35

Baryonic correlators

35.1 Light baryons

35.1.1 The decuplet

The simplest interpolating field for the $\Delta(\frac{3}{2})$ is [424–430]:

$$\Delta_\mu = \frac{1}{\sqrt{2}} : (\psi^T C \psi) \left(g_{\mu\lambda} - \frac{1}{4}\gamma_\mu \gamma_\lambda \right) \psi : \ , \tag{35.1}$$

where C is the charge conjugation matrix and colour indices. The corresponding correlator is:

$$\begin{aligned} S_{\mu\nu} &= i \int d^4q\, e^{iqx} \langle 0|\mathcal{T}\Delta_\mu(x)\Delta_\mu(0)^\dagger|0\rangle \\ &= (\hat{q} F_1 + F_2)\, g_{\mu\nu} + \cdots \end{aligned} \tag{35.2}$$

Using the SVZ-expansion, the form factor can be expressed as:

$$\begin{aligned} -F_1 &= q^4 A_1 \log -\frac{q^2}{\nu^2} + A_2\pi \log -\frac{q^2}{\nu^2}\langle \alpha_s G^2\rangle + m_s A_3 \pi^2 \langle\bar\psi\psi\rangle + A_4 \frac{\pi^4}{q^2}\langle\bar\psi\psi\rangle^2 \\ -F_2 &= m_s B_1 q^4 \log -\frac{q^2}{\nu^2} + B_2\pi^2 \log -\frac{q^2}{\nu^2}\langle\bar\psi\psi\rangle q^2 + B_3\pi^2 \left\langle \bar\psi\sigma^{\mu\nu}\frac{\lambda_a}{2}G^a_{\mu\nu}\psi \right\rangle , \end{aligned} \tag{35.3}$$

where A_i and B_i are Wilson coefficients determined from perturbative calculation of the QCD diagrams shown in Fig. 35.1. The different expressions of these Wilson coefficients compiled in [426] are given in Tables 35.1 and 35.2 to lowest order of α_s. In the table, we also introduce the parameters controlling the $SU(3)$ breaking of the condensates:

$$\chi_3 = \frac{\langle\bar s s\rangle}{\langle\bar u u\rangle} , \quad \text{and} \quad \chi_5 = \frac{\langle \bar s \sigma^{\mu\nu}\lambda_a G^a_{\mu\nu} s\rangle}{\langle \bar u \sigma^{\mu\nu}\lambda_a G^a_{\mu\nu} u\rangle} . \tag{35.4}$$

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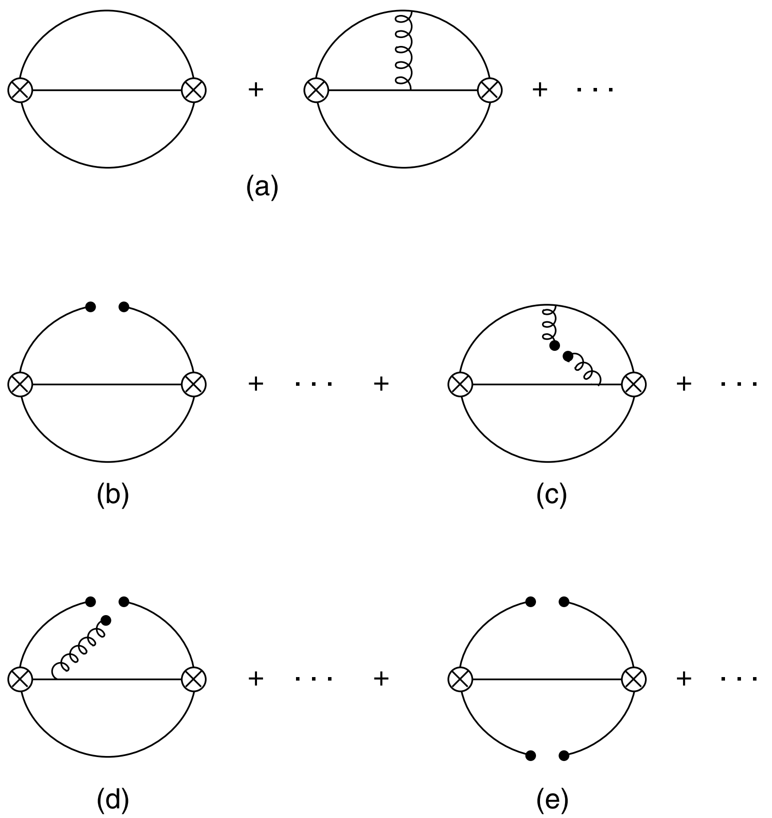

Fig. 35.1. Feynman diagrams corresponding to the OPE of the baryon correlator: (a) perturbative; (b) quark condensate; (c) gluon condensate; (d) mixed condensate; (e) four-quark condensate.

35.1.2 The octet

The nucleon can be in general interpolated by the lowest dimension operators:

$$N = \frac{1}{\sqrt{2}} : [\,(\psi C \gamma_5 \psi)\,\psi + b\,(\psi C \psi)\,\gamma_5 \psi\,] : \tag{35.5}$$

where b is an arbitrary parameter. We shall discuss the different choices of b in the sum rule analysis. The corresponding correlator is:

$$\begin{aligned} S_{\mu\nu} &= i \int d^4 q\, e^{iqx} \langle 0|\mathcal{T} N(x) N(0)^\dagger|0\rangle \\ &= \hat{q} F_1 + F_2 + \cdots , \end{aligned} \tag{35.6}$$

Using the SVZ-expansion, the form factor can be expressed as in Eq. (35.3). The corresponding Wilson coefficients are given Tables 35.1 and 35.2.

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Table 35.1. *Wilson coefficients in the OPE of the form factor F_1.*

Type	A_1	A_2	A_3	A_4
3/2				
Δ	$\frac{1}{20}$	$-\frac{5}{36}$	0	$\frac{32}{3}$
Σ^*	$\frac{1}{20}$	$-\frac{5}{36}$	$-\frac{2}{3}(4-\chi_3)$	$\frac{32}{9}(1+2\chi_3)$
Ξ^*	$\frac{1}{20}$	$-\frac{5}{36}$	$-\frac{4}{3}(2+\chi_3)$	$\frac{32}{9}\chi_3(2+\chi_3)$
Ω	$\frac{1}{20}$	$-\frac{5}{36}$	$-6\chi_3$	$\frac{32}{3}\chi_3^2$
1/2				
N	$\frac{1}{256}(5+2b+5b^2)$	$\frac{1}{256}(5+2b+5b^2)$	0	$\frac{2}{6}(7-2b-5b^2)$
Λ	$\frac{1}{256}(5+2b+5b^2)$	$\frac{1}{256}(5+2b+5b^2)$	$-\frac{1}{48}[4(5-4b-b^2)$ $-3(5+2b+5b^2)\chi_3]$	$\frac{1}{9}[(11+2b-13b^2)$ $+2(5-4b-b^2)\chi_3]$
Σ	$\frac{1}{256}(5+2b+5b^2)$	$\frac{1}{256}(5+2b+5b^2)$	$-\frac{1}{16}[12(1-b^2)$ $-(5+2b+5b^2)\chi_3]$	$\frac{1}{3}[(1-b)^2$ $+6(1-b^2)\chi_3]$
Ξ	$\frac{1}{256}(5+2b+5b^2)$	$\frac{1}{256}(5+2b+5b^2)$	$-\frac{3}{8}[2(1-b^2)$ $-(1+b)^2\chi_3]$	$\frac{1}{3}\chi_3[6(1-b^2)$ $+(1-b)^2\chi_3]$

Table 35.2. *Wilson coefficients in the OPE of the form factor F_2.*

Type	B_1	B_2	B_3
3/2			
Δ	0	$-\frac{8}{3}$	$\frac{4}{3}$
Σ^*	$\frac{1}{8}$	$-\frac{8}{9}(2+\chi_3)$	$\frac{4}{9}(2+\chi_5)$
Ξ^*	$\frac{1}{4}$	$-\frac{8}{9}(1+2\chi_3)$	$-\frac{4}{9}(1+2\chi_5)$
Ω	$\frac{3}{8}$	$-\frac{8}{3}\chi_3$	$\frac{4}{3}\chi_5$
1/2			
N	0	$-\frac{1}{8}(7-2b-5b^2)$	$\frac{3}{8}(1-b^2)$
Λ	$\frac{1}{192}(11+2b-13b^2)$	$-\frac{1}{24}[2(5-4b-b^2)$ $+(11+2b-13b^2)\chi_3]$	$\frac{1}{32}(1-b^2)(10+4\chi_5)$
Σ	$\frac{1}{64}(1-b)^2$	$-\frac{1}{8}[6(1-b^2)$ $+(1-b)^2\chi_3]$	$\frac{3}{9}(1-b^2)$
Ξ	$\frac{3}{32}(1-b^2)$	$-\frac{1}{8}[(1-b)^2$ $+6(1-b^2)\chi_3]$	$\frac{3}{8}(1-b^2)\chi_5$

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35.1.3 Radiative corrections

As the current gets renormalized, the previous correlators have anomalous dimensions. These anomalous dimensions are equal for the Δ and N and read:

$$\gamma = -2 \times \left(\frac{2}{3}\right) . \tag{35.7}$$

Radiative corrections to these lowest-order terms have been first obtained in the chiral limit in [424] and corrected in [428,425]. For the nucleon, one has [428,425]:

$$A_1 = \frac{1}{256}(5 + 2b + 5b^2)\left[1 + \frac{71}{12}\left(\frac{\alpha_s}{\pi}\right) - \frac{1}{2}\log -\frac{q^2}{\nu^2}\right]$$
$$B_2 = -\frac{1}{8}\left[7\left[1 + \left(\frac{\alpha_s}{\pi}\right)\frac{15}{14}\right] - 2b\left[1 + \left(\frac{\alpha_s}{\pi}\right)\frac{3}{2}\right] - 5b^2\left[1 + \left(\frac{\alpha_s}{\pi}\right)\frac{9}{10}\right]\right] . \tag{35.8}$$

35.2 Heavy baryons

Analogous correlators but for baryons containing heavy quarks have been evaluated in [453,454,731].

35.2.1 Spin 1/2 baryons

Let us consider the baryonic current:

$$J = r_1\,(u^t \mathcal{C}\gamma^5 c)b + r_2\,(u^t \mathcal{C} c)\gamma^5 b + r_3\,(u^t \mathcal{C}\gamma^5\gamma^\mu c)\gamma_\mu b\ , \tag{35.9}$$

which has the quantum numbers of the $\Lambda(bcu)$; r_1, r_2 and r_3 are arbitrary mixing parameters where, in terms of the b parameter used in [454]:

$$r_1 = (5 + b)/\,2\sqrt{6};\quad r_2 = (1 + 5b)/\,2\sqrt{6};\quad r_3 = (1 - b)/\,2\sqrt{6}\,. \tag{35.10}$$

The choice of operators in [453] is recovered in the particular case where:

$$r_1 = 1;\quad r_2 = k;\quad r_3 = 0\,. \tag{35.11}$$

The associated two-point correlator is:

$$i\int d^4x\; e^{ip\cdot x}\langle 0|\mathbf{T}J(x)\bar{J}(0)|0\rangle = \not{p}F_1 + F_2\,. \tag{35.12}$$

The QCD expressions of the form factors F_1 and F_2 can be parametrized as:

$$F_i = F_i^{\text{Pert}} + F_i^{G} + F_i^{\text{Mix}}\ , \tag{35.13}$$

where:

$$\text{Im}\ F_2^{\text{Pert}}(t) = \frac{1}{128\pi^3 t}\left\{\left(2r_3^2 + r_2^2 - r_1^2\right) m_b\ \left\{6\left[m_b^2 t^2 + \left(m_b^4 - 2m_b^2 m_c^2 - m_c^4\right)t\right.\right.\right.$$
$$\left. + 2m_b^2 m_c^4\right]\mathcal{L}_1 - 6t\left[m_b^2\,t + \left(m_b^2 - m_c^2\right)^2\right]\mathcal{L}_2$$

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$$- \left[t^2 + 5\left(2m_b^2 - m_c^2\right)t + m_b^4 - 5m_b^2 m_c^2 - 2m_c^4\right] \lambda_{bc}^{1/2}\Big\}$$
$$- 2r_1 r_3\, m_c \Big\{6\left[m_c^2 t^2 + \left(m_c^4 - 2m_c^2 m_b^2 - m_b^4\right)t + 2m_c^2 m_b^4\right]\mathcal{L}_1$$
$$+ 6t\left[m_c^2\, t + \left(m_c^2 - m_b^2\right)^2\right]\mathcal{L}_2$$
$$- \left[t^2 + 5\left(2m_c^2 - m_b^2\right)t + m_c^4 - 5m_c^2 m_b^2 - 2m_b^4\right] \lambda_{cb}\Big\}\Big\} \qquad (35.14)$$

$$\text{Im}\ F_2^{\psi}(t) = \frac{\langle\bar\psi\psi\rangle}{8\pi t}\lambda_{bc}^{1/2}\left\{-\left(r_1^2 + r_2^2 + 4r_3^2\right)m_b m_c + r_1 r_3\, \left(m_b^2 + m_c^2 - t\right)\right\} \qquad (35.15)$$

$$\text{Im}\ F_2^{G}(t) = \frac{\langle\alpha_s G^2\rangle}{384\pi^2 t}\left\{\left[2\frac{r_3^2}{m_b}\left(-2t + 7m_b^2 + 2m_c^2\right)\right.\right.$$
$$+ \frac{r_1^2 - r_2^2}{m_b}\left(2t + 5m_b^2 - 2m_c^2\right) + 2\frac{r_1 r_3}{m_c}\left(2t - 2m_b^2 - m_c^2\right)$$
$$\left. + 12 r_2 r_3\, m_c\right]\lambda_{bc}^{1/2}$$
$$+ 6\left[\left(r_2^2 - r_1^2\right) m_b\, t + 2r_3^2 m_b m_c^2 - r_1 r_3 m_c t - r_2 r_3 m_c \left(t - 2m_b^2\right)\right]\mathcal{L}_1$$
$$\left. - 6t\left[\left(r_2^2 - r_1^2\right)m_b + (r_1 + r_2) r_3 m_c\right]\mathcal{L}_2\right\} \qquad (35.16)$$

$$\text{Im}\ F_2^{\text{Mix}}(t) = \frac{M_0^2\langle\bar\psi\psi\rangle}{64\pi t\, \lambda_{bc}^{3/2}}\Big\{2\left(r_1^2 + r_2^2\right) m_b m_c\left[-t^3 + t^2\left(m_b^2 + 3m_c^2\right)\right.$$
$$\left. + t\left(m_b^2 + m_c^2\right)\left(m_b^2 - 3m_c^2\right) - \left(m_b^2 - m_c^2\right)^3\right]$$
$$+ 4r_3^2 m_b m_c\left[-t^3 + t^2\left(3m_b^2 + m_c^2\right)\right.$$
$$\left. + \ t\left(-3m_b^4 - 6m_b^2 m_c^2 + m_c^4\right) + \left(m_b^2 - m_c^2\right)^3\right]$$
$$+ 2r_1 r_3\left[t^4 + t^3\left(-3m_b^2 - 2m_c^2\right) + 3t^2 m_b^2\left(m_b^2 - m_c^2\right)\right.$$
$$\left. + t\left(-m_b^6 + 4m_b^4 m_c^2 + 3m_b^2 m_c^4 + 2m_c^6\right) + m_c^2\left(m_b^2 - m_c^2\right)^3\right]$$
$$+ 2r_2 r_3\left[t^4 + t^3\left(-4m_b^2 - 3m_c^2\right) + 3t^2\left(2m_b^4 + m_b^2 m_c^2 + m_c^4\right)\right.$$
$$\left. - t\left(m_b^2 - m_c^2\right)\left(4m_b^4 + m_b^2 m_c^2 - m_c^4\right) + m_b^2\left(m_b^2 - m_c^2\right)^3\right]\Big\} \qquad (35.17)$$

$$\text{Im}\ F_1^{\text{Pert}}(t) = \frac{1}{512\pi^3 t^2}\Big\{\left(r_1^2 + r_2^2 + 4r_3^2\right)\Big\{12\left[t^2\left(m_b^4 + m_c^4\right) - 2m_b^4 m_c^4\right]\mathcal{L}_1$$
$$- 12t^2\left(m_b^4 - m_c^4\right)\mathcal{L}_2$$
$$+ \left[t^3 - 7t^2\left(m_b^2 + m_c^2\right) + t\left(-7m_b^4 + 12m_b^2 m_c^2 - 7m_c^4\right)\right.$$
$$\left. + m_b^6 - 7m_b^4 m_c^2 - 7m_b^2 m_c^4 + m_c^6\right]\lambda_{bc}^{1/2}\Big\}$$
$$- 4r_1 r_3\, m_b m_c\Big\{12\left[t^2\left(m_b^2 + m_c^2\right) - 4t m_b^2 m_c^2 + 2m_b^2 m_c^2\left(m_b^2 + m_c^2\right)\right]\mathcal{L}_1$$
$$- 12t^2\left(m_b^2 - m_c^2\right)\mathcal{L}_2$$
$$- \ 2\left[2t^2 + 5t\left(m_b^2 + m_c^2\right) - m_b^4 - 10m_b^2 m_c^2 - m_c^4\right]\lambda_{cb}\Big\}\Big\} \qquad (35.18)$$

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$$\text{Im } F_1^{\psi}(t) = \frac{\langle\bar{\psi}\psi\rangle}{16\pi t^2}\lambda_{bc}^{1/2}\left\{\left(2r_3^2+r_2^2-r_1^2\right)m_c\left(t+m_b^2-m_c^2\right)\right.$$
$$\left.+2r_1r_3\,m_b\left(m_b^2-m_c^2-t\right)\right\} \tag{35.19}$$

$$\text{Im } F_1^{G}(t) = \frac{\langle\alpha_s G^2\rangle}{768\pi^2t^2}\left\{\left[-4r_3^2\left(t+3m_b^2\right)-\left(r_2^2+r_1^2\right)\left(t-3m_b^2+3m_c^2\right)\right.\right.$$
$$+4\frac{r_1r_3}{m_bm_c}\left(2t\left(m_b^2+m_c^2\right)-2m_b^4-11m_b^2m_c^2-2m_c^4\right)$$
$$\left.-36r_2r_3\,m_bm_c\right]\lambda_{bc}^{1/2}$$
$$+12m_bm_c\left[-2r_3^2\,m_bm_c+2r_1r_3\left(t-2m_b^2-3m_c^2\right)\right.$$
$$\left.\left.+\;2r_2r_3\left(t-m_b^2-2m_c^2\right)\right]\mathcal{L}_1\right\} \tag{35.20}$$

$$\text{Im } F_1^{\text{Mix}}(t) = \frac{M_0^2\langle\bar{\psi}\psi\rangle}{64\pi t^2\,\lambda_{bc}^{3/2}}\left\{2\left(r_1^2-r_2^2\right)m_c\left[-t^4+t^3\left(2m_b^2+5m_c^2\right)\right.\right.$$
$$-t^2\left(2m_b^4+3m_b^2m_c^2+9m_c^4\right)+t\left(m_b^2-m_c^2\right)\left(2m_b^4-m_b^2m_c^2-7m_c^4\right)$$
$$\left.-\left(m_b^2-m_c^2\right)^3\left(m_b^2-2m_c^2\right)\right]$$
$$+2r_3^2m_c\left[t^3\left(m_b^2-m_c^2\right)+t^2\left(-3m_b^4+4m_b^2m_c^2+3m_c^4\right)\right.$$
$$\left.+3t\left(m_b^2-m_c^2\right)\left(m_b^2+m_c^2\right)^2-\left(m_b^2-m_c^2\right)^3\left(m_b^2+m_c^2\right)\right]$$
$$+2r_1r_3\,m_b\left[-t^4+t^3\left(5m_b^2+m_c^2\right)+t^2\left(-9m_b^4-4m_b^2m_c^2+m_c^4\right)\right.$$
$$\left.+t\left(m_b^2-m_c^2\right)\left(7m_b^4+4m_b^2m_c^2+m_c^4\right)-2m_b^2\left(m_b^2-m_c^2\right)^3\right]$$
$$+2r_2r_3m_b\left[-t^4+2t^3\left(2m_b^2+m_c^2\right)-2t^2\left(3m_b^4+m_c^4\right)\right.$$
$$\left.\left.+2t\left(2m_b^6-3m_b^4m_c^2+m_c^6\right)-\left(m_b^2-m_c^2\right)^4\right]\right\}, \tag{35.21}$$

with:

$$\mathcal{L}_1(t) = \frac{1}{2}\log\frac{1+v}{1-v}; \qquad v = \sqrt{1-\frac{4m_b^2m_c^2}{\left(t-m_b^2-m_c^2\right)^2}}$$
$$\lambda_{bc}^{1/2} = \left(t-m_b^2-m_c^2\right)v; \quad \mathcal{L}_2 = \log\frac{\left(m_b^2+m_c^2\right)t+\left(m_b^2-m_c^2\right)\left(\lambda_{bc}^{1/2}-m_b^2+m_c^2\right)}{2m_bm_c\,t}.$$
$$= \lambda^{1/2}\left(m_b^2,m_c^2,t\right) \tag{35.22}$$

35.2.2 Spin 3/2 baryon

Let us consider the two-point correlator:

$$S_{\mu\nu}(q^2) = i\int d^4x\;\langle 0|\mathcal{T}J_\mu(x)J_\nu(0)|0\rangle \equiv g_{\mu\nu}(\hat{q}F_1+F_2)+\cdots \tag{35.23}$$

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built from the *simplest* interpolating Ξ^*_Q operator:

$$J_\mu^{\Xi^*_Q} = \frac{1}{\sqrt{3}}\epsilon^{\alpha\beta\lambda} : 2\big(Q_\alpha^T C\gamma^\mu u_\beta\big)Q_\lambda + \big(Q_\alpha^T C\gamma_\mu Q_\beta\big)u_\lambda : \ , \tag{35.24}$$

where Q and u are respectively the heavy- and light-quark fields. The QCD expressions of the form factors for a heavy quark of mass M are [453]:

$$\begin{aligned}
\mathrm{Im}F_1^{\mathrm{pert}}(x) &= \frac{M^4}{480\pi^3}\Big[60(-1+4x-4x^2-2x^3)\mathcal{L}_v \\
&\quad + \frac{v}{x^2}(3-19x+98x^2-130x^3-60x^4)\Big] , \\
\mathrm{Im}F_2^{\mathrm{pert}}(x) &= \frac{M^5}{288\pi^3x^2}[24(-2+3x-5x^3)\mathcal{L}_v + v(9+34x-10x^2-60x^3)] , \\
\mathrm{Im}F_1^{G}(x) &= -\frac{\langle\alpha_s G^2\rangle}{12\pi^2}\Big[x^2(2+x)\mathcal{L}_v + \frac{v}{24}(1+26x+12x^2)\Big] , \\
\mathrm{Im}F_2^{G}(x) &= -\frac{\langle\alpha_s G^2\rangle}{36\pi^2}M\left[(2+3x^2)\mathcal{L}_v + \frac{v}{12}\left(11+18x-\frac{8}{x}\right)\right] , \\
\mathrm{Im}F_1^{\psi}(x) &= -M\langle\bar\psi\psi\rangle\frac{v}{3\pi} , \quad \mathrm{Im}F_2^{\psi}(x) = -M^2\langle\bar\psi\psi\rangle\frac{v}{18\pi}\left(7+\frac{2}{x}\right) , \\
\mathrm{Im}F_1^{\mathrm{mix}}(x) &= M_0^2\frac{\langle\bar\psi\psi\rangle}{9\pi}\frac{x^2}{Mv^3}(2-11x) , \\
\mathrm{Im}F_2^{\mathrm{mix}}(x) &= M_0^2\frac{\langle\bar\psi\psi\rangle}{36\pi}\frac{1}{v^3}(2-11x+12x^2-30x^3) ,
\end{aligned} \tag{35.25}$$

with:

$$x = \frac{M^2}{t} , \quad v \equiv \sqrt{1-4x} , \quad \mathcal{L}_v = \ln\left(\frac{1+v}{1-v}\right) . \tag{35.26}$$

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