SPHERICAL STELLAR SYSTEMS: STRUCTURE AND EVOLUTION

L. M. Ozernoy
Harvard-Smithsonian Center for Astrophysics
Cambridge, MA 02138
and
V. A. Volodin
Lebedev Physical Institute
Moscow 117924, U.S.S.R.

ABSTRACT. We analyze a class of spherical stellar systems with an isotropic, non-isothermal, one-particle distribution function which is truncated in energy ϵ as $(\epsilon_b - \epsilon)^\nu$ $(\nu > 0)$ near the energy of truncating ϵ_b and is the Maxwell-Boltzmannian at $|\epsilon| \gg |\epsilon_b|$. The stellar systems which correspond to this distribution function appear to be isothermal in their central parts and polytropic in the external parts. Some qualitative results which follow from such a "polytropic-isothermal" distribution function (PIDF) are briefly discussed. A further extention of PIDF enables us to analyze in a new way the evolution of spherical stellar systems.

Why are the King (1966) models, in spite their simplicity (spherical symmetry, isotropy, one-particle distribution function), capable of reproducing many properties of real stellar systems in a surprisingly wide domain of applications – from globulars to elliptical galaxies and regular clusters of galaxies? Apparently the key element is that the distribution function taken even in the simplest conceivable form – Maxwell-Boltzmannian – is truncated at energies greater than some maximum energy. Physically, this means accounting for the principal fact of rapid evaporation of all those particles which acquire at encounters velocities greater than the local escape velocity.

We propose here a more general distribution function which, being still one-particle, spherically symmetric, isotropic, and Maxwell-Boltzmannian at energies $|\epsilon| \gg |\epsilon_b|$, vanishes as $(\epsilon_b - \epsilon)^\nu$ as $\epsilon \to \epsilon_b$, $\nu > 0$ being a free parameter.

We have obtained numerical solutions for the potential and density runs. The shape of each solution is determined by two parameters: the power law index ν and the central dimensional potential $W_\circ$.

In the external parts of the stellar system where the local potential is small, it is described approximately by the Lane-Emden equation. The Lane-Emden polytrope of the index $\lambda = \nu + 3/2$ represents the exact solution for the density distribution in the limiting case when $W_\circ \to 0$.

In the central parts of the system, the density distribution tends, at sufficiently large $W_\circ$'s, to the isothermal (Emden) solution, independently of ν. The size of the isothermal region, at a given λ and a not too small $W_\circ$, does not depend on $W_\circ$

T. de Zeeuw (ed.), Structure and Dynamics of Elliptical Galaxies, 513–514.

and for the King models (they are a particular case of our solutions at $\lambda = 5/2$) it comprises about 2% of the overall size of the system. The central, gaussian density peak is typical for the models with great enough $W_\circ$'s (for King models, it appears only when $W_\circ \geq 6$).

The larger (at a fixed λ) is $W_\circ$, the closer is the density distribution in most of the system (except in the narrow $[\sim \exp(-W_\circ/2)]$, central core region) to a peculiar, "singular" solution. The latter has the logarithmic singularity at the center and is the limiting, λ-dependent solution which corresponds to $W_\circ \to \infty$. In particular, the King models are transformed at $W_\circ \to \infty$ into the singular solution of the index $\lambda = 5/2$, and not into an isothermal sphere.

All the solutions discussed here, including the limiting ones, are spatially limited and finite. The boundary radius of the system $r_b(\lambda, W_\circ)$ is found to be changing, for any λ and $W_\circ$ satisfying the inequalities $0 < \lambda \leq 3.5$ and $0 \leq W_\circ < \infty$, within some finite limits: $0 < r_{min}(\lambda) < r_b(\lambda, \infty) < r_{max}(\lambda) \equiv r_b(\lambda, 0)$. Note that the finiteness of $r_b(\lambda, W_\circ)$ is valid even for "free" (not tidally truncated) models, *i.e.* even in those cases when the interaction with an external tidal field is negligibly small. (Such systems which are bound due to their intrinsic properties evolve without an appreciable influence of the external tidal field as long as it keeps sufficiently small). As for a tidally truncated system, its boundary radius is merely fixed by the value of the tidal potential and does not depend on $W_\circ$. Thus, at any external conditions of the stellar system (whether in a tidal field or in a free state) its boundary radius remains finite and it is changing gradually during the secular evolution of the system, together with the change of its total energy and number of stars. Such a picture is valid as long as the system does not deviate from the state characterized by the "polytropic-isothermal" distribution function (PIDF) introduced above.

A natural extension of this distribution function, by introducing, as a first step, a scale factor, leads to a two-parameter family whose distribution functions are capable of describing quantitatively the spherical stellar systems at various evolutionary stages, including the core collapse (gravo-thermal catastrophe). This family enables us, in particular, to reproduce the evolutionary sequence for spherical stellar systems which was obtained by Cohn (1980) with the help of the numerical solution of the Fokker-Planck equation. The scale factor introduced into PIDF provides an excellent fit both for dependence of the distribution function on the binding energy and for the star density as a function of the dimensionless potential. However, this results in the fit for the potential run which appears to be insufficiently precise. This defect can be removed if the second step in the generalization of PIDF's is made by taking their linear combination. Already two terms are sufficient to reproduce fairly well all the dependencies mentioned above.

The extensions of PIDF proposed here reveal a new approach to consider both the early and the late stages in the evolution of spherical stellar systems. The King models appear to describe a particular, though a rather long, intermediate evolutionary stage.

A full account of the work will be published elsewhere.

REFERENCES

Cohn, H., 1980. *Astrophys. J.*, **242**, 765.
King, I. R. 1966. *Astron. J.*, **71**, 64.