Canad. Math. Bull. Vol. **56** (4), 2013 pp. 827–828
http://dx.doi.org/10.4153/CMB-2013-008-6

Erratum to 'Quantum Limits of Eisenstein Series and Scattering States'

Yiannis N. Petridis, Nicole Raulf, and Morten S. Risager

Abstract. This paper provides an erratum to Y. N. Petridis, N. Raulf, and M. S. Risager, "Quantum Limits of Eisenstein Series and Scattering States." Canad. Math. Bull. **56**(2013), 814–826 (this issue). http://dx.doi.org/10.4153/CMB-2011-200-2

In [1] there is a typo in Section 2.4 (iii) where the residue of $B(s)$ at $s = 2 - 2\sigma_t$ is computed. Instead of $\xi(1 - 4\sigma_t)$ it should read $\xi(3 - 4\sigma_t)$. As a consequence Theorem 1.3 is not correctly stated, while Theorems 1.1 and 1.4, and Corollary 1.5 need no modification. To correct Theorem 1.3 we need an extra condition on the rate with which we approach the critical line. Here is the corrected statement.

Theorem 1.3 *Assume that $\sigma_\infty = 1/2$ and $(\sigma_t - 1/2)\log t \to 0$. Let A, B be compact Jordan measurable subsets of X. Then*

$$\frac{\mu_{s(t)}(A)}{\mu_{s(t)}(B)} \to \frac{\mu(A)}{\mu(B)}$$

as $t \to \infty$. In fact we have

$$\mu_{s(t)}(A) \sim \mu(A)\frac{6}{\pi}\log t.$$

Consequently, the range of validity in the complex plane of the Quantum Unique Ergodicity Theorem needs to be modified from

$$\Im(\lambda) = o\big(\sqrt{\Re(\lambda)}\big) \quad \text{to} \quad \Im(\lambda) = o\big(\sqrt{\Re(\lambda)}/\log\Re(\lambda)\big)$$

in the region $\Re(\lambda) \geq 0$. This is only slightly smaller.

The correct statement of Lemma 2.2 in the case $\sigma_\infty = 1/2$ is as follows.

Lemma 2.2 *Let F_h be an incomplete Eisenstein series. If $(\sigma_t - 1/2)\log t \to 0$, then*

$$\int_{\Gamma\backslash\mathbb{H}^2} F_h(z)\left|E(z,\sigma_t+it)\right|^2 d\mu(z) \sim \int_{\Gamma\backslash\mathbb{H}^2} F_h(z)d\mu(z)6\pi^{-1}\log t, \quad t\to\infty.$$

Received by the editors January 23, 2013.
Published electronically April 23, 2013.
AMS subject classification: **11F72**, 8G25,35P25.
Keywords: quantum limits, Eisenstein series, scattering poles.

The combined contributions of (iii) and (iv) from Section 2.4 in [1] can be dealt with using the estimate

$$\frac{1-|\phi(s(t))|^2}{4\xi(2)(\sigma_t-1/2)} \sim \frac{6}{\pi}\log t. \tag{1.1}$$

This estimate is seen as follows. By using the mean value theorem twice on the function $\sigma \mapsto \phi(\sigma+it)\phi(\sigma-it)$ we find that

$$\frac{1-|\phi(\sigma+it)|^2}{1/2-\sigma} = \Big(1-(1/2-\sigma')\,|\phi(\sigma''+it)|^2\,\frac{\phi'}{\phi}(\sigma''\pm it)\Big)\,\frac{\phi'}{\phi}(\sigma'\pm it)$$

for some $1/2 \le \sigma'' \le \sigma' \le \sigma$, where $\frac{\phi'}{\phi}(\sigma\pm it)$ means $\frac{\phi'}{\phi}(\sigma+it)+\frac{\phi'}{\phi}(\sigma-it)$. The claim now follows from $(\sigma_t-1/2)\log t \to 0$ and the well-known fact that

$$\frac{\phi'}{\phi}(\sigma\pm it) \sim -4\log t \tag{1.2}$$

as $t\to\infty$ for $\sigma \ge 1/2$ and that $|\phi(\sigma+it)|$ is bounded for $\Re(s)\ge 1/2$ and $t>1$. The estimate (1.2) follows from the bounds on the zeta function [2, Theorem 5.17], combined with the Stirling asymptotics on the Gamma function.

While we can also deal with other ranges for the rate of approach of the critical line, *e.g.*, in the case that $(\sigma_t-1/2)\log|t|\to\infty$, the conclusion of [1, Theorem 1.3] needs no modification, the "intermediate" region seems difficult to deal with, since it is hard to get clean asymptotics for the left-hand side of (1.1).

References

[1] Y. Petridis, N. Raulf, and M. S. Risager, *Quantum limits of Eisenstein series and scattering states.* Canad. Math. Bull. **56**(2013), 814–826. http://dx.doi.org/10.4153/CMB-2011-200-2

[2] E. Titchmarsh, *The theory of the Riemann zeta-function*. Second ed., The Clarendon Press, Oxford University Press, New York, 1986.

Department of Mathematics, University College London, Gower Street, London WC1E 6BT, United Kingdom
e-mail: i.petridis@ucl.ac.uk

Laboratoire Paul Painlevé, U.F.R. de Mathématiques, Université Lille 1 Sciences et Technologies, 59 655 Villeneuve d'Ascq Cédex, France
e-mail: raulf@math.univ-lille1.fr

Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen Ø, Denmark
e-mail: risager@math.ku.dk