THE SIGNAL TO NOISE RATIO IN SPECKLE INTERFEROMETRY

J.C.Dainty and A.H. Greenaway*

Dept. of Physics, Queen Elizabeth College, Campden Hill Road, London W8 7AH.

*Dept. d'Astrophysique, Universite de Nice, Parc Valrose, 06034 Nice, France.

ABSTRACT

Recent theoretical studies of the signal to noise ratio (SNR) of photon limited speckle (image plane) interferometry are reviewed. The SNR of an estimate of the object power spectrum is evaluated for both the single and double aperture cases, for arbitrary light levels. The SNR for the auto-correlation function method of analysis is also given for the low light level case and applied to the special case of binary star observations. The SNRs for the power spectrum and autocorrelation function analyses are compared and a comparison is also made between speckle (image plane) and amplitude (pupil or aperture plane) interferometry. Limiting observable magnitudes are estimated for some relevant cases.

1. INTRODUCTION

The aim of stellar interferometry is to determine information about the intensity distribution, $o(x)$, across an object. In the methods described here this information is either an estimate of the object power spectrum, $\phi_o(\nu)$, or of the object autocorrelation function $C_o(x)$. Given an interferometer which is instrumentally perfect, the error in an estimate of the power spectrum or autocorrelation function is determined by the fluctuations of atmospheric seeing and by the quantum nature of the radiation. One way of expressing the error in an estimated quantity Q is by the signal to noise ratio (SNR) defined as

$$\mathrm{SNR} = \frac{\text{expected value of quantity}}{\text{standard deviation of estimate}}$$

$$= \frac{\langle Q \rangle}{(\langle Q^2 \rangle - \langle Q \rangle^2)^{\frac{1}{2}}} \qquad (1)$$

The estimated quantity Q might, for example, be the value of the power spectrum at a point, an integrated value of the power spectrum or the autocorrelation function at a point. In all cases the SNRs given here relate to an estimate of Q based on a single record(frame) of data. Normally, one would repeat the experiment many times in order to improve the overall SNR. The SNR obtained using M statistically independent frames, denoted $(SNR)_M$, is greater than $(SNR)_1$ by a factor of $M^{\frac{1}{2}}$.

General reviews of the technique of speckle interferometry are given in references 1-3. In this review one dimensional notation will be used for simplicity, the extension to two dimensions being trivial.

2. POWER SPECTRUM ANALYSIS

2.1 The Transfer Function

In speckle interferometry many short exposure, narrow bandwidth records of the image intensity, i(x), are taken. At high light levels each record is a realisation of a random process that may be assumed to be ergodic (and hence stationary) in time, but nonstationary in space. The power, or Wiener, spectrum of the process is defined as

$$\phi_i(\nu) = \langle |I(\nu)|^2 \rangle \quad (2)$$

where I(ν) is the Fourier transform of i(x) and the ensemble average <.> may also be interpreted as a time average. The power spectrum of the image intensity, normalised to unity at the origin (ν=0) and written as $\hat{\phi}_i(\nu)$, is related to the normalised object power spectrum, $\hat{\phi}_o(\nu)$, by

$$\hat{\phi}_i(\nu) = \hat{\phi}_o(\nu) \langle |T(\nu)|^2 \rangle \quad (3)$$

where T(ν) is the instantaneous transfer function defined by

$$T(\nu) = \int_{-\infty}^{\infty} H^*(\lambda f\nu')\, Z^*(\lambda f\nu')\, H(\lambda f[\nu'+\nu])\, Z(\lambda f[\nu'+\nu])\, d\nu' \quad (4)$$

where

- $H(\ )$ is the pupil function of the telescope aperture
- $Z(\ )$ is the complex amplitude fluctuation in the telescope pupil due to atmospheric turbulence
- λ is the wavelength of the light and
- f is the focal length of the telescope

The form of the speckle transfer function, $<|T(\nu)|^2>$, has been investigated by several authors and, in general, depends upon the model used for $Z(\ )$

If we model $Z(\ )$ as a complex Gaussian process, then providing that $(D/r_o)^2 >> 1$ and telescope aberrations are negligible over regions of dimension r_o, the normalised transfer function is given by[4]

$$<|T(\nu)|^2> \simeq |T_s(\nu)|^2 \; |T_o(\nu)|^2 + 0.435 \left(\frac{r_o}{D}\right)^2 T_D(\nu) \qquad (5)$$

where

- $T_s(\nu)$ is the long exposure seeing transfer function
- $T_o(\nu)$ is the OTF of the telescope
- $T_D(\nu)$ is the diffraction limited OTF of the telescope
- r_o is Fried's seeing coherence diameter[5] and
- D is the diameter of the (unobscured) telescope pupil

Since $T_s(\nu)$ decreases rapidly with increasing spatial frequency and is effectively zero for frequencies very much greater than the seeing limit (i.e. $\nu >> r_o/\lambda f$), we may write

$$<|T(\nu)|^2> \simeq 0.435 \left(\frac{r_o}{D}\right)^2 T_D(\nu) \qquad ; \; D/r_o >> 1, \; \nu >> r_o/\lambda f \qquad (6)$$

Although analysis using the more realistic log-normal distribution for $Z(\ )$ yields an expression for $<|T(\nu)|^2>$ differing from equation(5), the result in the limit $D/r_o >> 1$, $\nu >> r_o/\lambda f$ is identical to equation (6) (see equation (37) of reference 6).

It is sometimes helpful to define the average number of speckles per frame, n_s, as

$$n_s = \left(\frac{D}{r_o}\right)^2 \cdot \frac{1}{0.435} \tag{7}$$

giving

$$\langle |T(\nu)|^2 \rangle \simeq \frac{1}{n_s} \cdot T_D(\nu) \qquad ; n_s >> 1, \nu >> r_o/\lambda f \tag{8}$$

2.2 SNR at a Point - The General Expression

The SNR at a point in the power spectrum in speckle interferometry was first derived by Roddier[7] for the case in which $\overline{N}$, the average number of detected photons per frame, satisfies $\overline{N} >> 1$. A more detailed analysis is given by Goodman and Belsher[8,9] and the problem is also discussed in [10-12].

The jth image record, $d_j(x)$, is modelled as an inhomogeneous, or compound, Poisson process which has a rate proportional to the classical image intensity $i(x)$.

$$d_j(x) = \sum_{k=1}^{N_j} \delta(x-x_{jk}) \tag{9}$$

where each delta function represents a photon event; x_{jk} is the location of the kth event in the jth frame and N_j is the total number of detected photons in the jth frame. Experimentally, the squared modulus of the Fourier transform, $|D_j(\nu)|^2$, is computed for each frame.

Several authors[8-11] have evaluated SNRs for the estimate Q_1, defined by

$$Q_1 = |D_j(\nu)|^2 - \overline{N} \tag{10}$$

With this definition of the measured quantity one has

$$\langle Q_1 \rangle = \overline{N}^2 \hat{\phi}_i(\nu) \tag{11}$$

and

$$\sigma^2_{Q_1} = \overline{N} + \overline{N}^2 + 2(2+\overline{N})\overline{N}^2 \hat{\phi}_i(\nu) + \overline{N}^2 \hat{\phi}_i(2\nu) + \overline{N}^4 \hat{\phi}^2_i(\nu) \tag{12}$$

As in all problems of this type, it should be noted that the fluctuations of the estimate at spatial frequency ν are influenced by the value of the power spectrum at 2ν. At exceedingly low light levels, $\overline{N} << 1$, the SNR per frame for estimate Q_1 is given by

$$(SNR)_1 = \overline{N}^{3/2} \hat{\phi}_i(\nu) \qquad ; \overline{N} << 1 \tag{13}$$

The use of definition (10) for the experimentally estimated quantity has the disadvantage that the noise associated with Q_1 contains contributions arising from the fluctuations in N_j, the actual number of photons detected per frame. These fluctuations are related to the brightness of the object but not to its structure. If one is interested in the morphology of the object a better estimate is Q_2, defined by

$$Q_2 = |D_j(\nu)|^2 - N_j \tag{14}$$

With this definition we find that[12]

$$\langle Q_2 \rangle = \overline{N}^2 \hat{\phi}_i(\nu) \tag{15}$$

and

$$\sigma^2_{Q_2} = \overline{N}^2 + \overline{N}^2 \hat{\phi}_i(2\nu) + 2\overline{N}^3 \hat{\phi}_i(\nu) + \overline{N}^4 \hat{\phi}^2_i(\nu) \tag{16}$$

yielding a SNR per frame given by

$$(SNR)_1 = \frac{\overline{N} \hat{\phi}_i(\nu)}{\{(1+\overline{N} \hat{\phi}_i(\nu))^2 + \hat{\phi}_i(2\nu)\}^{\frac{1}{2}}} \tag{17}$$

We shall now apply equation (17) to single aperture and double aperture (long baseline) speckle interferometry using the transfer function relationships (3) and (8).

2.3 Single Aperture Case

Using equations (3) and (8) the relationship between the normalised image and object power spectra is

$$\hat{\phi}_i(\nu) \simeq \frac{T_D(\nu)}{n_s} \hat{\phi}_o(\nu) \qquad ; n_s >> 1,\ \nu >> r_o/\lambda f \tag{18}$$

Substituting equation (18) into (17), ignoring the 2ν term (valid for $\nu \geqq D/2\lambda f$) and defining the average number of detected photons per speckle as

$$\bar{n} = \frac{\bar{N}}{n_s} \tag{19}$$

the expression for the SNR per frame becomes

$$(SNR)_1 = \frac{\bar{n}\ T_D(\nu)\ \hat{\phi}_o(\nu)}{1 + \bar{n}\ T_D(\nu)\ \hat{\phi}_o(\nu)} \qquad ; n_s >> 1,\ \nu \geqq D/2\lambda f \tag{20}$$

Two limiting cases are of interest :

(i) $\bar{n}\ T_D(\nu)\ \hat{\phi}_o(\nu) >> 1$, e.g. very bright objects

$$(SNR)_1 \rightarrow 1 \tag{21}$$

(ii) $\bar{n}\ T_D(\nu)\ \hat{\phi}_o(\nu) << 1$, e.g. faint objects yielding, on average, less than one detected photon per speckle.

$$(SNR)_1 \rightarrow \bar{n}\ T_D(\nu)\ \hat{\phi}_o(\nu) \tag{22}$$

Note that the 3/2 power law (equation (13)) predicted by the Q_1 measure in the low limit, does not occur when Q_2 is the estimated quantity: this limit $\bar{N}$ <<1, is in any case of academic interest in speckle interferometry since there is insufficient observing time available in a single night to be able to observe such faint objects.

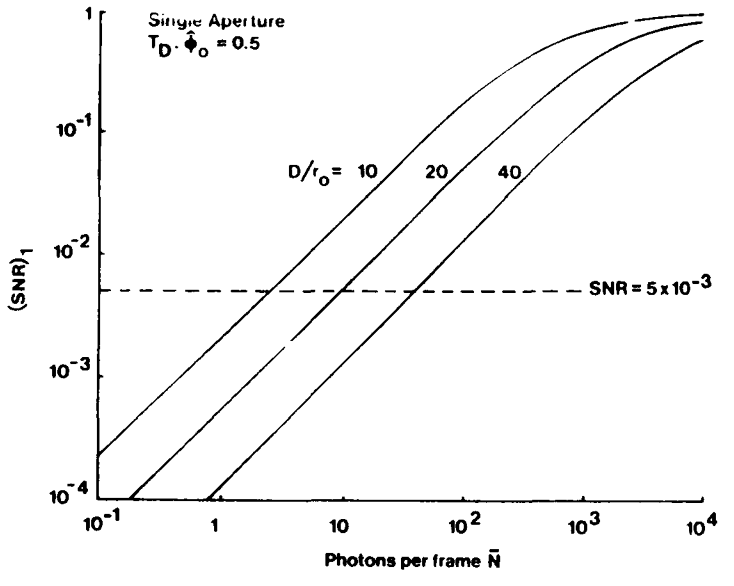

Figure 1

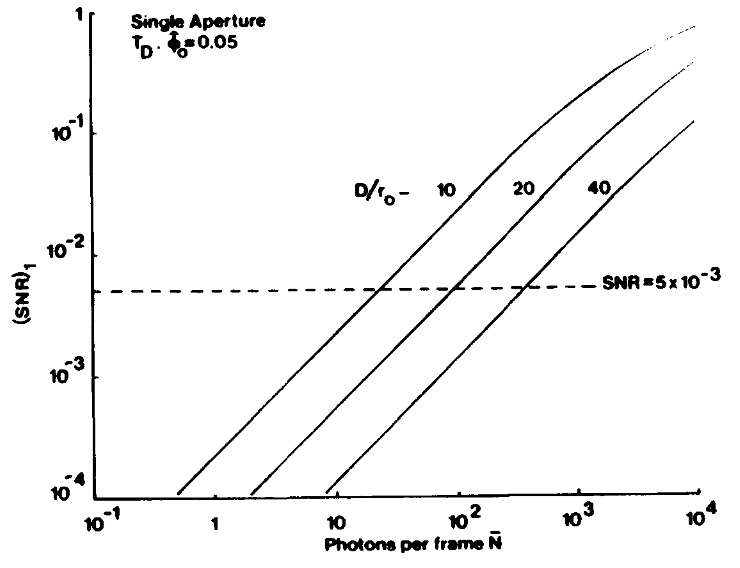

Figure 2

Equation (20) is plotted in figures 1 and 2 for $T_D(\nu)\ \hat{\phi}_o(\nu)$ equal to 0.5 and 0.05, and for values of (D/r_o) equal to 10, 20 and 40 in each case.

Other definitions of Q are possible. For example, frames containing less than q detected photons (i.e. $N_j < q$) may be excluded from the analysis: since clearly any frames containing either zero or one detected photons cannot contribute usefully in a spatial power spectrum analysis. With q = 2 it can be shown[12] that the limiting equations (21) and (22) apply However, at intermediate levels the SNR with this analysis is marginally lower than for Q_2

2.4 Double Aperture Case

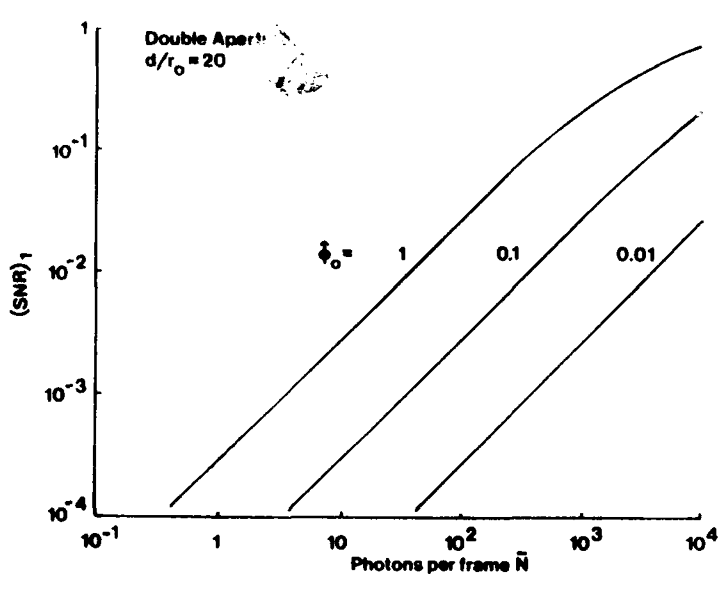

Figure 3

Equations (20) - (22) are equally applicable to double (large) aperture speckle interferometry of the type undertaken by Labeyrie and his group (ref 13), provided that $\bar{n}$ is defined in a suitable way. If d is the size (diameter) of each (unobscured) pupil then the total collector area is equal to $\pi\ d^2/2$, and thus the correct definition of $\bar{n}$ is given by equation (23)

$$\overline{n} = 0.22\ \overline{N}\ (r_o/d)^2 \tag{23}$$

The maximum SNR value occurs for the spatial frequency corresponding to the interferometer baseline, for which $T_D(\nu) = 0.5$ Using this value for T_D and equation (23) for $\overline{n}$, equation (22) has been plotted for the double aperture case in figure 3, with $(d/r_o) = 20$ and for $\hat{\phi}_o = 1$, 0.1 and 0.01.

2.5 The Integrated SNR

The SNR values given so far refer to a single point in the measured power spectrum. In practice, particularly in double aperture (long baseline) interferometry, one may wish to integrate over a region of the power spectrum either to increase the SNR, to obtain "seeing independent" measurements[14] or for the sake of operational simplicity In speckle (image plane) interferometry, integration over the image power spectrum leads to a smoothing of the estimate of the object power spectrum: in the results presented below it is assumed that $\hat{\phi}_o(\nu)$ is constant over the region of integration.

The SNR can be evaluated by a similar method to that used for the point-wise case, except that we now define the estimated quantity to be Q_3, where

$$Q_3 = \int (|D_j(\nu)|^2 - N_j)\ W(\nu)\ d\nu \tag{24}$$

where $W(\nu)$ is a weighting function that is applied to the measured result It can be shown that[15]

$$\langle Q_3 \rangle = \overline{N}^2 \int W(\nu)\ \hat{\phi}_i(\nu)\ d\nu \tag{25}$$

and

$$\sigma^2_{Q_3} = \int\int W(\nu)\ W(\nu')\ \{\langle |D(\nu)|^2\ |D(\nu')|^2 \rangle - \langle N_j\ |D(\nu)|^2 \rangle - \langle N_j\ |D(\nu')|^2 \rangle + \langle N_j^2 \rangle - \overline{N}^4\ \hat{\phi}_i(\nu)\ \hat{\phi}_i(\nu')\ \}\ d\nu\ d\nu' \tag{26}$$

Further evaluation of equation (26) is difficult. For very bright objects the SNR tends to the expression

$$(SNR)_1 \rightarrow n_s'^{\frac{1}{2}} \qquad ; \quad \bar{n} >> 1 \qquad (27)$$

where n_s' is a measure of the effective number of independent speckles within the weighting function. For faint objects, $\bar{n} << 1$, the SNR is given approximately by

$$(SNR)_1 \rightarrow \frac{\bar{n}\, n_s^{\frac{1}{2}}}{X} \overline{\hat{\phi}_o(\nu)} \qquad ; \quad \bar{n} << 1 \qquad (28)$$

where $X > 1$ is a factor that depends on the form of the weighting function; for $W(\nu) = T_D(\nu)$, the value of X is approximately 3-5.

The essential point to notice is that in both cases the improvement in SNR over the pointwise result is of the order of the square root of the effective number of speckles within the weighting function. If an integrated version of the Q_1 measure is used this result holds only for very bright objects.

3. AUTOCORRELATION FUNCTION ANALYSIS

3.1 General Low Light Level Case

Autocorrelation function analysis of sparse photon images is an operationally simple process that can be carried out in real time using either special hardware or a microprogrammed computer[16,17] The calculation of the SNR is therefore of particular interest at low light levels ($\bar{n} << 1$) and this case is considered below; several authors have studied this problem[1,2,18-21]

The essential features of the SNR of the autocorrelation technique are illustrated by the following much simplified analysis. Assume that the seeing disc is a top hat function containing n_s speckles (Airy discs) and n_s detector elements matched to the speckle size: if the object under study is small compared to the size of the seeing disc, the background in the autocorrelation function may be assumed to be independent of the lag τ

The value of the background is equal,at low light levels, to the probability that a photon is detected at each of two points 1 and 2 separated by a distance τ, multiplied by the number of times such separations occur in the image; this gives

$$B = \bar{n}^2 n_s \qquad ; \bar{n} << 1 \qquad (29)$$

Defining the normalised autocorrelation function of the image intensity,

$$\hat{C}_i(\tau) = \frac{< \int i(x)\, i(x+\tau)\, dx >}{< \int i(x)\, dx >^2}$$

with[22]

$$\frac{< I_1 \; I_2 >}{<I_1><I_2>} = \frac{< N_1 \; N_2 >}{<N_1><N_2>} \qquad ; 1 \neq 2$$

where N_1 and N_2 are the photon counts at the points 1 and 2, we may write the estimate of the photon-counting autocorrelation function as

$$C_N(\tau) = B\, \hat{C}_i(\tau) \qquad ; \tau \neq 0 \qquad (30)$$

The signal is equal to the value of $C_N(\tau)$ minus the background B and the noise is simply $\{C_N(\tau)\}^{\frac{1}{2}}$ at low light levels; thus

$$(SNR)_1 = \frac{B\, (\hat{C}_i(\tau) - 1)}{(B\, \hat{C}_i(\tau))^{\frac{1}{2}}} \qquad ; \tau \neq 0$$

or

$$(SNR)_1 = \bar{n}\, n_s^{\frac{1}{2}} \frac{\hat{C}_i(\tau) - 1}{(\hat{C}_i(\tau))^{\frac{1}{2}}} \qquad ; \tau \neq 0,\ \bar{n} << 1 \qquad (31)$$

In practice, the seeing disc has a Gaussian-like profile and therefore the background, B, is a function of the lag τ; consideration of this factor has the effect of altering the definition of the number of speckles per frame. Note that no assumptions about the atmospheric statistics, other than those given above, are necessary to derive equation (31) Other assumptions about the atmosphere are, however, required to relate $\hat{C}_i(\tau)$ to $\hat{C}_o(\tau)$.

3.2 Application to Binary Stars

The autocorrelation function of the instantaneous image intensity for binary star objects has a particularly simple form, consisting of peaks at $\tau = \pm a$, where a is the separation of the binary. Defining the signal as the height of these peaks above their local background and the noise as the standard deviation of the background, it can be shown that the single frame SNR for close binaries and Gaussian seeing disc is given by

$$(SNR)_1 = \frac{\bar{n}\, n_s^{\frac{1}{2}}}{2^{\frac{1}{2}}} \frac{f}{1 + 2f + f^2} \qquad ; \bar{n} << 1 \tag{32}$$

where f is related to the magnitude difference Δm by

$$f = (2.5)^{-\Delta m}$$

4. DISCUSSION

4.1 Comparison of Power Spectrum and Autocorrelation Function SNRs

In this section we compare the single frame SNRs for faint objects (i.e. $\bar{n} << 1$) for the power spectrum (Q_2, Q_3 estimates) and the autocorrelation function method of analysis. The pointwise SNRs are given by

$$(SNR)_{1,ps} \simeq \bar{n}\, T_D(\nu)\, \hat{\phi}_o(\nu) \tag{22}$$

$$(SNR)_{1,ac} \simeq \bar{n}\, n_s^{\frac{1}{2}} \frac{\hat{C}_i(\nu) - 1}{(\hat{C}_i(\nu))^{\frac{1}{2}}} \tag{31}$$

in which

$\bar{n}$	is the average number of detected photons per speckle
n_s	is the number of speckles per frame
$T_D(\nu)$	is the diffraction limited OTF of the telescope
$\hat{\phi}_o(\nu)$	is the normalised object power spectrum
$\hat{C}_i(\nu)$	is the normalised image autocorrelation function

The first point to note is that for both cases the SNR achieved depends upon the argument, τ or ν, thus on the point at which the autocorrelation or power spectrum is evaluated. Thus any comparison between these methods should be a global one. The value at a point in the autocorrelation function is equal to the Fourier transform of the power spectrum (and visa versa) Thus one possible comparison is between the SNR of the autocorrelation function at a point (equation (31)) and the SNR of the weighted integral of the power spectrum, where the weighting function $W(\nu)$ is given by

$$W(\nu) \simeq \exp(-2\pi i\nu\tau)$$

Unfortunately, the evaluation of the integrated SNR, using the Q_3 form as described in §2.5, is somewhat complicated[15].

Consider, however, the case of an unresolved binary object, such that

$$\hat{\phi}_o(\nu) \simeq 1 \qquad ; \text{ over the region of integration}$$

and

$$\hat{C}_i(\nu) \simeq 1 + \delta(\tau-\Delta)$$

where Δ is the (unresolvable) separation of the object components.

Using equation (28) of §2.5, i.e.

$$(\mathrm{SNR})_{1,ps,integrated} \simeq \frac{\bar{n}\, n_s^{\frac{1}{2}}\, \overline{\hat{\phi}_o(\nu)}}{X} \tag{28}$$

we see that the integrated SNR of a power spectrum measurement for this object is of the order of $\bar{n}\, n_s^{\frac{1}{2}}/X$, where X is in the range 3-5. The SNR at $\tau = \Delta$ in the autocorrelation function is of the order of $\bar{n}\, n_s^{\frac{1}{2}}/4\sqrt{2}$ thus for this object

$$(\mathrm{SNR})_{1,ps,integrated} \simeq (\mathrm{SNR})_{1,ac}$$

The above heuristic arguments are not strictly valid, but it is possible to show[15] that, in general, there is no essential difference in the SNR for the two types of analysis. Suboptimum data analysis, such as that using the

Q_1 measure in the power spectral analysis, would change this conclusion.

4.2 Comparison with Pupil Plane Interferometers

A detailed comparison of large aperture, long-baseline speckle (image plane) and amplitude (pupil plane) interferometry has been given in[11].

If the statistics of the complex amplitude in the pupil plane are taken to be complex Gaussian, the expression for the pointwise SNRs in long-baseline image and pupil plane interferometers are essentially the same, differing only in the form of the transfer function; at the central frequency of the bandpass islands the expressions are identical for this model.

A more realistic, but mathematically still convenient model, is the low-scintillation, log-normal model[11] for the atmospheric fluctuations, in which the phase is a zero-mean Gaussian process and the intensity is a non-zero-mean Gaussian process. For bright objects this leads to single frame SNRs greater than unity in pupil plane interferometry (as predicted in[9]) with a limiting value of approximately $(2c)^{-\frac{1}{2}}$, where $c \ll 1$ is the contrast of the scintillation. The SNR at low light levels, i.e. those of particular interest in stellar interferometry, is unaffected.

Thus, apart from minor factors, the SNRs of image and pupil plane interferometers are very similar for equal collecting apertures and equal numbers of detected photons; in this context it is worth noting that image plane interferometry is commonly believed to be able to tolerate longer exposure times than its pupil space rival. On the other hand, the pupil space design may be able to tolerate wider bandwidths as correction for chromatic dispersion is easier in the pupil space design. The pupil plane interferometer also has the advantages of a more favourable transfer function and that integration over the pupil does not lead to a smoothing of the estimate of the object spectrum. It is believed that wavefront-folding interferometers[9,24] give a similar SNR to that of speckle interferometry

4.3 Limiting Magnitudes

There are as many different estimates of the limiting magnitude of speckle interferometry as there publications on the subject. We shall consider three typical cases: (i) single aperture binary star observation, (ii) single aperture power spectrum measurement and (iii) long baseline integrated power spectrum measurements.

(i) Single aperture binary star observation. In this case we shall use the criterion that the information is just measurable if the SNR at a point in the autocorrelation function is equal to 5; equation (32) is the relevant equation for the SNR. If F is defined as the product of the optical bandwidth $\Delta\lambda$ in nm, the exposure time Δt in seconds and the quantum efficiency q of the detector (where $0 \leq q \leq 1$), i.e

$$F = \Delta t \; \Delta\lambda \; q$$

then a source of visual magnitude m_V gives rise to[25]

$$N_A = F.10^{(8 - 0.4m_V)} \tag{33}$$

detected photons per m^2 per frame. Assuming a clear aperture of diameter D, equation (32) may be re-arranged to give, for an equal magnitude binary (f = 1)

$$m_V \simeq 17.8 + 2.5 \log F - 2.5 \log (SNR)_M - 1.25 \log n_s + 1.25 \log M + 5 \log D \tag{34}$$

where $(SNR)_M$ is the signal to noise ratio for M independent frames at the limiting magnitude m_V. For $D = 4m$, $n_s = 10^3$, $M = 10^5$, $\Delta t = 0.02s$, $\Delta\lambda = 25nm$, $q = 0.1$ and a limiting $(SNR)_M = 5$, this gives a limiting magnitude of approx. $m_V \simeq 18$, corresponding to an average of 3 detected photons per frame ($\bar{N} = 3$). By increasing the number of independent frames to 10^6 and slightly increasing the bandwidth or exposure time (as experimental conditions permit) binaries as faint as 20th magnitude may be observable.

(ii) Single aperture, power spectrum measurements. A reliable estimate at every point in the power spectrum clearly requires a brighter object than for the single-parameter measurement of binary stars Equation (22) is the relevant equation for $\bar{n} << 1$ and using equations (7) and (33) it may be written as

$$m_v \simeq 18.8 + 2.5 \log F - 2.5 \log (SNR)_M + 1.25 \log M + 2.5 \log (T_D(\nu).\hat{\phi}_o(\nu)) + 5 \log r_o \qquad (35)$$

Note that the limiting magnitude is independent of the diameter of the telescope in this case; of course, a larger telescope gives more independent estimates (and higher resolution) for a given seeing condition. For $r_o = 0.1$m $M = 10^5$, $\Delta t = 0.02$s, $\Delta\lambda = 25$nm, $q = 0.1$, $T_D(\nu).\hat{\phi}_o(\nu) = 0.2$ and a limiting $(SNR)_M = 5$, this yields a limiting magnitude of approximately $m_v = 13.2$, which corresponds to 320 detected photons per frame ($\bar{N} = 320$) in a 4m telescope (or $\bar{N} = 20$ in a 1m telescope).

(iii) Long baseline integrated power spectrum measurements. Equation (28) may be used for estimating the limiting magnitude; with the factor X = 5 we obtain

$$m_v \simeq 18 + 2.5 \log F - 2.5 \log (SNR)_M + 1.25 \log M + 2.5 \log (\overline{\hat{\phi}_o(\nu)}) - 1.25 \log n_s + 5 \log D \qquad (36)$$

In practice, the usable bandwidth $\Delta\lambda$ might be as small as 0.1nm for a 100m baseline[26]. For $r_o = 0.1$m, D = 1.5m (the diameter for Labeyrie's interferometer[13]), $M = 10^5$, $\Delta t = 0.02$s, $q = 0.1$, $\overline{\hat{\phi}_o(\nu)} = 0.5$ and a limiting $(SNR)_M = 5$, the limiting magnitude is approximately $m_v \simeq 9.6$, corresponding to 5 detected photons per frame ($\bar{N} = 5$).

It should be noted that in all of the above examples an increase in the quantity $F = \Delta t\ \Delta\lambda\ q$ by a factor of 10 will increase the limiting magnitude by 2.5 magnitudes; for example, in the long baseline case a limiting magnitude of approximately $m_V \simeq 12.1$ could be expected if the bandwidth used could be increased to 1nm.

4.4 Space-Time analysis

Throughout this review we have assumed that the separate frames are analysed individually and that no attempt is made to cross-correlate frames in the time domain. Intuitively, we would expect that a knowledge of the spatio-temporal power spectrum of the seeing could be used in a processing scheme in which the full space-time correlation function or power spectrum is evaluated, to ultimately achieve fainter limiting magnitudes or higher SNR for a given object.

ACKNOWLEDGEMENTS

We thank the Science Research Council in the U.K. for supporting the major part of this study, which was also supported by the CNRS in France under the European Scientific Exchange Scheme.

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DISCUSSION

L. Mertz: What is the effect of sky background for the limiting magnitude at which the presence of a star can be detected?

J.C. Dainty: It is not too bad; see Ref 20.

L. Mertz: There is nothing sacred about power spectrum or autocorrelation averaging. Can you comment on how the SNR might compare using logarithmic averaging of the spectrum, à la homomorphic (cepstrum) filtering?

J.C. Dainty: I don't know how it would compare. It would certainly be interesting to investigate it.

J.E. Walker: How are you defining the signal, in the autocorrelation function, for an unresolved object?

J.C. Dainty: It's a hand-waving argument, but we look near, but not at, the origin. A rigorous analysis is in preparation.

J.E. Walker: As the value of the SNR you use to define the limiting magnitude is somewhat arbitrary, might a probability type analysis be more appropriate?

J.C. Dainty: Perhaps. It would certainly be interesting.