=0$ and $x=-d_2$, $y=z=0$, respectively (where $d_1>0$, $d_2=0$, since the mass of the Galaxy $m_2 \gg m_\odot$).

The Oort cloud is formed by particles of infinitely small mass located in the vicinity

Chebotarev et al. (eds.), The Motion, Evolution of Orbits, and Origin of Comets, 335–340.

of the Sun within the Hill surface, and for which the Jacobi constant $C > C_0$ (where C_0 is the value of the Jacobi constant at the libration point L_1).

3. Mechanism for Sweeping Particles from the Oort Cloud

The passage of stars through the Oort cloud or near it causes variations in the Jacobi constants of the particles. Particles whose Jacobi constants C_1 become less than C_0 are unstable in Hill's sense (Moulton, 1914; Subbotin, 1968; Chebotarev, 1969) and therefore will not be regarded as belonging to the cloud, an assumption allowing us to evaluate the lower limit of the half-life of the Oort cloud.

If a particle belonged to the Oort cloud prior to the passage of a star and then left the cloud, the following inequality can be written:

$$C > C_0 > C_1; \tag{1}$$

whence

$$(C_0 - C) + (C - C_1) > 0, \tag{2}$$

where

$$C = 2\Omega - v^2,$$

$$C_1 = 2\Omega_1 - v_1^2, \tag{3}$$

$$\Omega = \tfrac{1}{2}n^2(x^2 + y^2) + k^2\left(\frac{m_\odot}{r_1} + \frac{m_2}{r_2}\right), \tag{4}$$

x, y, z being the coordinates of a particle prior to the star's passage; x_1, y_1, z_1 the coordinates of the particle after the star's passage; $r_1 = \sqrt{[(x-d_1)^2 + y^2 + z^2]}$, $r_2 = \sqrt{(x^2 + y^2 + z^2)}$, $\dot{v}^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$; k^2 is the gravitational constant, and n the angular velocity of the Sun. [In a more general case, if the Galaxy is considered to be more than a point mass (Antonov and Latyshev, 1971), the factor m_2/r_2 in Equation (4) is replaced by $\varphi(r_2, z)$.] Inserting Equations (3) and (4) into (2) we obtain

$$(C_0 - C) + 2(\Omega - \Omega_1) - (v^2 - v_1^2) > 0. \tag{5}$$

During the passage of a star through the cloud the coordinates of the particle change but little (since the velocity of the particle is several orders lower than that of the passing star). It can therefore be assumed that the perturbing star gives the particle a sudden impulse, changing only the latter's velocity:

$$\Omega - \Omega_1 = 0, \tag{6}$$

$$v_1 = v + \Delta v. \tag{7}$$

The inequality (5) can then be rewritten

$$(C_0 - C) + 2v\,\Delta v + (\Delta v)^2 > 0. \tag{8}$$

Obviously, the change in the Jacobi constant of the particle depends on the distance at which the star passes the particle. Let us therefore estimate the distance that would cause the latter's Jacobi constant C_1 to become less than C_0 and the particle to leave the cloud.

If $\Delta v \gg v$, the principal terms in inequality (8) are

$$(C_0 - C) + (\Delta v)^2 > 0. \tag{9}$$

Variation in the velocity of the particle (Δv) can be expressed (Ogorodnikov, 1958) in terms of the mass m and velocity V of the passing star, and the least distance p to it:

$$\Delta v = \frac{2k^2 m}{pV}. \tag{10}$$

Inserting this into the inequality (9) gives

$$p^2 < \left(\frac{2k^2 m}{V}\right)^2 (C - C_0)^{-1}. \tag{11}$$

We have thus obtained the radius of the tunnel all the particles of which are swept out by the passing star.

Let us now assume that a certain star consistently remains within the Oort cloud and moves with an average velocity V, sweeping particles out of a tunnel of average radius p. The density $\nu(t)$ is the same throughout the cloud but it changes with time. It should be noted that as soon as the star has passed the tunnel is again filled with particles.

Designating by T=const and $M(t)$ the volume and mass, respectively, of the Oort cloud, we have

$$\begin{aligned} -\frac{dM}{dt} &= \pi p^2 V \nu, \\ M &= \nu T \\ -T\frac{d\nu}{dt} &= \pi p^2 V \nu, \end{aligned} \tag{12}$$

and integrating,

$$\nu = \nu_0 \exp\left(-\frac{\pi V p^2}{T} t\right).$$

If $q\nu = \nu_0 = \nu(0)$, then

$$t = \frac{T}{\pi V p^2} \ln q. \tag{13}$$

Equation (13) shows how long will elapse, under the above conditions, before the density of the cloud decreases by a factor of q.

If we designate by τ the average time spent by a star in crossing the Oort cloud, we may estimate, using formulae (11) and (13), the average number r of stars that would pass through the cloud before its density would decrease by a factor of q as the result of the sweeping out of particles. The result is

$$r = \frac{TV \ln q}{4\pi k^4 m^2 \tau \overline{(C - C_0)^{-1}}}, \tag{14}$$

where $\overline{(C-C_0)^{-1}}$ is the average value of $(C-C_0)^{-1}$ for the particles of the Oort cloud.

4. Cumulative Mechanism for Dispersion of the Cloud

A passing star usually has a negligible effect on the velocity of most particles, and thus their Jacobi constants C usually remain larger than C_0. However, the cumulative effect of the passage of several stars can eject the particle from the Oort cloud.

It can be seen from the inequality (8) that in order for a considerable fraction of the particles to leave the cloud the mean square value of the increments in the particle velocities should be of the order

$$(\delta v)^2 = \bar{C} - C_0. \tag{15}$$

We now derive a formula for $(\delta v)^2$. (For more detail see Ogorodnikov, 1958.) For greater simplicity, let the distribution function $f(V)$ with respect to the relative velocities of stars have spherical symmetry, i.e., be independent of direction. Then in time t there will be on the average

$$2\pi p t V \,\mathrm{d}p\, f(V)\,\mathrm{d}V$$

passages of stars with velocities from V to $V+\mathrm{d}V$ at distances from p to $p+\mathrm{d}p$ from the particle. Assuming that the masses of all the passing stars are similar, we have

$$(\delta v)^2 = \int_{p_1}^{p_2}\int_0^{\infty} (\Delta v)^2 2\pi p t V \,\mathrm{d}p\, f(V)\,\mathrm{d}V$$

or, using Equation (10),

$$(\delta v)^2 = 8\pi k^4 m^2 t \int_0^{\infty} \frac{f(V)\,\mathrm{d}V}{V} \int_{p_1}^{p_2} \frac{\mathrm{d}p}{p}, \tag{16}$$

where p_1 and p_2 are the minimum and maximum distances, respectively, to the passing star. Let us assume that $p_1 = p$ is the average radius of the tunnel given by inequality (11) and $p_2 = d$ is the average distance between stars. (If $p_2 > d$, multiple passages occur, i.e., several stars pass by a particle simultaneously at similar distances. It may therefore be supposed that such multiple passages do not produce any significant variations in the Jacobi constant.)

In so far as we are interested only in order of magnitude results, we may take

$$\int_0^{\infty} \frac{f(V)}{V}\,\mathrm{d}V = \frac{\nu}{\hat{v}},$$

https://doi.org/10.1017/S0074180900006720 Published online by Cambridge University Press

where $\hat{v}$ is the root mean square residual velocity of the stars. Then

$$(\delta v)^2 = 8\pi k^4 m^2 t\nu \frac{1}{\hat{v}} \ln \frac{p_2}{p_1}. \tag{17}$$

Using Equations (15) and (17) we find the cumulative half-life of the cloud to be

$$t = \frac{(\bar{C} - C_0)\hat{v}}{8\pi k^4 m^2 \nu \ln (p_2/p_1)}. \tag{18}$$

5. Numerical Estimates

We shall now estimate the half-life of the Oort cloud. For this purpose we take the following data (Ogorodnikov, 1958; Antonov and Latyshev, 1972): $d_2 = 10^4$ pc, $d = 2.15$ pc, $m = 10^{33}$ g, $m_\odot = 2 \times 10^{33}$ g, $m_2 = 2.6 \times 10^{44}$ g, $\nu = 0.1$ st pc^{-3}. The mean radius of the Oort cloud is 1 pc, $T = 3.35$ pc^3, $V = 20$ km s^{-1}, $\hat{v} = 20$ km s^{-1}, $\tau = 6.4 \times 10^4$ yr, $q = 2$.

Using Equation (14) we see that the density of the cloud will be decreased by half as the result of sweeping if 5.5×10^4 stars pass through the cloud. This will occur (Ogorodnikov, 1958) in 8.6×10^9 yr. On the other hand, from Equation (18) we find that the cumulative half-life of the Oort cloud is at least 1.1×10^9 yr. The cumulative effect is thus the more important mechanism, although more precise estimates will undoubtedly result in a much greater half-life.

Acknowledgments

I find it my pleasant duty to express my gratitude to G. A. Chebotarev for suggesting this problem and for the interest he has shown, and also to V. A. Antonov for his counsel and comments in the course of my work.

References

Antonov, V. A. and Latyshev, I. N.: 1972, this Symposium, p. 341.
Chebotarev, G. A.: 1964, *Astron. Zh.* **41**, 983.
Chebotarev, G. A.: 1966, *Astron. Zh.* **43**, 435.
Chebotarev, G. A.: 1969, *Byull. Inst. Teor. Astron.* **11**, 625.
Moulton, F. R.: 1914, *An Introduction to Celestial Mechanics*, Macmillan, New York.
Ogorodnikov, K. F.: 1958, *Dinamika Zvezdnykh Sistem*, Moscow.
Ogorodnikov, K. F. and Latyshev, I. N.: 1969, *Astron. Zh.* **46**, 1190.
Sekanina, Z.: 1968, *Bull. Astron. Inst. Czech.* **19**, 223, 291.
Subbotin, M. F.: 1968, *Vvedenie b Teoreticheskuyu Astronomiyu*, Moscow.
Vsekhsvyatskij, S. K.: 1967, *Priroda i Proiskhozhdenie Komet i Meteornogo Veshchestva*, Prosveshchenie, Moscow.
Zal'kalne, I. E.: 1969, *Astrometr. Astrofiz.* No. 4.

Discussion

G. N. Duboshin: Your assumption that the passing stars do not influence the Sun would seem to be a very rough approximation.

E. M. Nezhinskij: If a large number of stars pass the Sun at considerable distances in various directions, they can be considered to have no net influence on the Sun's motion.

B. Yu. Levin: If each star produces a narrow tunnel, whereas total destruction stems from the passage of a large number of stars, it seems to me that Duboshin's objection is not important, because a star has a decisive effect only on the region very close to it.