THE MOTION OF A GEOSYNCHRONOUS SATELLITE

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ABSTRACT : The motion of a geosynchronous satellite has been studied under the combined gravitational effects of the oblate Earth (including its equatorial ellipticity), the Sun, the Moon and the solar-radiation pressure. It is observed that the orbital plane rotates with an angular velocity the maximum value of which is 0.058°/yr. and regresses with a period which increases both as the orbital inclination and the altitude increase. The effect of earth's equatorial ellipticity on the regression period is oscillatory whereas that of Solar-radiation pressure is to decrease it.

The synchronism is achieved when the angular velocity of the satellite is equal to the difference between the spin-rate of the Earth and the regression rate of the orbital plane. With this angular velocity of the satellite, the ground trace is in the shape of figure eight, though its size and position relative to the Earth change as the time elapses. The major effect of earth's equatorial ellipticity is to produce a change in the relative angular position of the satellite as seen from the Earth. If the satellite is allowed to execute large angle oscillations the mid-point of oscillation would be at the position of the minor axis of the earth's equatorial section. The oscillatory period T has been determined in terms of the amplitude Γ and the tesseral harmonic $J_2^{(2)}$. From this result we can determine the value of $J_2^{(2)}$ as T and Γ can be observed accurately.

The paper presents the results of an investigation done on the motion of a geosynchronous satellite under the gravitational forces of the earth (including the elliplicity of the earth equator) the sun, the moon and the solar radiation pressure. It has been assumed that the centre of the earth-moon system moves around the sun in the plane of the ecliptic, the moon moves in a plane at an angle of 5°8' to the plane of the ecliptic and the earth is spinning about its axis.

The study presented here can be applied to satellites in near circular orbits at any inclination and with orbital radii less than ten times the earth's radius.

From the general equations of motion we have deduced the equations of motion in the following two cases.

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 293–295.

Case 1 When the orbit is in the equatorial plane
Case 2 When the satellite is geocentric synchronous

We have studied the case 2 in detail. In this case the equation of motion take the form

$$\alpha = \dot{\alpha}_\circ + \sum_{i=1}^{182} A_i \sin w_i t \tag{1}$$

$$\dot{\psi} = \dot{\psi}_0 + \frac{1}{\sin \alpha_0} \sum_{i=1}^{182} B_i \cos w_i t \tag{2}$$

$$\ddot{r} - r\theta^2 + \frac{GMF}{r^2} = \xi + \sum_{i=1}^{179} C_i \cos w_i t \tag{3}$$

$$\frac{d}{dt}(r^2\theta) = \eta + \sum_{i=1}^{179} D_i \sin w_i t \tag{4}$$

where α, ψ determine the orbital plane and r, θ the position of the satellite in the orbital plane A_i, B_i and C_i, D_i are functions of any or all of the quantities α_0, α_I, $\rightarrow\alpha$ m, r_0, J_2, $J_2(^2)$ and the frequencies w_i are linear combination of θ_0, θ_m, ψ_m and ψ_0.

It is shown that an orbit of given radius and orbital orientation can be found which remains invariant related to inertial space. It has a common line of intersection with the earth equatorial plane and the plane of the ecliptic. The plane is taken as the reference plane. For low altitude orbits, the reference plane is very close to the equator and approaches the plane of the ecliptic for extremely high orbits. Due to earth equatorial ellipticity it oscillates between the position corresponding to $\Gamma = 0°$ and $\Gamma = 90°$ where Γ is the ellipticity parameter and due to solar radiation present it moves away from the ecliptic as q decreases where q is the solar radiation parameter.

We would like to emphasise the presence of the term $\dot{\alpha}_0$ on the R.H.S. of equation (1). This term is not available in the current literature and is due to the earth's equatorial ellipticity. Due to this term the orbital plane rotates at the rate lying between 0.042° and .058° for year for various inclinations for a synchronous satellite.

The oscillatory terms in the equation of motion effect both the regression rate and the orbital inclination. It is seen that the displacement ΔD (called drift) due to these terms is less than half a degree for values of T which are significantly greater than the oscillatory periods included and are independent of T. ΔD Oscillates, due to equatorial ellipticity between the values corresponding to $\Gamma = 0°$ and $\Gamma = 90°$ with mean position at Γ equal to 45°. The effect of the solar radiation pressure in general is to decrease ΔD.

The regression period T_R increases both as orbital inclination and altitude increase. For fixed α_0 it is maximum in the vicinity of nine times earth radius. The effect of ellipticity is oscillatory and of solar radiation pressure is to decrease it as q decreases. T_R lies between .0978 years and 411.16 years for various altitudes and inclinations.

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The ground trace has been studied in the following two cases :
Case 1 Orbit in reference plane
Case 2 Orbit in equatorial plane

In both the cases, the effect of ellipticity is oscillatory and due to solar radiation pressure it shrinks as q decreases. For orbit in equatorial plane the ground grows from a single point (0,0) at T = 0 and attains the maximum size in half the regression period. The ground trace is just the reverse in the second half of the regression period. First the equatorial crossing point moves towards the east and then after some time, it starts moving towards the west. Due to ellipticity the change in a latitude is ± .2° and in longitude ± .02° and due to solar radiation pressure the change in latitude is ± .3° and in longitude ± .04°.

The change in altitude of a synchronous satellite for various inclination is about 1-1/2 km.

The effect of equatorial ellipticity is to produce a change in the relative angular position of the satellite as seen from the earth. The positions in the direction of the minor axis are stable and in the directions of major axis unstable. For large angle oscillations (when station propulsion is absent), the time period is given by

$$T_0 = \frac{2}{3\, K_2'\theta E} \quad \frac{d\psi_1}{(1 - \sin^2 \Gamma_0 \sin^2 \psi_1)^{1/2}}$$

Here K_2' depends upon the second tesseral harmonic $J_2^{(2)}$ of the earth. The value of the oscillatory period T_0 and amplitude Γ_0 can be measured very accurately for any geo-centric synchronous satellite. These values can be used in the above result to determine the value of K_2 and thereby the parameter $J_2^{(2)}$ which occurs in the potential function of the earth.